GOVERNMENT OF TAMIL NADU

STANDARD EIGHT

MATHEMATICS

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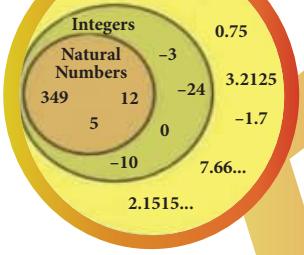
Department of School Education

Untouchability is Inhuman and a Crime

1 NUMBERS



Learning Objectives

- To understand the necessity for extending fractions to rational numbers, to represent rational numbers on the number line and to know that between any two given rational numbers, there lies many rational numbers.
- To learn and perform the four basic arithmetic operations and solve word problems on rational numbers and simplify expressions with atmost three brackets.
- To understand the properties of rational numbers.
- To compute the square, the square root, the cube and the cube root of numbers.
- To make a rough estimate of the square roots and the cube roots of numbers.
- To express numbers in exponential form and understand the laws of exponents with integral powers.
- To identify and express the numbers in scientific notation.

1.1 Introduction

Let us recall the different types of numbers which we have already learnt in our earlier classes. When we want to count, it is natural to start with numbers 1, 2, 3, 4, 5, … Isn't it?

These are all called as **Counting numbers** or **Natural numbers** and their collection is denoted by **ℕ**. The use of three dots at the end of the list is a notation to show that the list keeps going forever.

The natural numbers can be visualized as a ray marked with these numbers:

Fig 1.1

Consider the situation that yesterday my purse had money, say ₹8, but today the purse may be empty. How many rupees are there in the purse now? How to denote this emptiness? Here comes the concept of zero which evolved to symbolize the idea of emptiness. The concept of zero, though quite natural now, was not normal to early humans. Only after hundreds of years people started thinking of it as an actual number. The difficulty was solved when the Indian Mathematicians provided the symbol for zero. The natural numbers system

with this additional number zero became **Whole numbers.** The whole numbers can be visualized now as follows:

Fig 1.2

The system of whole numbers is denoted by $\mathbb{W}$.

Even zero was not sufficient to solve all problems. Think what happens when 4 is taken away from 6?

Draw a number line up to, say 9. Mark 6 by a dot on it.

Fig 1.3

We know that, to subtract 4, we need to go 4 steps to the left side from 6. We will land on 2 and so the answer is 2. But what will happen if we want to subtract 6 from 4? This situation is where the humans needed (and created) negative numbers.

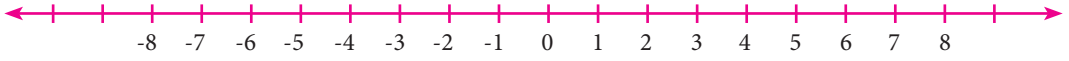
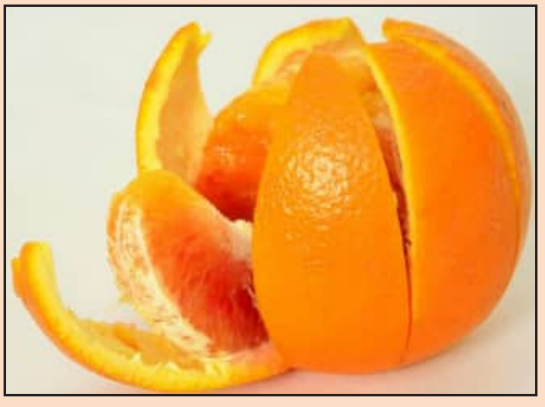


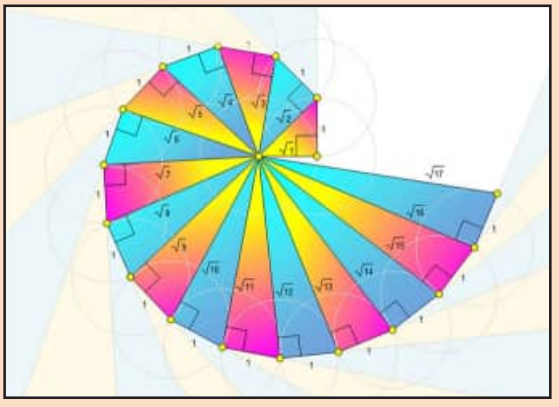
Fig 1.4

But, how can a number be negative? Simple! Just think of them as numbers less than zero. Including the negative integers with the whole numbers, we get the list of numbers called the **Integers.** The integers consist of zero, the natural numbers and the negatives of the natural numbers and it consists a list of numbers that stretch in either direction without end. The entire collection of integers is denoted by $\mathbb{Z}$.

MATHEMATICS ALIVE - NUMBERS IN REAL LIFE



If an orange is peeled off and 8 carpels are found, then one carpel represents the rational number $\frac{1}{8}$.



Finding the square root of numbers in the form spiral using the Pythagoras Theorem from Geometry

1.2 Rational Numbers

Even after coining integers, one could not relax! $10 \div 5$ is no doubt fine, giving the answer 2 but is $8 \div 5$ comfortable? Numbers between numbers are needed. $8 \div 5$ seen as 1.6, is a number between 1 and 2. But, where does $(-3) \div 4$ lie? Between 0 and –1. Similarly, where do you find

$\frac{-12}{5}$ on the number line? Between –2 and –3. Thus, a ratio made by dividing an integer by another integer is called a **rational number**. (Remember, we should not divide by zero!)

Formally speaking, a rational number is a number of the fractional (ratio) form $\frac{a}{b}$, where a and b are integers and $b \neq 0$. The collection of all rational numbers is denoted by $\mathbb{Q}$. Non-negative rational numbers may be thought of as fractions. They can also be expressed as decimals and percents.

Since operations on rational numbers demand a basic knowledge of operations on fractions, let us recall through an exercise some of the basic ideas related to fractions.

Recap Exercise

1. The simplest form of $\frac{125}{200}$ is _____.
2. Which of the following is not an equivalent fraction of $\frac{8}{12}$?
   (A) $\frac{2}{3}$ (B) $\frac{16}{24}$ (C) $\frac{32}{60}$ (D) $\frac{24}{36}$
3. Which is bigger: $\frac{4}{5}$ or $\frac{8}{9}$?
4. Add the fractions: $\frac{3}{5}+\frac{5}{8}+\frac{7}{10}$
5. Simplify: $\frac{1}{8}-\left(\frac{1}{6}-\frac{1}{4}\right)$
6. Multiply: $2\frac{3}{5}$ and $1\frac{4}{7}$.
7. Divide: $\frac{7}{36}$ by $\frac{35}{81}$.
8. Fill in the boxes: $\frac{\square}{66}=\frac{70}{\square}=\frac{28}{44}=\frac{\square}{121}=\frac{7}{\square}$
9. In a city $\frac{7}{20}$ of the population are women and $\frac{1}{4}$ are children. Find the fraction of the population of men.
10. Represent $\left(\frac{1}{2}+\frac{1}{4}\right)$ by a diagram.

Try these

1. Is the number –7 a rational number? Why?
2. Write any 6 rational numbers between 0 and 1.

Note

The word 'ratio' in Math refers to the comparison of the sizes of two different quantities of any kind. For example, if there is one teacher for every 20 students in a class, then the ratio of teachers to students is 1:20. Ratios are often written as fractions and so $1:20=\frac{1}{20}$. For this reason, numbers in the fractions form are called rational numbers.

1.2.1 Rational numbers on a number line

Locating the rational numbers on a number line is an important skill. For example, to represent the number $\frac{-3}{4}$ on the number line, $\frac{-3}{4}$ being negative would be marked to the left of 0 and it is between 0 and –1. We know that the integers, 1 and –1 are equidistant from 0 and so are the numbers 2 and –2, 3 and –3 from 0. This concept remains the same for rational numbers too. Now, as we mark $\frac{3}{4}$ to the right of zero, at 3 parts out of 4 between 0 and 1, the same way, we will mark $\frac{-3}{4}$ to the left of zero, at 3 parts out of 4 between 0 and –1 as shown below.

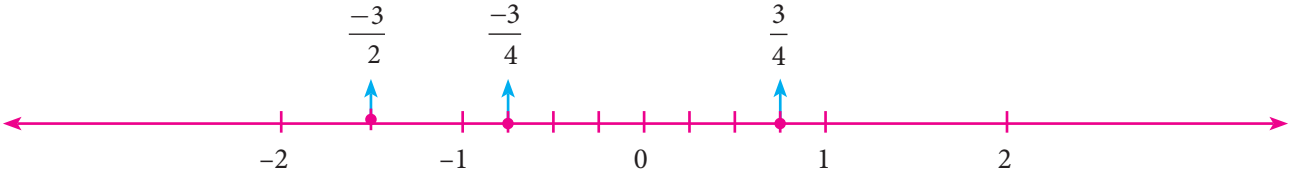


Fig. 1.5

Similarly, it is easy to find $\frac{-3}{2}$ between –1 and –2 since $\frac{-3}{2} = -1\frac{1}{2}$.

Now, on the following number line what rational numbers do the letters A and B represent?

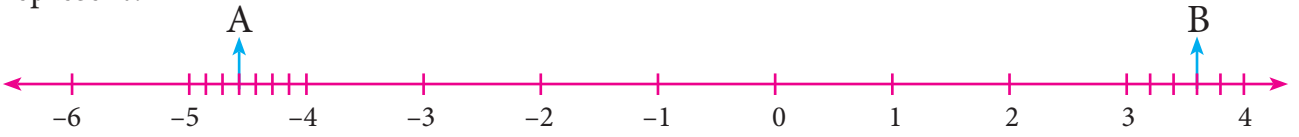


Fig. 1.6

You will now be able to say easily the rational numbers marked by A and B on the number line as shown above. Isn't it? Here, A represents the rational number $-4\frac{4}{7}\left(\frac{-32}{7}\right)$ and B represents the rational number $3\frac{3}{5}\left(\frac{18}{5}\right)$.

1.2.2 Decimal representation of a rational number

A rational number can be nicely represented in decimal form rather than in the usual fractional form. Given a rational number in the form $\frac{a}{b}$ ($b \neq 0$), just divide the numerator a by the denominator b and we can see that it can be expressed as a terminating or non-terminating, recurring decimal.

Activity

Use a string as a number line and fix it on the wall, for the length of the class room. Just mark the integers spaciously and ask the students to pick the rational number cards from a box and fix it roughly at the right place on the number line string. This can be played between teams and the team which fixes more number of cards correctly (by marking) will be the winner.

Example 1.1

Write the decimal forms of the following rational numbers:

i) $\frac{1}{4}$ ii) $1\frac{3}{20}$ iii) $-5\frac{4}{5}$ iv) 3 v) $\frac{1}{3}$

Solution:

(i) $\frac{1}{4}=\frac{25}{100}=0.25$ (ii) $1\frac{3}{20}=\frac{23}{20}=\frac{115}{100}=1.15$ (iii) $-5\frac{4}{5}=\frac{-29}{5}=\frac{-58}{10}=-5.8$

(iv) $3=\frac{3}{1}=\frac{30}{10}=3.0$ (v) $\frac{1}{3}=0.3333...$ (by actual division and it is recurring and non-terminating)

Note

❖ The above examples show how a rational number may be given in decimal form. The reverse process of converting the decimal form of a rational number to the fractional form may be seen in the higher classes.

❖ There are decimal numbers which are non-terminating and non-recurring such as

π = 3.14159265358979323846264 3.....

$\sqrt{2}$ = 1.41421356237309504880168..... etc.

They are not rational numbers and one can study more about them in the higher classes.

Try these

Write the decimal forms of the following rational numbers:

1. $\frac{4}{5}$ 2. $\frac{6}{25}$ 3. $\frac{486}{1000}$ 4. $\frac{1}{9}$ 5. $3\frac{1}{4}$ 6. $-2\frac{3}{5}$

1.2.3 Positive and Negative rational numbers

Rational numbers may be classified as positive and negative rational numbers.

If both the numerator and the denominator of the fraction representing a rational number are of the same sign, then the rational number is positive.

For example, numbers like $\frac{3}{4}, \frac{-11}{-6}$ etc., are positive rational numbers.

If either the numerator or the denominator of the fraction representing a rational number is negative, then the rational number is negative.

For example, numbers like $\frac{-3}{4}, \frac{11}{-6}$ etc., are negative rational numbers.

Note

❖ 0 is a rational number which is neither positive nor negative.

❖ Note that $\frac{-11}{6}=\frac{11}{-6}=-\frac{11}{6}$

1.2.4 Equivalent rational numbers

We know how to write equivalent fractions when a fraction is given. Since a rational number can be represented by a fraction, we can think of equivalent rational numbers, duly obtained through equivalent fractions.

Suppose a rational number is in fractional form. Multiply its numerator and denominator by the same non-zero integer to obtain a rational number which is equivalent to it.

For example,

$-\frac{2}{3}$ is equivalent to $\frac{-6}{9}$ since $\frac{-2}{3}=\frac{-2\times 3}{3\times 3}=\frac{-6}{9}$

$-\frac{2}{3}$ is also equivalent to $\frac{10}{-15}$ since $\frac{2}{-3}=\frac{2\times 5}{-3\times 5}=\frac{10}{-15}$

Thus, $-\frac{2}{3}=\frac{-6}{9}=\frac{10}{-15}$.

Try these

1. $\frac{7}{3}=\frac{?}{9}=\frac{49}{?}=\frac{-21}{?}$
2. $\frac{-2}{5}=\frac{?}{10}=\frac{6}{?}=\frac{-8}{?}$

1.2.5 Rational numbers in Standard form

Observe the following rational numbers: $\frac{4}{5},\frac{-3}{7},\frac{1}{6},\frac{-4}{13},\frac{-50}{51}$. Here, we see that

i) the denominators of these rational numbers are all positive integers

ii) 1 is the only common factor between the numerator and the denominator of each of them and

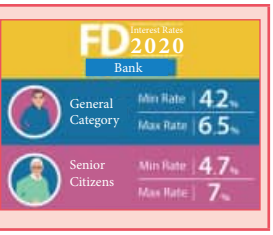
iii) the negative sign occurs only in the numerator.

Such rational numbers are said to be in standard form.

A rational number is said to be in standard form, if its denominator is a positive integer and both the numerator and denominator have no common factor other than 1.

If a rational number is not in the standard form, then it can be simplified to arrive at the standard form.

The collection of rational numbers is denoted by the letter $\mathbb{Q}$ because it is formed by considering all quotients, except those involving division by 0. Decimal numbers can be put in quotient form and hence they are also rational numbers.



Example 1.2

Reduce to the standard form: (i) $\frac{48}{-84}$ (ii) $\frac{-18}{-42}$

Solution:

(i) **Method 1:**

$$\frac{48}{-84}=\frac{48\div(-2)}{-84\div(-2)}=\frac{-24\div 2}{42\div 2}=\frac{-12\div 3}{21\div 3}=\frac{-4}{7}$$ (dividing by –2, 2 and 3 successively)

Method 2:

The HCF of 48 and 84 is 12 (Find it!). Thus, we can get its standard form by dividing it by –12.

$$\frac{48}{-84}=\frac{48\div(-12)}{-84\div(-12)}=\frac{-4}{7}$$

(ii) **Method 1:**

$$\frac{-18}{-42}=\frac{-18\div(-2)}{-42\div(-2)}=\frac{9\div3}{21\div3}=\frac{3}{7}$$ (dividing by –2 and 3 successively)

Method 2:

The HCF of 18 and 42 is 6 (Find it!). Thus, we can get its standard form by dividing it by 6.

$$\frac{-18}{-42}=\frac{-18\times(-1)}{-42\times(-1)}=\frac{18}{42}=\frac{18\div6}{42\div6}=\frac{3}{7}$$

Try these

1. Which of the following pairs represents equivalent rational numbers?

   (i) $\frac{-6}{4}, \frac{18}{-12}$ (ii) $\frac{-4}{-20}, \frac{1}{-5}$ (iii) $\frac{-12}{-17}, \frac{60}{85}$

2. Find the standard form of:

   (i) $\frac{36}{-96}$ (ii) $\frac{-56}{-72}$ (iii) $\frac{27}{18}$

3. Mark the following rational numbers on a number line.

   (i) $\frac{-2}{3}$ (ii) $\frac{-8}{-5}$ (iii) $\frac{5}{-4}$

1.2.6 Comparison of rational numbers

It is useful to remember the following points:

- Every positive number is greater than zero.
- Every negative number is smaller than zero.
- Every positive number is greater than every negative number.
- Every number on the right of a number on a number line is greater than that number.

When two integers or fractions are given, we know how to compare them and say which is greater or smaller. Now, in the same way, we can compare a pair of rational numbers.

Type 1 : Comparing two rational numbers with opposite signs

Example 1.3

Compare $\frac{5}{17}$ and $\frac{-10}{19}$.

Solution:

Since every positive number is greater than every negative number,

we conclude that $\frac{5}{17} > \frac{-10}{19}$.

Type 2 : Comparing two rational numbers represented by two fractions with same denominators

Example 1.4

Compare $\frac{1}{3}$ and $\frac{4}{3}$.

Solution:

Since the denominators are the same, just compare the numerators.

Since $1 < 4$, we conclude that $\frac{1}{3} < \frac{4}{3}$.

Type 3 : Comparing two rational numbers represented by two fractions with different denominators

Example 1.5

Compare $\frac{3}{4}$ and $\frac{5}{6}$.

Solution:

The LCM of the denominators is 12 (Find it!). Consider for each rational number an equivalent rational number with the LCM 12 as denominator.

We get $\frac{3}{4} = \frac{9}{12}$ and $\frac{5}{6} = \frac{10}{12}$, which become like fractions now.

Here, $\frac{9}{12} < \frac{10}{12}$. Hence, we conclude that $\frac{3}{4} < \frac{5}{6}$.

Type 4 : Comparing two rational numbers that are not in standard form

Example 1.6

Compare $\frac{9}{-4}$ and $\frac{-2}{3}$.

Solution:

The number $\frac{9}{-4}$ is not in standard form. First put it in the standard form.

$$\frac{9}{-4} = \frac{9}{-4} \times \frac{-1}{-1} \text{(to get a positive denominator)} = \frac{-9}{4}$$

Now, we shall compare the fractions $\frac{-9}{4}$ and $\frac{-2}{3}$. We find that these two fractions are unlike fractions. To make them as like fractions, we make use of their LCM, which is 12.

We can now compare their equivalent fractions $\frac{-9}{4} = \frac{-27}{12}$ and $\frac{-2}{3} = \frac{-8}{12}$ (How?). We find that the denominators are the same and so just comparing the numerators –27 and –8 are enough.

Visualizing these numbers on the number line, we see that

Fig. 1.7

-8 is to the right of -27 and hence $(-8) > (-27)$. This leads to the result that $\frac{-8}{12} > \frac{-27}{12}$ and consequently we conclude that $\frac{-2}{3} > \frac{9}{-4}$.

Example 1.7

Write the following rational numbers in ascending and descending order.

$$\frac{-3}{5}, \frac{7}{-10}, \frac{-15}{20}, \frac{14}{-30}, \frac{-8}{15}$$

Solution:

First make the denominators to be positive and write the numbers in standard form as $\frac{-3}{5}, \frac{-7}{10}, \frac{-15}{20}, \frac{-14}{30}, \frac{-8}{15}$. Here, the LCM of 5,10,15,20 and 30 is 60 (Find it!). Change the given rational numbers in equivalent form with common denominator 60.

$\frac{-3}{5}$	$\frac{-7}{10}$	$\frac{-15}{20}$	$\frac{-14}{30}$	$\frac{-8}{15}$
$= \frac{-3}{5} \times \frac{12}{12}$	$= \frac{-7}{10} \times \frac{6}{6}$	$= \frac{-15}{20} \times \frac{3}{3}$	$= \frac{-14}{30} \times \frac{2}{2}$	$= \frac{-8}{15} \times \frac{4}{4}$
$= \frac{-36}{60}$	$= \frac{-42}{60}$	$= \frac{-45}{60}$	$= \frac{-28}{60}$	$= \frac{-32}{60}$

Comparing the numerators alone, that is, $-36, -42, -45, -28$ and -32 we see that

$$-45 < -42 < -36 < -32 < -28$$

Hence, $\frac{-45}{60} < \frac{-42}{60} < \frac{-36}{60} < \frac{-32}{60} < \frac{-28}{60}$ and so, $\frac{-15}{20} < \frac{7}{-10} < \frac{-3}{5} < \frac{-8}{15} < \frac{14}{-30}$.

So, the ascending order is $\frac{-15}{20}, \frac{7}{-10}, \frac{-3}{5}, \frac{-8}{15}$ and $\frac{14}{-30}$.

Also, its reverse order gives the descending order as $\frac{14}{-30}, \frac{-8}{15}, \frac{-3}{5}, \frac{7}{-10}$ and $\frac{-15}{20}$.

1.2.7 Rational numbers between any two given rational numbers

Consider the integers 4 and 10. We can locate five integers namely 5,6,7,8 and 9 (shown in dark dots) between them. Isn't it?

Fig. 1.8

How many integers can you find between 3 and -2? List them.

Are there any integers between -5 and -4? No, is the answer.

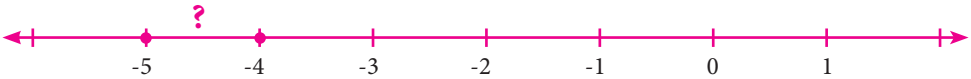


Fig. 1.9

This shows that the choice of integers between two given integers is limited. They are finite in number or may be nothing between them. Let us think what will happen, if we consider rational numbers instead of integers? We will see that we can have many rational numbers between any two rational numbers. There are at least two methods to find more rational numbers between any two rational numbers.

Method of Averages:

We know that that the average of any two numbers always lies at the middle of them. For example, the average of 2 and 8 is $\frac{2+8}{2}=5$ and this 5 lies at the middle of 2 and 8 as shown in the following number line.

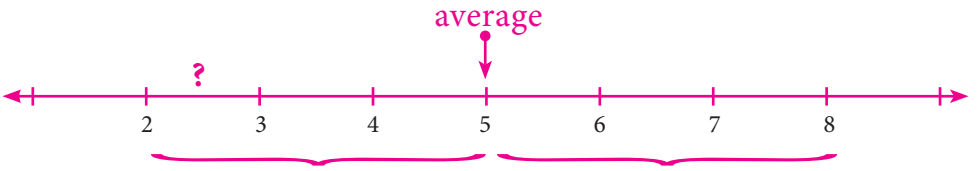


Fig. 1.10

We use this idea to find more rational numbers between any two rational numbers.

Example 1.8

Find a rational number between $\frac{1}{3}$ and $\frac{5}{9}$.

Solution:

The average of $\frac{1}{3}$ and $\frac{5}{9}=\frac{1}{2}\left(\frac{1}{3}+\frac{5}{9}\right)=\frac{1}{2}\left(\frac{1}{3}\times\frac{3}{3}+\frac{5}{9}\right)$ (Why?)

$$=\frac{1}{2}\left(\frac{3}{9}+\frac{5}{9}\right)=\frac{1}{2}\times\frac{8}{9}=\frac{4}{9}$$

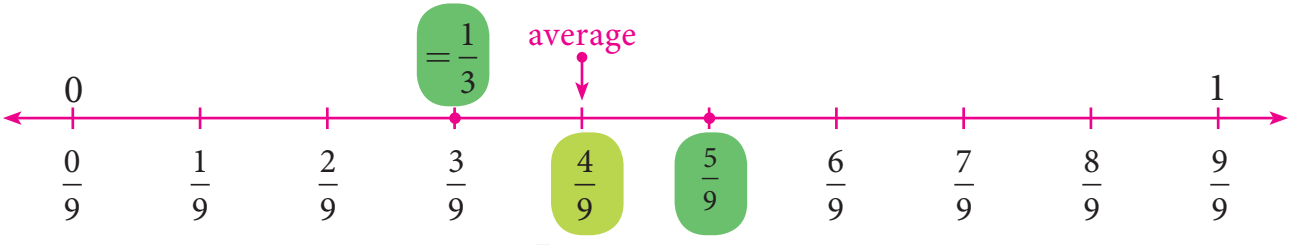


Fig. 1.11

Note that $\frac{4}{9}$ is one rational number we have found in between $\frac{1}{3}$ and $\frac{5}{9}$ and we can find many such numbers in between $\frac{1}{3}$ and $\frac{5}{9}$. This shows that between any two rational numbers there lie an unlimited number of rational numbers! Mathematically, we say that there lie an infinite number of rational numbers between any two rational numbers.

Think

Are there any rational numbers between $\frac{-7}{11}$ and $\frac{6}{-11}$?

Method of Equivalent rational numbers:

We can use the idea of equivalent fractions to get more rational numbers between any two rational numbers. This is clearly explained in the following illustration.

Illustration:

Let us now try to find more rational numbers say between $\frac{3}{7}$ and $\frac{4}{7}$ by the following visual explanation on the number line. If we get the multiples of the denominator of the equivalent rational numbers (the easy one will be to multiply by 10), then we can insert as many rational numbers as we want. We shall write $\frac{3}{7}$ as $\frac{30}{70}$ and $\frac{4}{7}$ as $\frac{40}{70}$ and see that there are 9 rational numbers between $\frac{3}{7}$ and $\frac{4}{7}$ as given in the number line below.

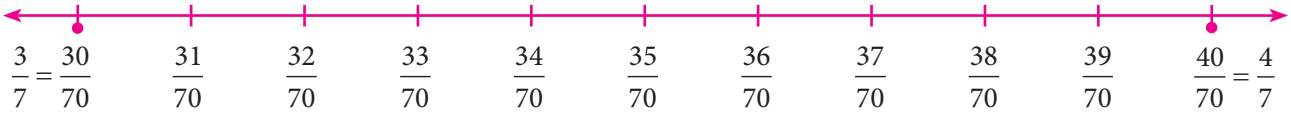


Fig. 1.12

Now, if we want more rational numbers between say $\frac{37}{70}$ and $\frac{38}{70}$ we can write $\frac{37}{70}$ as $\frac{370}{700}$ and $\frac{38}{70}$ as $\frac{380}{700}$. Then again, we will get nine rational numbers between $\frac{37}{70}$ and $\frac{38}{70}$ as $\frac{371}{700}, \frac{372}{700}, \frac{373}{700}, \frac{374}{700}, \frac{375}{700}, \frac{376}{700}, \frac{377}{700}, \frac{378}{700}$ and $\frac{379}{700}$.

The following diagram helps us to understand this nicely with a magnifying lens used between 0 and 1 and further zoomed into the fractional parts also.

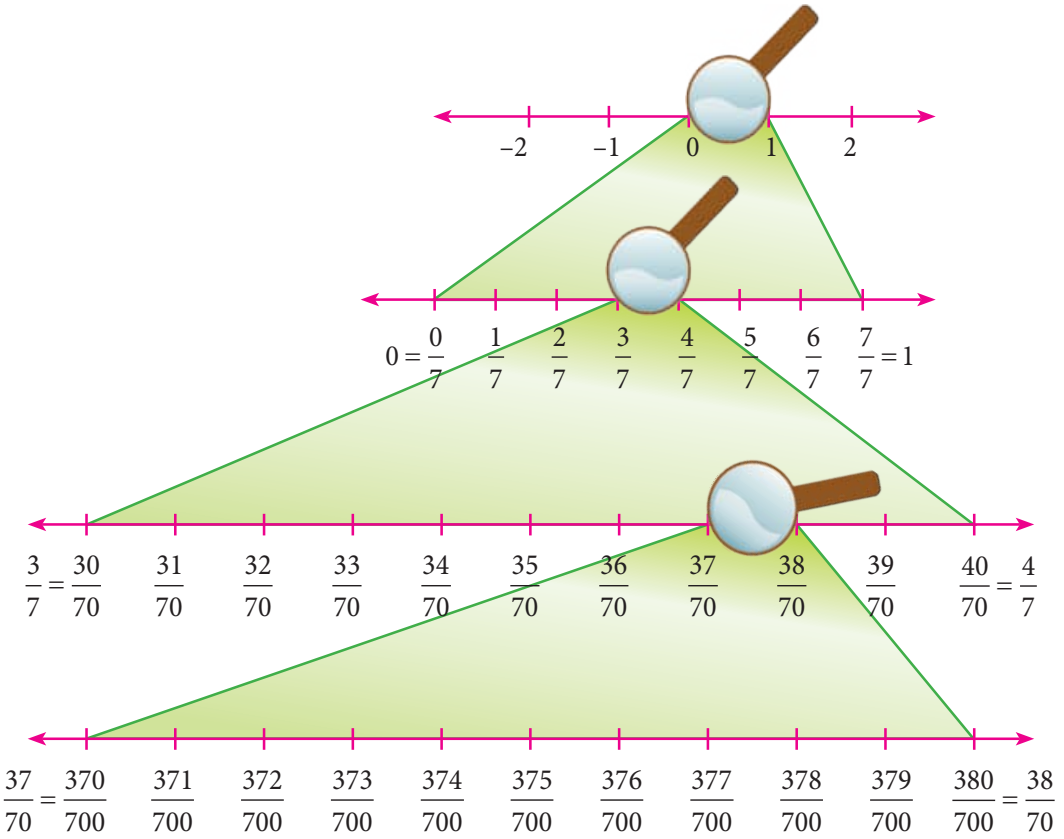


Fig. 1.13

Thus, we can see that there are an unlimited number of rational numbers between any two given rational numbers.

Example 1.9

Find atleast two rational numbers between $\frac{-3}{4}$ and $\frac{-2}{5}$.

Solution:

The denominators are different for the given rational numbers. The LCM of the denominators 4 and 5 is 20. Make the rational numbers such that they have common denominators as 20. Here,

$\frac{-3}{4}=\frac{-3}{4}\times\frac{5}{5}=\frac{-15}{20}$ and $\frac{-2}{5}=\frac{-2}{5}\times\frac{4}{4}=\frac{-8}{20}$.

It is easy now to find and insert rational numbers between $\frac{-15}{20}$ and $\frac{-8}{20}$ as shown below.

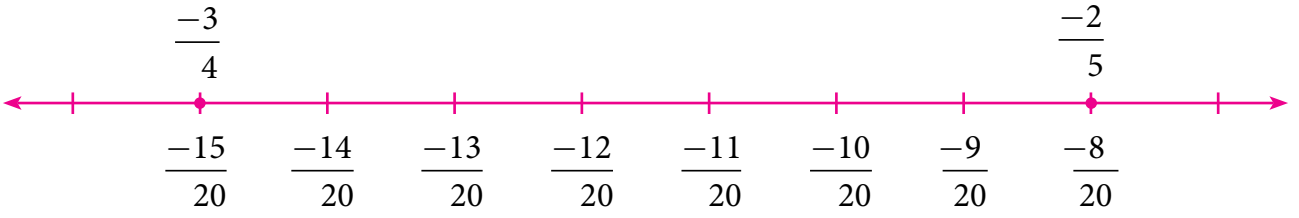


Fig. 1.14

We can list a few rational numbers as $\frac{-14}{20}, \frac{-13}{20}, \frac{-12}{20}, \frac{-11}{20}, \frac{-10}{20}$ and $\frac{-9}{20}$ between $\frac{-15}{20}$ and $\frac{-8}{20}$.

Are these the only rational numbers between $\frac{-15}{20}$ and $\frac{-8}{20}$? Think! Try to find 10 more rational numbers between them, if possible!

Note

We can find many rational numbers between $\frac{-7}{11}$ and $\frac{5}{-9}$ quickly as given below: The range of rational numbers can be got by the cross multiplication of denominators with the numerators after writing the given fractions in standard form. The cross multiplication here $\frac{-7}{11}\ \frac{-5}{9}$ gives the range of rational numbers from –63 to –55 with the denominator 99. This is nothing but making the given rational numbers equivalent with the denominator 99!

Exercise 1.1

1. **Fill in the blanks:**

   (i) $\frac{-19}{5}$ lies between the integers _____ and _____.

   (ii) The decimal form of the rational number $\frac{15}{-4}$ is _____.

   (iii) The rational numbers $\frac{-8}{3}$ and $\frac{8}{3}$ are equidistant from _____.

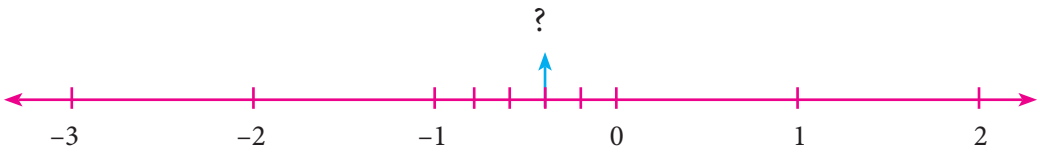
   (iv) The next rational number in the sequence $\frac{-15}{24}, \frac{20}{-32}, \frac{-25}{40}$ is _____.

   (v) The standard form of $\frac{58}{-78}$ is _____.

2. **Say True or False:**
   (i) 0 is the smallest rational number.
   (ii) $\frac{-4}{5}$ lies to the left of $\frac{-3}{4}$.
   (iii) $\frac{-19}{5}$ is greater than $\frac{15}{-4}$.
   (iv) The average of two rational numbers lies between them.
   (v) There are an unlimited number of rational numbers between 10 and 11.
3. Find the rational numbers represented by each of the question marks marked on the following number lines.

(i)

(ii)



(iii)

4. The points S, Y, N, C, R, A, T, I and O on the number line are such that CN=NY=YS and RA=AT=TI=IO. Find the rational numbers represented by the letters Y, N, A, T and I.

5. Draw a number line and represent the following rational numbers on it.
   (i) $\frac{9}{4}$ (ii) $\frac{-8}{3}$ (iii) $\frac{-17}{-5}$ (iv) $\frac{15}{-4}$
6. Write the decimal form of the following rational numbers.
   (i) $\frac{1}{11}$ (ii) $\frac{13}{4}$ (iii) $\frac{-18}{7}$ (iv) $1\frac{2}{5}$ (v) $-3\frac{1}{2}$
7. List any five rational numbers between the given rational numbers.
   (i) −2 and 0 (ii) $\frac{-1}{2}$ and $\frac{3}{5}$ (iii) $\frac{1}{4}$ and $\frac{7}{20}$ (iv) $\frac{-6}{4}$ and $\frac{-23}{10}$
8. Use the method of averages to write 2 rational numbers between $\frac{14}{5}$ and $\frac{16}{3}$.
9. Compare the following pairs of rational numbers.
   (i) $\frac{-11}{5}, \frac{-21}{8}$ (ii) $\frac{3}{-4}, \frac{-1}{2}$ (iii) $\frac{2}{3}, \frac{4}{5}$.
10. Arrange the following rational numbers in ascending and descending order.
    (i) $\frac{-5}{12}, \frac{-11}{8}, \frac{-15}{24}, \frac{-7}{-9}, \frac{12}{36}$ (ii) $\frac{-17}{10}, \frac{-7}{5}, 0, \frac{-2}{4}, \frac{-19}{20}$

11. The number which is subtracted from $\frac{-6}{11}$ to get $\frac{8}{9}$ is _____.

(A) $\frac{34}{99}$ (B) $\frac{-142}{99}$ (C) $\frac{142}{99}$ (D) $\frac{-34}{99}$

12. Which of the following pairs is equivalent?

(A) $\frac{-20}{12}, \frac{5}{3}$ (B) $\frac{16}{-30}, \frac{-8}{15}$ (C) $\frac{-18}{36}, \frac{-20}{44}$ (D) $\frac{7}{-5}, \frac{-5}{7}$

13. $\frac{-5}{4}$ is a rational number which lies between _____ .

(A) 0 and $\frac{-5}{4}$ (B) –1 and 0 (C) –1 and –2 (D) –4 and –5

14. Which of the following rational numbers is the greatest?

(A) $\frac{-17}{24}$ (B) $\frac{-13}{16}$ (C) $\frac{7}{-8}$ (D) $\frac{-31}{32}$

15. The sum of the digits of the denominator in the simplest form of $\frac{112}{528}$ is _____ .

(A) 4 (B) 5 (C) 6 (D) 7

1.3 Basic Arithmetic Operations on Rational Numbers

All the rules and principles that govern fractions in the basic operations apply to rational numbers also.

1.3.1 Addition

There can be four different situations while doing addition.

Type 1 : Adding numbers that have same denominators

This is simply like adding like fractions and the result is the sum of the numerators divided by their common denominator.

Example 1.10

Add : $\frac{-6}{11}, \frac{8}{11}, \frac{-12}{11}$

Solution:

Write the given rational numbers in the standard form and then add them.

So, $\frac{-6}{11} + \frac{8}{11} + \frac{-12}{11} = \frac{-6+8-12}{11} = \frac{-10}{11}$

Type 2 : Adding numbers that have different denominators

After writing the given rational numbers in the standard form, use the LCM of their denominators to convert the numbers into equivalent rational numbers with a common denominator so that this reduces to Type1.

Example 1.11

Add : $\frac{-5}{9}, \frac{-4}{3}, \frac{7}{12}$

Solution:

LCM of 9,3,12 = 36

So, $\frac{-5}{9}+\frac{-4}{3}+\frac{6}{12}=\frac{-5}{9}\times\frac{4}{4}+\frac{-4}{3}\times\frac{12}{12}+\frac{7}{12}\times\frac{3}{3}$

$=\frac{-20}{36}+\frac{-48}{36}+\frac{21}{36}=\frac{-20-48+21}{36}=\frac{-47}{36}$

1.3.2 Additive Inverse

The additive inverse of a rational number is another rational number which when added to the given number, gives zero.

For example, $\frac{4}{3}$ and $\frac{-4}{3}$ are additive inverses of each other, since their sum is zero.

Think

Is zero a rational number? If so, what is its additive inverse?

1.3.3 Subtraction

Subtraction is simply adding the additive inverse.

Example 1.12

Subtract : $\frac{9}{17}$ from $\frac{-12}{17}$

Solution:

Now, $\frac{-12}{17}-\frac{9}{17}=\frac{-12}{17}+\left(\frac{-9}{17}\right)=\frac{-12-9}{17}=\frac{-21}{17}$

Example 1.13

Subtract : $\left(-2\frac{6}{11}\right)$ from $\left(-4\frac{5}{22}\right)$

Solution:

Now, $\left(-4\frac{5}{22}\right)-\left(-2\frac{6}{11}\right)$

$=\frac{-93}{22}-\left(\frac{-28}{11}\right)$

$=\frac{-93}{22}+\frac{28}{11}=\frac{-93+28\times2}{22}$

$=\frac{-93+56}{22}=\frac{-37}{22}=-1\frac{15}{22}$

1.3.4 Multiplication

Product of two or more rational numbers is found by multiplying the corresponding numerators and denominators of the numbers and then writing them in the standard form.

Example 1.14

Evaluate : (i) $\frac{-5}{8}\times\frac{7}{3}$ (ii) $\frac{-6}{-11}\times(-4)$

Solution:

(i) $\frac{-5}{8}\times\frac{7}{3}=\frac{-5\times7}{8\times3}=\frac{-35}{24}$ (ii) $\frac{-6}{-11}\times(-4)=\frac{6}{11}\times\frac{(-4)}{1}=\frac{6\times(-4)}{11\times1}=\frac{-24}{11}$

1.3.5 Multiplicative Inverse

If the product of two rational numbers is 1, then each of them is said to be the reciprocal or the multiplicative inverse of the other.

Think

What is the multiplicative inverse of 1 and –1?

For the rational number a, its reciprocal is $\frac{1}{a}$ and vice versa since $a\times\frac{1}{a}=\frac{1}{a}\times a=1$.

For the rational number $\frac{a}{b}$, its multiplicative inverse is $\frac{b}{a}$ and vice versa since $\frac{a}{b}\times\frac{b}{a}=\frac{b}{a}\times\frac{a}{b}=1$.

1.3.6 Division

The idea of reciprocals of fractions is extended to the division of rational numbers also. To divide a given rational number by another rational number, we have to multiply the given rational number by the reciprocal of the second rational number. That is, division is simply multiplying by the multiplicative inverse of the divisor.

Example 1.15

Divide : $\frac{7}{-8}$ by $\frac{-3}{4}$

Solution:

Here, $\frac{7}{-8}\div\frac{-3}{4}=\frac{-7}{8}\times\frac{-4}{3}=\frac{7}{6}$

Try these

Divide :

(i) $\frac{-7}{3}$ by 5 (ii) 5 by $\frac{-7}{3}$ (iii) $\frac{-7}{3}$ by $\frac{35}{6}$

Note

For any non-zero b, c, and d, we have

(i) $\left(\frac{a}{b}\right)\div c=\frac{a}{bc}$ (ii) $a\div\left(\frac{b}{c}\right)=\frac{ac}{b}$ (iii) $\left(\frac{a}{b}\right)\div\left(\frac{c}{d}\right)=\frac{ad}{bc}$

1.4 Word Problems on the basic operations

Example 1.16

The sum of two rational numbers is $\frac{4}{5}$. If one number is $\frac{2}{15}$, then find the other.

Solution:

Let the other number be x.

Given, $\frac{2}{15}+x=\frac{4}{5}$

$\Rightarrow x=\frac{4}{5}-\frac{2}{15}=\frac{12-2}{15}=\frac{10}{15}\Rightarrow x=\frac{2}{3}$

Example 1.17

The product of two rational numbers is $\frac{-2}{3}$. If one number is $\frac{3}{7}$, then find the other.

Solution:

Let the other number be x.

Given, $\frac{3}{7} \times x = \frac{-2}{3}$

Multiplying by the reciprocal of $\frac{3}{7}$, that is $\frac{7}{3}$ on both sides,

$$\Rightarrow \frac{7}{3} \times \frac{3}{7} \times x = \frac{7}{3} \times \frac{-2}{3}$$

$$\Rightarrow x = \frac{-14}{9}$$

Aliter

$\frac{3}{7} \times x = \frac{-2}{3}$

$\Rightarrow x = \frac{-2}{3} \times \frac{7}{3} = \frac{-14}{9}$

(by cross multiplication)

Example 1.18

One roll of ribbon is $18\frac{3}{4}$ m long. Sankari has four full rolls and one–third of another roll. How many metres of ribbon does Sankari have in total?

Fig. 1.15

Solution:

Number of metres of ribbon Sankari has in total

$$= 18\frac{3}{4} \times 4\frac{1}{3}$$

$$= \frac{75}{4} \times \frac{13}{3} = \frac{325}{4} = 81\frac{1}{4} \text{ m}$$

Example 1.19

Find the rational numbers that should be added and subtracted so that they will make the sum $3\frac{1}{2} + 1\frac{3}{4} + 2\frac{3}{8}$ to the nearest whole number.

Solution:

Now, $3\frac{1}{2} + 1\frac{3}{4} + 2\frac{3}{8}$

$$= \frac{7}{2} + \frac{7}{4} + \frac{19}{8} = \frac{7 \times 4 + 7 \times 2 + 19 \times 1}{8} = \frac{28 + 14 + 19}{8}$$

$= \frac{61}{8} = 7\frac{5}{8}$, which lies between the whole numbers 7 and 8.

If we subtract $\frac{5}{8}$ from $7\frac{5}{8}$, it becomes 7. If we add $\frac{3}{8}$ to $7\frac{5}{8}$, it becomes $7 + \frac{5}{8} + \frac{3}{8} = 7 + 1 = 8$.

Example 1.20

A student instead of multiplying a number by $\frac{8}{9}$, by mistake divided it by $\frac{8}{9}$. If the difference between the correct answer and the answer got by him is 34, then find the number.

Solution:

Let the number be x.

The student had to find $\frac{8x}{9}$ but, he had found $\frac{x}{\left(\frac{8}{9}\right)}$, that is $\frac{9x}{8}$.

Now, $\frac{9x}{8}-\frac{8x}{9} = 34$ (given)

$$\frac{81x-64x}{72} = 34 \Rightarrow \frac{17x}{72} = 34$$

$$x = \frac{34\times 72}{17} = 144$$

Example 1.21

Simplify: $\left(\frac{4}{3}-\left(\frac{-3}{2}\right)\right)+\left(\frac{-5}{3}\div\frac{30}{12}\right)+\left(\frac{-12}{9}\times\frac{-27}{16}\right)$

Solution:

Here, $\left(\frac{4}{3}-\left(\frac{-3}{2}\right)\right)+\left(\frac{-5}{3}\div\frac{30}{12}\right)+\left(\frac{-12}{9}\times\frac{-27}{16}\right) = \left(\frac{4}{3}+\frac{3}{2}\right)+\left(\frac{-5}{3}\times\frac{12}{30}\right)+\left(\frac{-12}{9}\times\frac{-27}{16}\right)$

$$= \left(\frac{8}{6}+\frac{9}{6}\right)+\left(\frac{-1}{1}\times\frac{4}{6}\right)+\left(\frac{-3}{1}\times\frac{-3}{4}\right)$$

$$= \left(\frac{17}{6}\right)+\left(\frac{-4}{6}\right)+\left(\frac{9}{4}\right)$$

$$= \left(\frac{17-4}{6}\right)+\frac{9}{4} = \frac{13}{6}+\frac{9}{4}$$

$$= \frac{26+27}{12} = \frac{53}{12}$$

Exercise 1.2

1. **Fill in the blanks:**
   (i) The value of $\frac{-5}{12}+\frac{7}{15}=$ _____.
   (ii) The value of $\left(\frac{-3}{6}\right)\times\left(\frac{18}{-9}\right)$ is _____.
   (iii) The value of $\left(\frac{-15}{23}\right)\div\left(\frac{30}{-46}\right)$ is _____.
   (iv) The rational number _____ does not have a reciprocal.
   (v) The multiplicative inverse of –1 is _____.
2. **Say True or False:**
   (i) All rational numbers have an additive inverse.
   (ii) The rational numbers that are equal to their additive inverses are 0 and –1.
   (iii) The additive inverse of $\frac{-11}{-17}$ is $\frac{11}{17}$.

(iv) The rational number which is its own reciprocal is –1.

(v) The multiplicative inverse exists for all rational numbers.

3. Find the sum:

(i) $\frac{7}{5}+\frac{3}{5}$ (ii) $\frac{7}{5}+\frac{5}{7}$ (iii) $\frac{6}{5}+\left(\frac{-14}{15}\right)$ (iv) $-4\frac{2}{3}+7\frac{5}{12}$

4. Subtract : $\frac{-8}{44}$ from $\frac{-17}{11}$.

5. Evaluate : (i) $\frac{9}{132}\times\frac{-11}{3}$ (ii) $\frac{-7}{27}\times\frac{24}{-35}$

6. Divide : (i) $\frac{-21}{5}$ by $\frac{-7}{-10}$ (ii) $\frac{-3}{13}$ by –3 (iii) –2 by $\frac{-6}{15}$

7. Find $(a + b) \div (a - b)$ if

(i) $a=\frac{1}{2}, b=\frac{2}{3}$ (ii) $a=\frac{-3}{5}, b=\frac{2}{15}$

8. Simplify : $\frac{1}{2}+\left(\frac{3}{2}-\frac{2}{5}\right)\div\frac{3}{10}\times 3$ and show that it is a rational number between 11 and 12.

9. Simplify :

(i) $\left[\frac{11}{8}\times\left(\frac{-6}{33}\right)\right]+\left[\frac{1}{3}+\left(\frac{3}{5}\div\frac{9}{20}\right)\right]-\left[\frac{4}{7}\times\frac{-7}{5}\right]$ (ii) $\left[\frac{4}{3}\div\left(\frac{8}{-7}\right)\right]-\left[\frac{3}{4}\times\frac{4}{3}\right]+\left[\frac{4}{3}\times\left(\frac{-1}{4}\right)\right]$

10. A student had multiplied a number by $\frac{4}{3}$ instead of dividing it by $\frac{4}{3}$ and got 70 more than the correct answer. Find the number.

Objective Type Questions

11. The standard form of the sum $\frac{3}{4}+\frac{5}{6}+\left(\frac{-7}{12}\right)$ is _____ .

(A) 1 (B) $\frac{-1}{2}$ (C) $\frac{1}{12}$ (D) $\frac{1}{22}$

12. $\left(\frac{3}{4}-\frac{5}{8}\right)+\frac{1}{2}=$ _____ .

(A) $\frac{15}{64}$ (B) 1 (C) $\frac{5}{8}$ (D) $\frac{1}{16}$

13. $\frac{3}{4}\div\left(\frac{5}{8}+\frac{1}{2}\right)=$ _____ .

(A) $\frac{13}{10}$ (B) $\frac{2}{3}$ (C) $\frac{3}{2}$ (D) $\frac{5}{8}$

14. $\frac{3}{4}\times\left(\frac{5}{8}\div\frac{1}{2}\right)=$ _____ .

(A) $\frac{5}{8}$ (B) $\frac{2}{3}$ (C) $\frac{15}{32}$ (D) $\frac{15}{16}$

15. Which of these rational numbers which have additive inverse?

(A) 7 (B) $\frac{-5}{7}$ (C) 0 (D) all of these

Note

Why $\frac{a}{b}+\frac{c}{d} \neq \frac{a+c}{b+d}$?

A mathematical statement is true only if it is 100% true without any exception. If we do the addition, say $\frac{3}{4}+\frac{1}{4}$ as $\frac{3+1}{4+4}=\frac{4}{8}=\frac{1}{2}$. This is wrong, since $\frac{3}{4}+\frac{1}{4}=\frac{3+1}{4}=\frac{4}{4}=1$.

1.5 Properties of Rational Numbers

Some properties listed here below will be of good use in solving problems.

1.5.1 Closure property/law for the collection $\mathbb{Q}$ of rational numbers

i) Closure property for Addition

For any two rational numbers ***a*** and ***b***, the sum ***a + b*** is also a rational number.

ii) Closure property for Multiplication

For any two rational numbers ***a*** and ***b***, the product ***ab*** is also a rational number.

Illustration

Take $a=\frac{3}{4}$ and $b=\frac{-1}{2}$

Now, $a+b=\frac{3}{4}+\frac{-1}{2}=\frac{3}{4}+\frac{-2}{4}=\frac{3-2}{4}=\frac{1}{4}$ is in $\mathbb{Q}$

Also, $a\times b=\frac{3}{4}\times\frac{-1}{2}=\frac{-3}{8}$ is in $\mathbb{Q}$

Try this

The closure property on integers holds for subtraction and not for division. What about rational numbers? Verify.

1.5.2 Commutative property/law for the collection $\mathbb{Q}$ of rational numbers

i) Commutative property for Addition

For any two rational numbers ***a*** and ***b***, ***a + b = b + a***.

ii) Commutative property for Multiplication

For any two rational numbers ***a*** and ***b***, ***ab = ba*** (ab means $a\times b$ and ba means $b\times a$).

Illustration

Take $a=\frac{-7}{8}$ and $b=\frac{3}{5}$

Now, $a+b=\frac{-7}{8}+\frac{3}{5}=\frac{-7\times5+3\times8}{40}=\frac{-35+24}{40}=\frac{-11}{40}$

Also, $b+a=\frac{3}{5}+\frac{-7}{8}=\frac{3\times8+-7\times5}{40}=\frac{24-35}{40}=\frac{-11}{40}$

Here, we find that $a+b=b+a$ and hence addition is commutative.

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Operations	Fill in the blanks in the table given below of properties of Integers. (If a, b, c are integers, then $-a, -b, -c$ are also integers)					
	Closure	**Commutative**	**Associative**	**Identity**	**Inverse**	**Distributive**
Addition	$a+b$ is in $\mathbb{Z}$ E.g. $5+(-3)=2$ $\Rightarrow 2$ is in $\mathbb{Z}$	$a+b=b+a$ E.g. $5+(-3)=(-3)+5$ $\Rightarrow 2=2$	$(a+b)+c$ $=a+(b+c)$ E.g. $(2+3)+(-4)=1$ $2+[3+(-4)]=1$	$a+0$ $=0+a=a$ E.g. $(-4)+0$ $=0+(-4)=-4$	$a+(-a)$ $=(-a)+a=0$ E.g. $5+(-5)$ $=(-5)+5=0$	$a\times(b+c)$ $=(a\times b)+(a\times c)$ E.g. $2\times[3+(-5)]=-4$ $(2\times 3)+[2\times(-5)]$ $=-4$
Multiplication	ab is in $\mathbb{Z}$ E.g._____	$a\times b=b\times a$ E.g._____	$(a\times b)\times c$ $=a\times(b\times c)$ E.g. $(2\times 3)\times(-6)=-36$ $2\times[3\times(-6)]=-36$	$a\times 1$ $=1\times a=a$ E.g._____	Does not exist	Not Applicable
Subtraction	$a-b$ is in $\mathbb{Z}$ E.g._____	Fails $a-b\neq b-a$ E.g._____	Fails $(a-b)-c$ $\neq a-(b-a)$ E.g._____	Fails $a-0\neq 0-a$ E.g. $5-0=5$ $0-5=-5$ $5\neq -5$	Fails $a-(-a)$ $\neq(-a)-a$ E.g. $2-(-2)=4$ $(-2)-2=-4$ $4\neq -4$	$a\times(b-c)$ $=(a\times b)-(a\times c)$ E.g._____
Division	Fails $a\div b$ is not in $\mathbb{Z}$ E.g. $3\div 5=\frac{3}{5}$ does not belong to $\mathbb{Z}$	Fails	Fails	Fails	Fails	Not applicable

Further,

$$a \times b = \frac{-7}{8} \times \frac{3}{5} = \frac{-7 \times 3}{8 \times 5} = \frac{-21}{40}$$

Also, $$b \times a = \frac{3}{5} \times \frac{-7}{8} = \frac{3 \times -7}{5 \times 8} = \frac{-21}{40}$$

Here, we find that $a \times b = b \times a$ and hence multiplication is commutative.

Try these

(i) Is $\frac{3}{5} - \frac{7}{8} = \frac{7}{8} - \frac{3}{5}$?

(ii) Is $\frac{3}{5} \div \frac{7}{8} = \frac{7}{8} \div \frac{5}{3}$?

So, what do you conclude?

1.5.3 Associative property/law for the collection $\mathbb{Q}$ of rational numbers

i) Associative property for Addition

For any three rational numbers ***a***, ***b***, and ***c***, ***a + (b + c) = (a + b) + c***

ii) Associative property for Multiplication

For any three rational numbers ***a***, ***b***, and ***c***, ***a(bc) = (ab)c***

Illustration

Take rational numbers a, b, c as $a = \frac{-1}{2}$, $b = \frac{3}{5}$ and $c = \frac{-7}{10}$

Now, $a + b = \frac{-1}{2} + \frac{3}{5} = \frac{-5}{10} + \frac{6}{10}$ (equivalent rationals with common denominators)

$$a + b = \frac{-5+6}{10} = \frac{1}{10}$$

$$(a+b)+c = \frac{1}{10} + \left(\frac{-7}{10}\right) = \frac{1-7}{10} = \frac{-6}{10} = \frac{-3}{5} \quad \ldots(1)$$

$$\text{Also, } b + c = \frac{3}{5} + \frac{-7}{10} = \frac{6}{10} + \frac{-7}{10} = \frac{6-7}{10} = \frac{-1}{10}$$

$$a+(b+c) = \frac{-1}{2} + \frac{-1}{10} = \frac{-5}{10} + \frac{-1}{10} = \frac{-5-1}{10} = \frac{-6}{10} = \frac{-3}{5} \quad \ldots(2)$$

(1) and (2) shows that $(a+b)+c = a+(b+c)$ is true for rational numbers.

$$\text{Similarly, } a \times b = \frac{-1}{2} \times \frac{3}{5} = \frac{-1 \times 3}{2 \times 5} = \frac{-3}{10}$$

$$(a \times b) \times c = \frac{-3}{10} \times \frac{-7}{10} = \frac{-3 \times -7}{10 \times 10} = \frac{21}{100} \quad \ldots(3)$$

$$\text{Also, } b \times c = \frac{3}{5} \times \frac{-7}{10} = \frac{3 \times -7}{5 \times 10} = \frac{-21}{50}$$

$$a \times (b \times c) = \frac{-1}{2} \times \frac{-21}{50} = \frac{-1 \times -21}{2 \times 50} = \frac{21}{100} \quad \ldots(4)$$

(3) and (4) shows that

$(a \times b) \times c = a \times (b \times c)$ is true for rational numbers.

Thus, the associative property is true for addition and multiplication of rational numbers.

Try this

Check whether associative property holds for subtraction and division.

1.5.4 Identity property/law for the collection $\mathbb{Q}$ of rational numbers

i) Identity property for Addition

For any rational number a, there exists a unique rational number 0 such that $0 + a = a = 0 + a$.

ii) Identity property for Multiplication

For any rational number a, there exists a unique rational number 1 such that $1 \times a = a = a \times 1$.

Illustration

Take $a = \frac{3}{-7}$ that is, $a = \frac{-3}{7}$. Now $\frac{-3}{7} + 0 = \frac{-3}{7} = 0 + \frac{-3}{7}$ (Isn't it?)

Hence, 0 is the additive identity for $\frac{-3}{7}$

Also, $\frac{-3}{7} \times 1 = \frac{-3}{7} = 1 \times \frac{-3}{7}$ (Isn't it?)

Hence, 1 is the multiplicative identity for $\frac{-3}{7}$

1.5.5 Inverse property/law for the collection $\mathbb{Q}$ of rational numbers

i) Additive Inverse property

For any rational number a, there exists a unique rational number $-a$ such that $a + (-a) = (-a) + a = 0$. Here, 0 is the additive identity.

ii) Multiplicative Inverse property

For any rational number b, there exists a unique rational number $\frac{1}{b}$ such that $b \times \frac{1}{b} = \frac{1}{b} \times b = 1$. Here, 1 is the multiplicative identity.

Illustration

Take $a = \frac{-11}{23}$ Now, $-a = -\left(\frac{-11}{23}\right) = \frac{11}{23}$

So, $a + (-a) = \frac{-11}{23} + \frac{11}{23} = \frac{-11+11}{23} = \frac{0}{23} = 0$

Also, $(-a) + a = \frac{11}{23} + \frac{-11}{23} = \frac{11-11}{23} = \frac{0}{23} = 0$

$\therefore a + (-a) = (-a) + a = 0$ is true.

Also, take $b = \frac{-17}{29}$. Now, $\frac{1}{b} = \frac{29}{-17} = \frac{-29}{17}$

$b \times \frac{1}{b} = \frac{-17}{29} \times \frac{-29}{17} = 1$. Also, $\frac{1}{b} \times b = \frac{-29}{17} \times \frac{-17}{29} = 1$

$\therefore b \times \frac{1}{b} = \frac{1}{b} \times b = 1$ is true.

1.5.6 Distributive property/law for the collection $\mathbb{Q}$ of rational numbers

Multiplication is distributive over addition for the collection of rational numbers.

For any three rational numbers ***a***, ***b*** and ***c***, the distributive law is $a\times(b+c)=(a\times b)+(a\times c)$

Illustration

Take rational numbers a,b,c as $a=\frac{-7}{9}$, $b=\frac{11}{18}$ and $c=\frac{-14}{27}$

$$\text{Now, } b+c = b+c=\frac{11}{18}+\frac{-14}{27}=\frac{33}{54}+\frac{-28}{54}=\frac{33-28}{54}=\frac{5}{54}$$

(equivalent rationals with common denominators)

$$\therefore\ a\times(b+c) = \frac{-7}{9}\times\frac{5}{54}=\frac{-7\times5}{9\times54}=\frac{-35}{486} \qquad \ldots(1)$$

$$\text{Also, } a\times b = \frac{-7}{9}\times\frac{11}{18}=\frac{-7\times11}{9\times18}=\frac{-77}{9\times9\times2}$$

$$a\times c = \frac{-7}{9}\times\frac{-14}{27}=\frac{7\times14}{9\times9\times3}=\frac{98}{9\times9\times3}$$

$$\therefore\ (a\times b)+(a\times c) = \frac{-77}{9\times9\times2}+\frac{98}{9\times9\times3}$$

$$=\frac{-77\times3+98\times2}{9\times9\times2\times3}=\frac{-231+196}{486}=\frac{-35}{486} \qquad \ldots(2)$$

(1) and (2) shows that $a\times(b+c)=(a\times b)+(a\times c)$.

Hence, multiplication is distributive over addition for the collection $\mathbb{Q}$ of rational numbers.

Exercise 1.3

1. Verify the closure property for addition and multiplication for the rational numbers $\frac{-5}{7}$ and $\frac{8}{9}$.
2. Verify the commutative property for addition and multiplication for the rational numbers $\frac{-10}{11}$ and $\frac{-8}{33}$.
3. Verify the associative property for addition and multiplication for the rational numbers $\frac{-7}{9},\frac{5}{6}$ and $\frac{-4}{3}$.
4. Verify the distributive property $a\times(b+c)=(a\times b)+(a+c)$ for the rational numbers $a=\frac{-1}{2}$, $b=\frac{2}{3}$ and $c=\frac{-5}{6}$.
5. Verify the identity property for addition and multiplication for the rational numbers $\frac{15}{19}$ and $\frac{-18}{25}$.

6. Verify the additive and multiplicative inverse property for the rational numbers $\frac{-7}{17}$ and $\frac{17}{27}$.

Objective Type Questions

7. Closure property is not true for division of rational numbers because of the number

(A) 1 (B) –1 (C) 0 (D) $\frac{1}{2}$

8. $\frac{1}{2}-\left(\frac{3}{4}-\frac{5}{6}\right)\neq\left(\frac{1}{2}-\frac{3}{4}\right)-\frac{5}{6}$ illustrates that subtraction does not satisfy the _____ property for rational numbers.

(A) commutative (B) closure (C) distributive (D) associative

9. Which of the following illustrates the inverse property for addition?

(A) $\frac{1}{8}-\frac{1}{8}=0$ (B) $\frac{1}{8}+\frac{1}{8}=\frac{1}{4}$ (C) $\frac{1}{8}+0=\frac{1}{8}$ (D) $\frac{1}{8}-0=\frac{1}{8}$

10. $\frac{3}{4}\times\left(\frac{1}{2}-\frac{1}{4}\right)=\frac{3}{4}\times\frac{1}{2}-\frac{3}{4}\times\frac{1}{4}$ illustrates that multiplication is distributive over

(A) addition (B) subtraction (C) multiplication (D) division

We know that different operations with the same pair of rational numbers usually give different answers. Check the following calculations which are some interesting exceptions in rational numbers.

(i) $\frac{13}{4}+\frac{13}{9}=\frac{13}{4}\times\frac{13}{9}$ (ii) $\frac{169}{30}+\frac{13}{15}=\frac{169}{30}\div\frac{13}{15}$

Amazing …! Isn't it? Try a few more like these, if possible.

Think

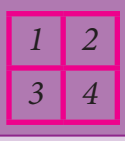
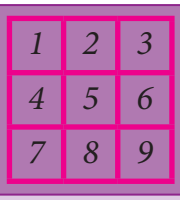
Observe that, $\frac{1}{1.2}+\frac{1}{2.3}=\frac{2}{3}$

$\frac{1}{1.2}+\frac{1}{2.3}+\frac{1}{3.4}=\frac{3}{4}$

$\frac{1}{1.2}+\frac{1}{2.3}+\frac{1}{3.4}+\frac{1}{4.5}=\frac{4}{5}$

Use your reasoning skills, to find the sum of the first 7 numbers in the pattern given above.

1.6 Introduction to Square Numbers

		
This is a square of side 1 unit. It is 1 squared. We write this as 1^2	This is a square of side 2 units. It is 2 squared. We write this as 2^2	This is a square of side 3 units. It is 3 squared. We write this as 3^2
$1^2 = 1 \times 1 = 1$	$2^2 = 2 \times 2 = 4$	$3^2 = 3 \times 3 = 9$

More often we write like this:

$4^2 = 16$

This says "4 squared is 16"

The 2 at the top stands for squared and it indicates the number of times the number 4 appears in the product ($4^2 = 4 \times 4 = 16$).

The numbers 1, 4, 9, 16, ... are all square numbers (also called perfect square numbers). Each of them is made up of the product of same two factors.

A natural number n is called a square number, if we can find another natural number m such that $n = m^2$.

Is 49 a square number? Yes, because it can be written as 7^2. Is 50 a square number?

The following table gives the squares of numbers up to 20.

Number	Its square	Number	Its square	Number	Its square	Number	Its square
1	1	6	36	11	121	16	256
2	4	7	49	12	144	17	289
3	9	8	64	13	169	18	324
4	16	9	81	14	196	19	361
5	25	10	100	15	225	20	400

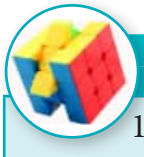
Try to extend the table up to 50 numbers!

We can now easily verify the following properties of square numbers by referring the table given above:

- The square numbers end with 0, 1, 4, 5, 6 and 9 only.
- If a number ends with 1 or 9, its square ends with 1.
- If a number ends with 2 or 8, its square ends with 4.
- If a number ends with 3 or 7, its square ends with 9.
- If a number ends with 4 or 6, its square ends with 6.
- If a number ends with 5 or 0, its square also ends with 5 or 0 respectively.
- Square of an odd number is always odd and the square of an even number is always even.
- Numbers that end with 2,3,7 and 8 are not perfect squares.

Think

1. Is the square of a prime number, prime?
2. Will the sum of two perfect squares always be a perfect square? What about their difference and their product?



Try these

1. Which among 256, 576, 960, 1025, 4096 are perfect square numbers? (Hint: Try to extend the table of squares already seen).
2. One can judge just by look that each of the following numbers 82, 113, 1972, 2057, 8888, 24353 is not a perfect square. Explain why?

1.6.1 Some more special properties of square numbers

(i) The square of a natural number other than 1, is either a multiple of 3 or exceeds a multiple of 3 by 1.

(ii) The square of a natural number, other than 1, is either a multiple of 4 or exceeds a multiple of 4 by 1.

(iii) The remainder of a perfect square when divided by 3, is either 0 or 1 but never 2.

(iv) The remainder of a perfect square, when divided by 4, is either 0 or 1 but never 2 and 3.

(v) When a perfect square number is divided by 8, the remainder is either 0 or 1 or 4, but will never be equal to 2, 3, 5, 6 or 7.

Perfect numbers such as 6, 28, 496, 8128 etc., are not square numbers.

Note

If a perfect square number ends in zero, then it must end with even number of zeroes always. We can verify this for a few numbers in the table given below.

Number	10	20	30	40	...	90	100	110	...	2000	...
Its Square	100	400	900	1600	...	8100	10000	12100	...	4000000	...

Think

Consider the claim: "Between the squares of the consecutive numbers n and $(n+1)$, there are $2n$ non-square numbers". Can it be true? How many non-square numbers are there between 2500 and 2601? Verify the claim.

1.7 Square Root

Squaring a number is another mathematical operation just like addition, subtraction, multiplication etc., Most mathematical operations have 'inverse' (meaning 'opposite') operations. For example, subtraction is the inverse of addition, division is the inverse of multiplication etc., Squaring also has an inverse operation namely finding the Square root.

The square root of a number n, written $\sqrt{n}$ (or) $n^{\frac{1}{2}}$, is the number that gives n when multiplied by itself. For example, $\sqrt{81}$ is 9, because $9 \times 9 = 81$.

In the adjacent table, we have square roots of all the perfect squares starting from 1 to 100.

If $11^2 = 121$, what is $\sqrt{121}$? If $529 = 23^2$, what is the square root of 529? If we know that $324 = 18^2$, we can immediately tell that $\sqrt{324}$ is 18.

We have, $1^2 = 1$ and so 1 is a square root of 1. Similarly, $(-1)^2 = 1$. So, (-1) is also a square root of 1.

$2^2 = 4$ and so 2 is a square root of 4. Similarly, $(-2)^2 = 4$. So, (-2) is also a square root of 4.

This continues as $3^2 = 9$ and so 3 is a square root of 9. Similarly, $(-3)^2 = 9$. So, (-3) is also a square root of 9.

The above examples suggest that there are two integral square roots for a perfect square number. However, in working out the problems, we will take up only positive integral square root. The positive square root of a number is always denoted by the symbol $\sqrt{\ }$. Thus, $\sqrt{4}$ is 2 (and not -2). Also, $\sqrt{9}$ is 3 (and not -3). We have to remember that this is a universally accepted notation.

Square root	Reason
$\sqrt{1} = 1$	$1^2 = 1$
$\sqrt{4} = 2$	$2^2 = 4$
$\sqrt{9} = 3$	$3^2 = 9$
$\sqrt{16} = 4$	$4^2 = 16$
$\sqrt{25} = 5$	$5^2 = 25$
$\sqrt{36} = 6$	$6^2 = 36$
$\sqrt{49} = 7$	$7^2 = 49$
$\sqrt{64} = 8$	$8^2 = 64$
$\sqrt{81} = 9$	$9^2 = 81$
$\sqrt{100} = 10$	$10^2 = 100$

1.7.1 Square root through Prime Factorisation

Study the following table giving the prime factors of numbers and those of their squares.

Numbers	Prime factorisation of the numbers	The square of given numbers	Prime factorisation of their squares
6	$6 = 2 \times 3$	$6^2 = 36$	$36 = 2 \times 2 \times 3 \times 3 = (2 \times 3)^2$
8	$8 = 2 \times 2 \times 2$	$8^2 = 64$	$64 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 = (2 \times 2 \times 2)^2$
12	$12 = 2 \times 2 \times 3$	$12^2 = 144$	$44 = 2 \times 2 \times 2 \times 2 \times 3 \times 3 = (2 \times 2 \times 3)^2$
15	$15 = 3 \times 5$	$15^2 = 225$	$225 = 3 \times 3 \times 5 \times 5 = (3 \times 5)^2$

Look at 6 and its prime factors. How many times do 2 and 3 occur in 36 in the list? Now, look at its square 36 and its prime factors. How many times do 2 and 3 occur in 36 here?

Repeat the above task in the case of other numbers 8, 12, and 15 also. (We may also choose our own numbers and their squares). What do we find? We find that,

The number of times a prime factor occurs in the square of a number = twice the number of times it occurs in the prime factorisation of the number.

We use this idea to find the square root of a square number. First, resolve the given number into prime factors. Group the identical factors in pairs and then take one from them to find the square root.

Example 1.22

Find the square root of 324 by prime factorisation.

Solution:

First, resolve the given number into prime factors. Group the identical factors in pairs and then take one from them to find the square root.

Now, $324 = 2\times2\times3\times3\times3\times3$

$= 2^2\times3^2\times3^2$

$= (2\times3\times3)^2$

$\therefore \sqrt{324} = \sqrt{(2\times3\times3)^2}$

$= 2\times3\times3$

$\therefore \sqrt{324} = 18$

2	324
2	162
3	81
3	27
3	9
3	3
	1

Example 1.23

Find the least number by which 250 is to be multiplied (or) divided so that the resulting number is a perfect square. Also, find the square root in that case.

Solution:

Here, $250 = 5\times5\times5\times2$

$= 5^2\times5\times2$

5	250
5	50
5	10
2	2
	1

Here, the prime factors 5 and 2 do not have pairs.

Therefore, we can either divide 250 by 10 (5×2) or multiply 250 by 10.

(i) If we multiply 250 by 10, we get $2500 = 5^2\times5\times2\times5\times2$ and therefore the square root of 2500 would be $5\times5\times2 = 50$.

(ii) If we divide 250 by 10, we get 25 and in this case we get $\sqrt{25} = \sqrt{5^2} = 5$.

Example 1.24

Is 108 a perfect square number?

Solution:

Here, $108 = 2\times2\times3\times3\times3$

$= 2^2\times3^2\times3$

2	108
2	54
3	27
3	9
3	3
	1

Here, the prime factor 3 does not have a second pair. Hence, 108 is not a perfect square number.

Think

In this case, if we want to find the smallest factor with which we can multiply or divide 108 to get a square number, what should we do?

1.7.2 Finding the square root of a number by Long Division Method

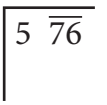
When we come across numbers with large number of digits, finding their square roots by factorisation becomes lengthy and difficult. Use of long division helps us in such cases. Let us look into the method with a couple of illustrations.

Illustration 1

Find the square root of 576 by long division method.

Step 1:

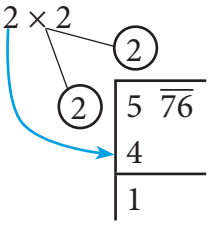
Group the digits in pairs, starting with the digit in the unit's place. Each pair and the remaining digit (if any) is called a period. Put a bar over every pair of digits starting from the right of the given number. If there are odd number of digits, the extreme left digit will be without a bar sign above it.



So, here we have **5 $\overline{76}$**

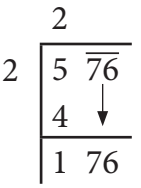
Step 2:

Think of the largest number whose square is equal to or just less than the first period. Take this number as the divisor and also as the quotient. The left extreme number here is 5. The largest number whose square is less than or equal to 5 is 2. This is our divisor and the quotient.



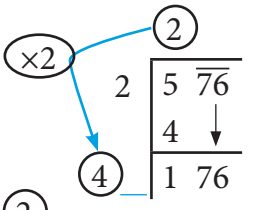
Step 3:

Bring 76 down and write it down to the right of the remainder 1.
Now, the new dividend is 176.



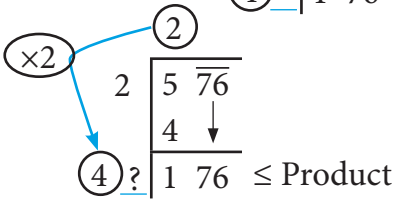
Step 4:

To find the new divisor, multiply the earlier quotient (2) by 2 (always) and write it leaving a blank space next to it.



Step 5:

The new divisor is 4 followed by a digit. We should choose this digit next to 4 such that the new quotient multiplied by the new divisor will be less than or equal to 176.

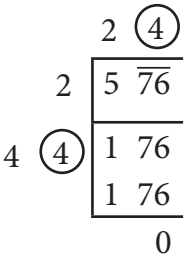


Step 6:

Clearly, the required digit here has to be 4 or 6. (Why?)

When we calculate, $46 \times 6 = 276$ whereas $44 \times 4 = 176$.

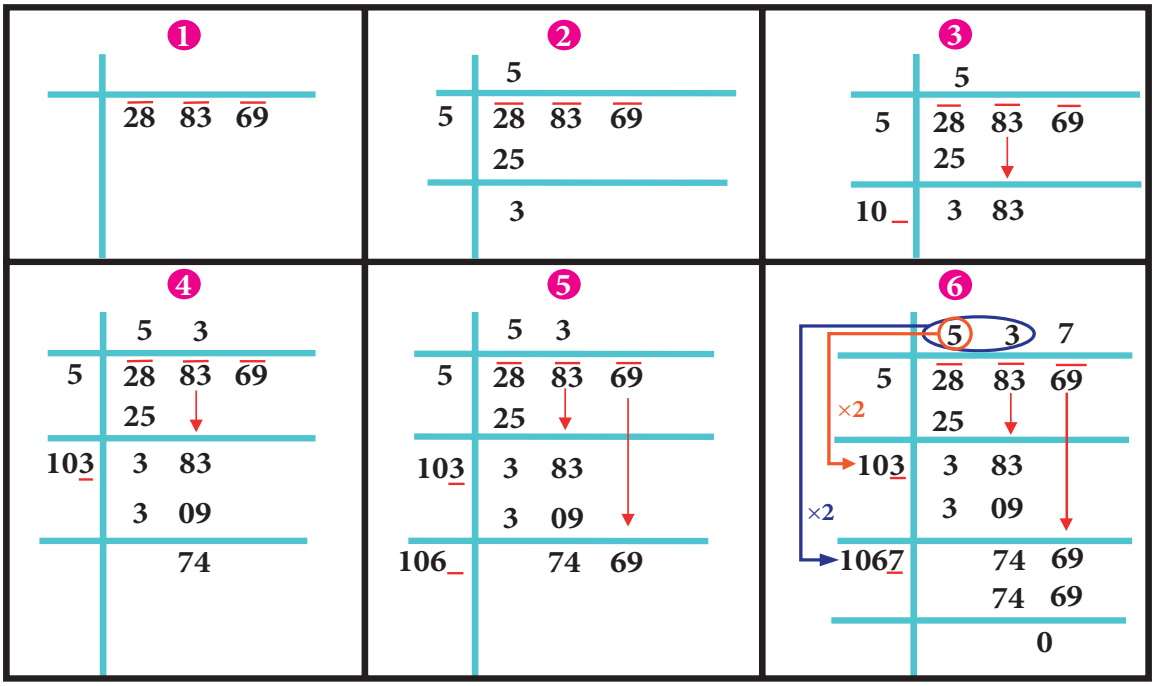
Therefore, we put 4 in the blank space and write $44 \times 4 = 176$ below 176 and subtract to get the remainder 0 and the quotient at the top, that is 24 is the square root of 576.



$\therefore \sqrt{576} = 24$.

Illustration 2

In the following example, follow the figures one after another and try to understand what each figure explains, the stage by stage and the gradual computation of computing the square root of 288369.



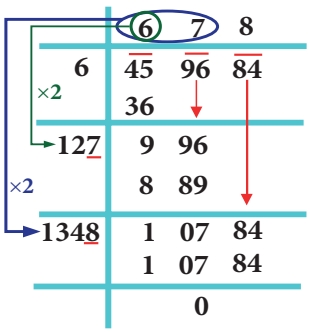
We find that $\sqrt{288369} = 537$.

Example 1.25

Find the square root of 459684 by long division method.

Solution:

By long division method, we can find the square root of 459684 as given below:



$\therefore \sqrt{459684} = 678$

Example 1.26

The area of a square field is 3136 m^2. Find its perimeter.

Solution:

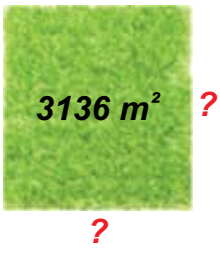
Given that the area of the square field = 3136 m^2.

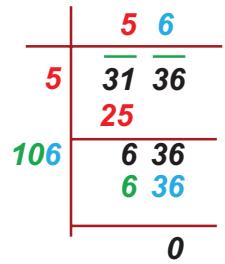
$\therefore$ The side of square field = $\sqrt{3136}$ = 56 m

$\therefore$ The perimeter of the square field = 4 × side

= 4 × 56

= 224 m



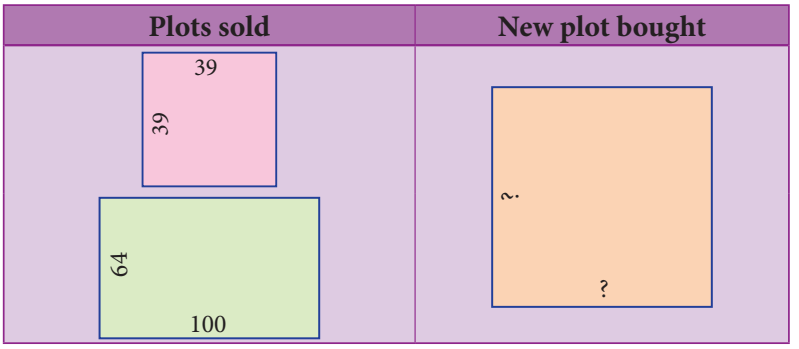


Example 1.27

A real estate owner had two plots, a square plot of side 39 m and a rectangular plot of dimensions 100 m length and 64 m breadth. He sells both of these plots and acquires a new square plot of the same area. What is the length of side of his new plot?

Solution:

The transactions can be visualised as follows:



$$\begin{aligned}\text{Area of the square plot bought} &= \text{Area of the square plot sold} + \text{Area of the rectangular plot sold}\\ &= 39 \times 39 + 100 \times 64\\ &= 1521 + 6400\\ &= 7921 \text{ m}^2\end{aligned}$$

$$\begin{array}{r|l} & 8\ \ 9 \\ \hline 8 & \overline{79}\ \ \overline{21} \\ & 64 \\ \hline 169 & 15\ \ 21 \\ & 15\ \ 21 \\ \hline & 0 \end{array}$$

Length of a side of the new square plot = $\sqrt{7921}$ = 89 m

Try these

Find the square root by long division method: 1. 400 2. 1764 3. 9801

1.7.3 Number of digits in the square root of a perfect square number

We made use of bars to find the square root in the division method. This marking bars help us to find the number of digits in the square root of a perfect square number. Observe the following examples (with bars shown as if we compute square root by division procedure).

Square root of numbers	No. of bars	No. of digits in the square root	Square root of numbers	No. of bars	No. of digits in the square root
$\sqrt{\overline{169}} = 13$	2	2	$\sqrt{\overline{4356}} = 66$	2	2
$\sqrt{\overline{441}} = 21$	2	2	$\sqrt{\overline{6084}} = 78$	2	2
$\sqrt{\overline{12544}} = 112$	3	3	$\sqrt{\overline{27225}} = 165$	3	3

Hence, we conclude that the number of bars indicates the number of digits in the square root.

Try these

Without calculating the square root, guess the number of digits in the square root of the following numbers: 1. 14400 2. 390625 3. 100000000

1.7.4 Square root of decimal numbers

To compute the square root of numbers in the decimal form, we simply follow the following steps:

Step 1:

Make even number of decimal places even by affixing a zero on the extreme right of the digit in the decimal part (only if required).

To find $\sqrt{42.25}$

Step 2:

The number has an integral part and a decimal part. In the integral part, mark the bars as done in the case of division method to find the square root of a perfect square number.

We put the bars as $\sqrt{\overline{42}.\overline{25}}$

Step 3:

In the decimal part, mark the bars on every pair of digits beginning with the first decimal place.

Step 4:

Now, calculate the square root by long division method.

Step 5:

Put the decimal point in the square root as soon as the integral part is exhausted. $\therefore \sqrt{42.25} = 6.5$

	6 5
6	42 25
	36
125	6 25
	6 25
	0

1.7.5 Square root of product and quotient of numbers

For any two positive numbers a and b. we have

(i) $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$ and (ii) $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$ $(b \neq 0)$

Example 1.28

Find the value of $\sqrt{256}$.

Solution:

$\sqrt{256} = \sqrt{16 \times 16} = \sqrt{16} \times \sqrt{16} = 4 \times 4 = 16$ (or) $\sqrt{256} = \sqrt{64 \times 4} = \sqrt{64} \times \sqrt{4} = 8 \times 2 = 16$.

Try these

Find the square root of

1. 5.4756 2. 19.36 3. 116.64

Think

Try to fill in the blanks using $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$.

$\sqrt{36} = 6$	$\sqrt{9} \times \sqrt{4} = 3 \times 2 = 6$	Is $\sqrt{36} = \sqrt{9} \times \sqrt{4}$?	$\sqrt{81} = ?$	$\sqrt{9} \times \sqrt{9} = \_\_ \times \_\_ = \_\_$	Is $\sqrt{81} = \sqrt{9} \times \sqrt{9}$?
$\sqrt{144} = ?$	$\sqrt{9} \times \sqrt{16} = \_\_ \times \_\_ = \_\_$	Is $\sqrt{144} = \sqrt{9} \times \sqrt{16}$?	$\sqrt{144} = ?$	$\sqrt{36} \times \sqrt{4} = \_\_ \times \_\_ = \_\_$	Is $\sqrt{144} = \sqrt{36} \times \sqrt{4}$?
$\sqrt{100} = ?$	$\sqrt{25} \times \sqrt{4} = \_\_ \times \_\_ = \_\_$	Is $\sqrt{100} = \sqrt{25} \times \sqrt{4}$?	$\sqrt{1225} = ?$	$\sqrt{25} \times \sqrt{49} = \_\_ \times \_\_ = \_\_$	Is $\sqrt{1225} = \sqrt{25} \times \sqrt{49}$?

Activity

Attempt to prepare a table of square root problems as in the above case to show that if a and b are two perfect square numbers, then $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$ $(b \neq 0)$. We can use this idea to compute certain square-root problems easily.

Example 1.29

Find the value of $\sqrt{42.25}$.

Solution:

We can write this as $\sqrt{42.25} = \sqrt{\frac{4225}{100}} = \frac{\sqrt{4225}}{\sqrt{100}}$

Now, it is easy to compute the square root of the whole number 4225 by long division method as $\sqrt{4225} = 65$ and so, we now get $\sqrt{42.25} = \sqrt{\frac{4225}{100}} = \frac{\sqrt{4225}}{\sqrt{100}} = \frac{65}{10} = 6.5$

This is another way of tackling problems of square root of decimal numbers without any botheration of decimal symbol.

Try these

Using this method, find the square root of the numbers 1.2321 and 11.9025.

Example 1.30

Simplify: (i) $\sqrt{12} \times \sqrt{3}$ (ii) $\sqrt{\frac{98}{162}}$

Solution:

(i) Remembering the rule, $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$

$\sqrt{12} \times \sqrt{3} = \sqrt{12 \times 3} = \sqrt{36} = \sqrt{6^2} = 6$

(ii) Remembering the rule, $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$ $(b \neq 0)$

$\sqrt{\frac{98}{162}} = \sqrt{\frac{2 \times 49}{2 \times 81}} = \sqrt{\frac{49}{81}} = \sqrt{\frac{7^2}{9^2}} = \frac{7}{9}$

Example 1.31

Simplify: (i) $\sqrt{2\frac{7}{9}}$ (ii) $\sqrt{1\frac{9}{16}}$

Solution:

(i) $\sqrt{2\frac{7}{9}} = \sqrt{\frac{25}{9}} = \sqrt{\frac{5^2}{3^2}} = \frac{5}{3} = 1\frac{2}{3}$ (ii) $\sqrt{1\frac{9}{16}} = \sqrt{\frac{25}{16}} = \sqrt{\frac{5^2}{4^2}} = \frac{5}{4} = 1\frac{1}{4}$

Remark: In the case of the (ii) problem one may be tempted to give the answer immediately as $1\frac{3}{4}$, but this is not correct since you have to convert the mixed fraction into an improper fraction and then use the rule $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$ $(b \neq 0)$

1.7.6 Approximating square roots

Can you write the given numbers $\sqrt{40}$, 6 and 7 in ascending order? Here $\sqrt{40}$ is not a square number and so we cannot determine its root easily. However, we can estimate an approximation to $\sqrt{40}$ and use it here.

Try these

Write the numbers in ascending order.

1. 4, $\sqrt{14}$, 5 2. 7, $\sqrt{65}$, 8

We know that the two closest squares surrounding 40 are 36 and 49.

Thus, $36 < 40 < 49$ which can be written as $6^2 < 40 < 7^2$.

Considering the square root, we have $6 < \sqrt{40} < 7$.

Exercise 1.4

1. **Fill in the blanks:**
   (i) The ones digit in the square of 77 is_____.
   (ii) The number of non-square numbers between 24^2 and 25^2 is _____.
   (iii) The number of perfect square numbers between 300 and 500 is _____.
   (iv) If a number has 5 or 6 digits in it, then its square root will have _____ digits.
   (v) The value of $\sqrt{180}$ lies between integers _____ and _____.

2. **Say True or False:**
   (i) When a square number ends in 6, its square root will have 6 in the unit's place.
   (ii) A square number will not have odd number of zeros at the end.
   (iii) The number of zeros in the square of 91000 is 9.
   (iv) The square of 75 is 4925.
   (v) The square root of 225 is 15.

3. Find the square of the following numbers.
   (i) 17 (ii) 203 (iii) 1098

4. Examine if each of the following is a perfect square.
   (i) 725 (ii) 190 (iii) 841 (iv) 1089

5. Find the square root by prime factorisation method.
   (i) 144 (ii) 256 (iii) 784 (iv) 1156 (v) 4761 (vi) 9025

6. Find the square root by long division method.
   (i) 1764 (ii) 6889 (iii) 11025 (iv) 17956 (v) 418609

7. Estimate the value of the following square roots to the nearest whole number:
   (i) $\sqrt{440}$ (ii) $\sqrt{800}$ (iii) $\sqrt{1020}$

8. Find the square root of the following decimal numbers and fractions.
   (i) 2.89 (ii) 67.24 (iii) 2.0164 (iv) $\frac{144}{225}$ (v) $7\frac{18}{49}$

9. Find the least number that must be subtracted to 6666 so that it becomes a perfect square. Also, find the square root of the perfect square thus obtained.

10. Find the least number by which 1800 should be multiplied so that it becomes a perfect square. Also, find the square root of the perfect square thus obtained.

Objective Type Questions

11. The square of 43 ends with the digit _____.
   (A) 9 (B) 6 (C) 4 (D) 3

12. _____ is added to 24^2 to get 25^2.

(A) 4^2 (B) 5^2 (C) 6^2 (D) 7^2

13. $\sqrt{48}$ is approximately equal to _____.

(A) 5 (B) 6 (C) 7 (D) 8

14. $\sqrt{128} - \sqrt{98} + \sqrt{18}$ = _____.

(A) $\sqrt{2}$ (B) $\sqrt{8}$ (C) $\sqrt{48}$ (D) $\sqrt{32}$

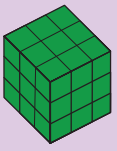
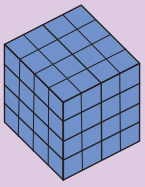
15. The number of digits in the square root of 123454321 is _____.

(A) 4 (B) 5 (C) 6 (D) 7

1.8 Cubes and Cube Roots

If you multiply a number by itself and then by itself again, the result is a cube number. This means that a cube number is a number that is the product of three identical numbers. If n is a number, its cube is represented by n^3.

Cube numbers can be represented visually as 3D cubes comprising of single unit cubes. Cube numbers are also called as perfect **cubes**. The perfect cubes of natural numbers are 1, 8, 27, 64, 125, 216, ... and so on.

Geometrical Representation	Product Representation	Notation	Perfect cube
	$1 \times 1 \times 1$	1^3	1
	$2 \times 2 \times 2$	2^3	8
	$3 \times 3 \times 3$	3^3	27
	$4 \times 4 \times 4$	4^3	64

Ramanujan Number - $1729 = 12^3+1^3 = 10^3+9^3$

Once Professor Hardy went to see Ramanujan when he was ill at Putney, riding in taxi cab number 1729 and said that the number seemed a dull one, and hoped it was not an unfavourable omen. "No," replied Ramanujan and he completed saying "It is a very interesting number. Infact, it is the smallest number expressible as the sum of two cubes in two different ways." 4104, 13832, 20683 are a few more examples of Ramanujan-Hardy numbers.

1.8.1 Properties of cubes of numbers

S. No.	Properties	Examples
1.	The cube of a positive number is positive.	$6^3 = 6\times6\times6 = 216.$
2.	The cube of a negative number is negative.	$(-7)^3 = (-7)\times(-7)\times(-7) = -343$
3.	The cube of every even number is even.	$8^3 = 8 \times 8 \times 8 = 512$ is even
4.	The cube of every odd number is odd.	$9^3 = 9 \times 9 \times 9 = 729$ is odd
5.	If a natural number ends with 0, 1, 4, 5, 6 or 9, its cube also ends with the same 0,1, 4, 5, 6 or 9 respectively.	$10^3 = 1000$, $11^3 = 1331$, $14^3 = 2744$ $15^3 = 3375$, $16^3 = 4096$, $19^3 = 6859$
6.	If a natural number ends with 2 or 8, its cube ends with 8 or 2 respectively.	$12^3 = 1728$, $18^3 = 5832$
7.	If a natural number ends with 3 or 7, its cube ends with 7 or 3 respectively.	$13^3 = 2197$, $17^3 = 4913$
8.	The sum of the cubes of first n natural numbers is equal to the square of their sum.	$1^3 + 2^3 + 3^3 + ... + n^3 = (1+2+3+.....+n)^2$ Check that, $1^3 + 2^3 + 3^3 + 4^3 = (1+2+3+4)^2$

Note

- A perfect cube does not end with two zeroes.
- The cube of a two digit number may have 4 or 5 or 6 digits in it.

Try these

Find the ones digit in the cubes of each of the following numbers.

1. 12 2. 27 3. 38
4. 53 5. 71 6. 84

1.8.2 Cube root

The cube root of a number is the value that when cubed gives the original number.

For example, the cube root of 27 is 3 because when 3 is cubed we get 27.

Notation:

The cube root of a number x is denoted as

$$\sqrt[3]{x} \text{ (or) } x^{\frac{1}{3}}.$$

Here are some more cubes and cube roots:

$\sqrt[3]{1} = 1$ since $1^3 = 1$, $\sqrt[3]{8} = 2$ since $2^3 = 8$,
$\sqrt[3]{27} = 3$ since $3^3 = 27$, $\sqrt[3]{64} = 4$ since $4^3 = 64$,
$\sqrt[3]{125} = 5$ since $5^3 = 125$ and so on.

Cubes	Cube Roots	Cubes	Cube Roots
1	1	729	9
8	2	1000	10
27	3	1331	11
64	4	1728	12
125	5	2197	13
216	6	2744	14
343	7	3375	15
512	8	4096	16

Example 1.32

Is 400 a perfect cube?

Solution:

By prime factorisation, we have $400 = \underline{2 \times 2 \times 2} \times 2 \times 5 \times 5$.

There is only one triplet. To make further triplets, we will need two more 2's and one more 5. Therefore, 400 is not a perfect cube.

Example 1.33

Find the smallest number by which 675 must be multiplied to obtain a perfect cube.

Solution:

3	675
3	225
3	75
5	25
5	1
	1

We find that, $675 = 3 \times 3 \times 3 \times 5 \times 5$ .....(1)

Grouping the prime factors of 675 as triplets, we are left over with 5×5.

We need one more 5 to make it a perfect cube.

To make 675 a perfect cube, multiply both sides of (1) by 5.

$675 \times 5 = 3 \times 3 \times 3 \times 5 \times 5 \times 5$

$3375 = 3 \times 3 \times 3 \times 5 \times 5 \times 5$

Now, 3375 is a perfect cube. Thus, the smallest required number to multiply 675 such that the new number perfect cube is 5.

Think

In this question, if the word 'multiplied' is replaced by the word 'divided', how will the solution vary?

1.8.3 Cube root of a given number by Prime Factorisation

Step 1: Resolve the given number into the product of prime factors.

Step 2: Make triplet groups of same primes.

Step 3: Choosing one from each triplet, find the product of primes to get the cube root.

Example 1.34

Find the cube root of 27000.

Solution:

By prime factorisation, we have $27000 = 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 5 \times 5 \times 5$

$$\therefore \sqrt[3]{27000} = 2 \times 3 \times 5 = 30.$$

Example 1.35

Evaluate: (i) $\sqrt[3]{\frac{9261}{8000}}$ (ii) $\sqrt[3]{\frac{1728}{729}}$

Solution:

(i) $\sqrt[3]{\frac{9261}{8000}} = \frac{\sqrt[3]{9261}}{\sqrt[3]{8000}} = \frac{(21 \times 21 \times 21)^{\frac{1}{3}}}{(20 \times 20 \times 20)^{\frac{1}{3}}} = \frac{\left(21^3\right)^{\frac{1}{3}}}{\left(20^3\right)^{\frac{1}{3}}} = \frac{21}{20} = 1\frac{1}{20}$

(ii) $\sqrt[3]{\frac{1728}{729}} = \frac{\sqrt[3]{1728}}{\sqrt[3]{729}} = \frac{(12 \times 12 \times 12)^{\frac{1}{3}}}{(9 \times 9 \times 9)^{\frac{1}{3}}} = \frac{\left(12^3\right)^{\frac{1}{3}}}{\left(9^3\right)^{\frac{1}{3}}} = \frac{12}{9} = \frac{4}{3} = 1\frac{1}{3}$

Exercise 1.5

1. **Fill in the blanks:**
   (i) The ones digits in the cube of 73 is_____.
   (ii) The maximum number of digits in the cube of a two digit number is _____.
   (iii) The smallest number to be added to 3333 to make it a perfect cube is _____.
   (iv) The cube root of 540×50 is _____.
   (v) The cube root of 0.000004913 is _____.

2. **Say True or False:**
   (i) The cube of 24 ends with the digit 4.
   (ii) Subtracting 10^3 from 1729 gives 9^3.
   (iii) The cube of 0.0012 is 0.000001728.
   (iv) 79570 is not a perfect cube.
   (v) The cube root of 250047 is 63.
3. Show that 1944 is not a perfect cube.
4. Find the smallest number by which 10985 should be divided so that the quotient is a perfect cube.
5. Find the smallest number by which 200 should be multiplied to make it a perfect cube.
6. Find the cube root of $24 \times 36 \times 80 \times 25$.
7. Find the cube root of 729 and 6859 by prime factorisation.
8. What is the square root of cube root of 46656?
9. If the cube of a squared number is 729, find the square root of that number.
10. Find the two smallest perfect square numbers which when multiplied together gives a perfect cube number.

Activity

Observe that

$2^3 - 1^3 = 1 + 2 \times 1 \times 3$

$3^3 - 2^3 = 1 + 3 \times 2 \times 3$

$4^3 - 3^3 = 1 + 4 \times 3 \times 3$

Find the value of $15^3 - 14^3$ in the above pattern.

Observe that

$1^3 = 1 = 1$

$2^3 = 8 = 3 + 5$

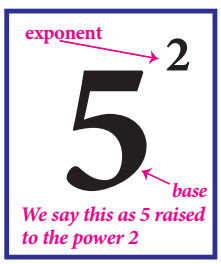
$3^3 = 27 = 7 + 9 + 11$

Continue this pattern to find the value of 7^3 as the sum of consecutive odd numbers.

1.9 Exponents and Powers

We know how to express some numbers as squares and cubes. For example, we write 5^2 for 25 and 5^3 for 125.

In general terms, an expression that represents repeated multiplication of the same factor is called a **power**. The number 5 is called the **base** and the number 2 is called the **exponent** (more often called as power). The exponent corresponds to the number of times the base is used as a factor.



1.9.1 Powers with positive exponents

Value of powers given by positive whole number exponents quite often increase rapidly. Observe the following example:

$2^1 = 2$

$2^2 = 2 \times 2 = 4$

$2^3 = 2 \times 2 \times 2 = 8$

$2^4 = 2 \times 2 \times 2 \times 2 = 16$

$2^5 = 2 \times 2 \times 2 \times 2 \times 2 = 32$

$2^6 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 64$

$2^7 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 128$

$2^8 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 256$

$2^9 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 512$

$2^{10} = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 1024$

At this rate of increase, what do you think 2^{100} will be?

In fact, $2^{100} = 1267650600228229401496703205376$

Thus, we understand that the positive exponential notation with positive power could be useful when we come across with large numbers.

1.9.2 Powers with zero and negative exponents

Observe this pattern:

$2^5 = 32$

$2^4 = 16$

$2^3 = 8$

$2^2 = 4$

$2^1 = 2$

$2^0 = ?$

Starting from the beginning, what happens in the successive steps? We find that the result is half of that of the previous step. So, what can we say about 2^0 ? If we prepare a table like this for 3^5, 3^4, 3^3, and so on what will it tell us about 3^0? We can use the same process as in this pattern, to conclude that any non-zero number raised to the zero exponent must result in 1. Thus,

$a^0 = 1$, where $a \neq 0$

Let us see what happens if we extend the above pattern further downward.

As before, starting from the beginning, in the successive steps, we find that the result is half of that of the previous step. Since $2^0 = 1$, the next step is 2^{-1}, whose value is the previous step's value 1, divided by 2, that is $\frac{1}{2}$. Next is 2^{-2}, which is the same as $\frac{1}{2}$ divided by 2, that is $\frac{1}{4}$ and so on. Thus,

$2^3 = 8$

$2^2 = 4$

$2^1 = 2$

$2^0 = 1$

$2^{-1} = \frac{1}{2}$

$2^{-2} = \frac{1}{4}$

$2^{-3} = \frac{1}{8}$

In general, $a^{-m} = \frac{1}{a^m}$, where m is an integer

1.9.3 Expanded form of numbers using exponents

In the lower classes, we have learnt how to write a whole number in the expanded form.

For example, $5832 = 5\times1000 + 8\times100 + 3\times10 + 2\times1$

$= 5\times10^3 + 8\times10^2 + 3\times10^1 + 2$ (when we use exponential notation).

What shall we do if we get decimal places? Powers of 10 with negative exponents come to our rescue!

Thus, $58.32 = 50 + 8 + \frac{3}{10} + \frac{2}{100}$

$= 5\times10 + 8\times1 + 3\times\frac{1}{10} + 2\times\frac{1}{100}$

$= 5\times10^1 + 8\times10^0 + 3\times10^{-1} + 2\times10^{-2}$

Try these

Expand the following numbers using exponents:

1. 8120
2. 20305
3. 3652.01
4. 9426.521

1.9.4 Laws of Exponents

Laws of exponents arise out of certain basic ideas. A **positive exponent** of a number indicate how many times we use that number in a multiplication whereas a **negative exponent** suggests us how many times we use that number in a division, since the opposite of multiplying is dividing.

- **Product law**

According to this law, when multiplying two powers that have the same base, we can add the exponents. That is,

$$a^m \times a^n = a^{m+n}$$

where a ($a\neq0$), m, n are integers. Note that the base should be the same in both the quantities.

Examples:

a)	$2^3\times2^2 = 2^5$ by law(meaning $8\times4 = 32$ and note that it is not $2^{3\times2}$)
b)	$(-2)^{-4}\times(-2)^{-3} = (-2)^{(-4)+(-3)}$ by law and so $= (-2)^{-7}$ (or) $(-2)^{-4}\times(-2)^{-3} = \frac{1}{(-2)^4}\times\frac{1}{(-2)^3} = \frac{1}{(-2)^4\times(-2)^3} = \frac{1}{(-2)^{4+3}} = \frac{1}{(-2)^7} = (-2)^{-7}$
c)	$(-5)^3\times(-5)^{-3} = (-5)^{3-3}$ by law and so $= (-5)^0 = 1$

- **Quotient law**

According to this law, when dividing two powers that have the same base we can subtract the exponents. That is,

$$\frac{a^m}{a^n} = a^{m-n}$$

where a ($a \neq 0$), m, n are integers. Note that the base should be the same in both the quantities. How does it work? Study the following examples.

Examples:

a) $\frac{(-3)^5}{(-3)^2} = (-3)^{5-2}$ by law and $(-3)^3 = -27$

(or)

$\frac{(-3)^5}{(-3)^2} = (-3)^5 \times (-3)^{-2} = (-3)^{5-2} = (-3)^3$

(or)

$\frac{(-3)^5}{(-3)^2} = \frac{(-3)\times(-3)\times(-3)\times(-3)\times(-3)}{(-3)\times(-3)} = (-3)\times(-3)\times(-3) = (-3)^3$

b) $\frac{(-7)^{100}}{(-7)^{98}} = (-7)^{100-98}$ by law and $(-7)^2 = 49$

(or)

$\frac{(-7)^{100}}{(-7)^{98}} = \frac{(-7)\times(-7)\times(-7)\times \ldots 100 \text{ times}}{(-7)\times(-7)\times(-7)\times \ldots 98 \text{ times}} = (-7)\times(-7) = 49$

• Power law

According to this law, when raising a power to another power, we can just multiply the exponents.

$$(a^m)^n = a^{mn}$$

where a ($a \neq 0$), m, n are integers.

Examples:

$[(-2)^3]^2 = (-2)^{3\times2}$ by law and $(-2)^6 = 64$

(or)

$[(-2)^3]^2 = [(-2)\times(-2)\times(-2)]^2 = [-8]^2 = 64$

Try these

Verify the following rules (as we did above). Here, a,b are non-zero integers and m, n are any integers.

1. Product of same powers to power of product rule: $a^m \times b^m = (ab)^m$
2. Quotient of same powers to power of quotient rule: $\frac{a^m}{b^m} = \left(\frac{a}{b}\right)^m$
3. Zero exponent rule: $a^0 = 1$.

Example 1.36

Find the value of (i) 4^{-3} (ii) $\frac{1}{2^{-3}}$ (iii) $(-2)^5 \times (-2)^{-3}$ (iv) $\frac{3^2}{3^{-2}}$

Solution:

(i) $4^{-3} = \frac{1}{4^3} = \frac{1}{4\times4\times4} = \frac{1}{64}$ (ii) $\frac{1}{2^{-3}} = 2^3 = 2\times2\times2 = 8$

(iii) $(-2)^5 \times (-2)^{-3} = (-2)^{5-3} = (-2)^2 = -2 \times -2 = 4$ (iv) $\frac{3^2}{3^{-2}} = 3^2 \times 3^2 = 9 \times 9 = 81$

Example 1.37

Simplify and write the answer in exponential form:

(i) $(3^5 \div 3^8)^5 \times 3^{-5}$ (ii) $(-3)^4 \times \left(\frac{5}{3}\right)^4$

Solution:

(i) $\left(\frac{3^5}{3^8}\right)^5 \times 3^{-5} = (3^{5-8})^5 \times 3^{-5} = (3^{-3})^5 \times 3^{-5} = 3^{-3\times5} \times 3^{-5} = 3^{-15} \times 3^{-5} = 3^{-15-5} = 3^{-20}$

(ii) $(-3)^4 \times \left(\frac{5}{3}\right)^4 = 3^4 \times \frac{5^4}{3^4} = 5^4$

Example 1.38

Find x so that $(-7)^{x+2} \times (-7)^5 = (-7)^{10}$

Solution:

$$(-7)^{x+2} \times (-7)^5 = (-7)^{10}$$

$$(-7)^{x+2+5} = (-7)^{10}$$

Since the bases are equal, we equate the exponents to get

$$x + 7 = 10$$

$$\Rightarrow x = 10 - 7 = 3$$

1.9.5 Standard Form and Scientific Notation

Standard form of a number is just the number as we normally write it. We use expanded notation to show the value of each digit. That is, it is exhibited as a sum of each digit duly multiplied by its matching place value (like ones, tens, hundreds etc.,). For example, **195** is in standard form. It can be expanded as **$195 = 1 \times 100 + 9 \times 10 + 5 \times 1$**.

Astronomers, biologists, engineers, physicists and many others come across quantities whose measures require very small or very large numbers. If they write the numbers in standard form, it may not help us to understand or make computations easily. **Scientific notation** is a way to make these numbers easier to work with.

To write in scientific notation, follow the form **$S \times 10^a$**. where **S** is a number (integer or integer with decimal) between 1 and 10, but not 10 itself, and a is a positive or negative integer. Thus, a number in scientific notation is written as the product of a number (integer or integer with decimal) and a power of 10. We move the decimal place forward or backward until we have a number from 1 to 9. Then, we add a power of ten that tells how many places you moved the decimal forward or backward.

Examples:

Standard Form	Scientific Notation	Standard Form	Scientific Notation
0.00123	1.23×10^{-3}	123	1.23×10^{2}
0.0123	1.23×10^{-2}	1230	1.23×10^{3}
0.123	1.23×10^{-1}	12300	1.23×10^{4}
1.23	1.23×10^{0}	123000	1.23×10^{5}
12.3	1.23×10^{1}	1230000	1.23×10^{6}

Some more examples:

(a) The diameter of the earth is 12756000 miles. This can be easily written in scientific form as 1.2756×10^{7} miles.

(b) The volume of Jupiter is about 143300000000000 km^3. This can be easily written in scientific form as 1.433×10^{14} km^3.

(c) The size of a bacterium is 0.00000085 mm. This can be easily written in scientific form as 8.5×10^{-7} mm.

Note

1. The positive exponent in 1.3×10^{12} indicates that it is a large number.
2. The negative exponent in 7.89×10^{-21} indicates that it is a small number.

Example 1.39

Combine the scientific notations: (i) $\left(7 \times 10^{2}\right)\left(5.2 \times 10^{7}\right)$ (ii) $\left(3.7 \times 10^{-5}\right)\left(2 \times 10^{-3}\right)$

Solution:

(i) $\left(7 \times 10^{2}\right)\left(5.2 \times 10^{7}\right) = 36.4 \times 10^{9} = 3.64 \times 10^{10}$

(ii) $\left(3.7 \times 10^{-5}\right)\left(2 \times 10^{-3}\right) = 7.4 \times 10^{-8}$

Example 1.40

Write the following scientific notations in standard form:
(i) 2.27×10^{-4} (ii) Light travels at 1.86×10^{5} miles per second.

Solution:

(i) $2.27 \times 10^{-4} = 0.000227$.

(ii) Light travels at 1.86×10^{5} miles per second = 186000 miles per second

Try these

1. Write in standard form: Mass of planet Uranus is 8.68×10^{25} *kg*.
2. Write in scientific notation: (i) 0.000012005 (ii) 4312.345 (iii) 0.10524
   (iv) The distance between the Sun and the planet Saturn 1.4335×10^{12} miles.

Exercise 1.6

1. **Fill in the blanks:**

   (i) $(-1)^{\text{even integer}}$ is _____.

   (ii) For $a \neq 0$, a^0 is _____.

   (iii) $4^{-3} \times 5^{-3} =$ _____.

   (iv) $(-2)^{-7} =$ _____.

   (v) $\left(-\frac{1}{3}\right)^{-5} =$ _____.

2. **Say True or False:**

   (i) If $8^x = \frac{1}{64}$, the value of x is -2.

   (ii) The simplified form of $(256)^{\frac{-1}{4}} \times 4^2$ is $\frac{1}{4}$.

   (iii) Using the power rule, $\left(3^7\right)^{-2} = 3^5$.

   (iv) The standard form of 2×10^{-4} is 0.0002.

   (v) The scientific form of 123.456 is 1.23456×10^{-2}.

3. Evaluate: (i) $\left(\frac{1}{2}\right)^3$ (ii) $\left(\frac{1}{2}\right)^{-5}$ (iii) $\left(\frac{-5}{6}\right)^{-3}$ (iv) $\left(2^{-5} \times 2^7\right) \div 2^{-2}$ (v) $\left(2^{-1} \times 3^{-1}\right) \div 6^{-2}$

4. Evaluate: (i) $\left(\frac{2}{5}\right)^4 \times \left(\frac{5}{2}\right)^{-2}$ (ii) $\left(\frac{4}{5}\right)^{-2} \div \left(\frac{4}{5}\right)^{-3}$ (iii) $2^7 \times \left(\frac{1}{2}\right)^{-3}$

5. Evaluate: (i) $\left(5^0 + 6^{-1}\right) \times 3^2$ (ii) $\left(2^{-1} + 3^{-1}\right) \div 6^{-1}$ (iii) $\left(3^{-1} + 4^{-2} + 5^{-3}\right)^0$

6. Simplify: (i) $(3^2)^3 \times (2 \times 3^5)^{-2} \times (18)^2$ (ii) $\frac{9^2 \times 7^3 \times 2^5}{84^3}$ (iii) $\frac{2^8 \times 2187}{3^5 \times 32}$

7. Solve for x : (i) $\frac{2^{2x-1}}{2^{x+2}} = 4$ (ii) $\frac{5^5 \times 5^{-4} \times 5^x}{5^{12}} = 5^{-5}$

8. Expand using exponents: (i) 6054.321 (ii) 897.14

9. Find the number in standard form for the following expansions:

   (i) $8 \times 10^4 + 7 \times 10^3 + 6 \times 10^2 + 5 \times 10^1 + 2 \times 1 + 4 \times 10^{-2} + 7 \times 10^{-4}$

   (ii) $5 \times 10^3 + 5 \times 10^1 + 5 \times 10^{-1} + 5 \times 10^{-3}$

   (iii) The radius of a hydrogen atom is 2.5×10^{-11} m

10. Write the following numbers in scientific notation:

    (i) 467800000000 (ii) 0.000001972 (iii) 1642.398

    (iv) Earth's volume is about 1,083,000,000,000 cubic kilometres

    (v) If you fill a bucket with dirt, the portion of the whole Earth that is in the bucket will be 0.00000000000000000000000016 kg

Objective Type Questions

11. By what number should $(-4)^{-1}$ be multiplied so that the product becomes 10^{-1}?

(A) $\frac{2}{3}$ (B) $\frac{-2}{5}$ (C) $\frac{5}{2}$ (D) $\frac{-5}{2}$

12. $(-2)^{-3} \times (-2)^{-2} =$ _____.

(A) $\frac{-1}{32}$ (B) $\frac{1}{32}$ (C) 32 (D) –32

13. Which is not correct?

(A) $\left(\frac{-1}{4}\right)^2 = 4^{-2}$ (B) $\left(\frac{-1}{4}\right)^2 = \left(\frac{1}{2}\right)^4$ (C) $\left(\frac{-1}{4}\right)^2 = 16^{-1}$ (D) $-\left(\frac{1}{4}\right)^2 = 16^{-1}$

14. If $\frac{10^x}{10^{-3}} = 10^9$, then x is _____.

(A) 4 (B) 5 (C) 6 (D) 7

15. 0.0000000002020 in scientific form is _____.

(A) 2.02×10^9 (B) 2.02×10^{-9} (C) 2.02×10^{-8} (D) 2.02×10^{-10}

Exercise 1.7

Miscellaneous Practice Problems

1. If $\frac{3}{4}$ of a box of apples weighs 3 kg and 225 gm, how much does a full box of apples weigh?
2. Mangalam buys a water jug of capacity $3\frac{4}{5}$ litre. If she buys another jug which is $2\frac{2}{3}$ times as large as the smaller jug, how many litre can the larger one hold?
3. Ravi multiplied $\frac{25}{8}$ and $\frac{16}{15}$ and he says that the simplest form of this product is $\frac{10}{3}$ and Chandru says the answer in the simplest form is $3\frac{1}{3}$. Who is correct? (or) Are they both correct? Explain.
4. Find the length of a room whose area is $\frac{153}{10}$ sq.m and whose breadth is $2\frac{11}{20}$ m.
5. There is a large square portrait of a leader that covers an area of 4489 cm^2. If each side has a 2 cm liner, what would be its area?
6. A greeting card has an area 90 cm^2. Between what two whole numbers is the length of its side?
7. 225 square shaped mosaic tiles, each of area 1 square decimetre exactly cover a square shaped verandah. How long is each side of the square shaped verandah?
8. If $\sqrt[3]{1906624} \times \sqrt{x} = 3100$, find x.
9. If $2^{m-1} + 2^{m+1} = 640$, then find m.

10. Give the answer in scientific notation:
A human heart beats at an average of 80 beats per minute. How many times does it beat in i) an hour? ii) a day? iii) a year? iv) 100 years?

Challenging Problems

11. In a map, if 1 inch refers to 120 km, then find the distance between two cities B and C which are $4\frac{1}{6}$ inches and $3\frac{1}{3}$ inches from the city A which lies between the cities B and C.

12. Give an example and verify each of the following statements.
    (i) The collection of all non-zero rational numbers is closed under division.
    (ii) Subtraction is not commutative for rational numbers.
    (iii) Division is not associative for rational numbers.
    (iv) Distributive property of multiplication over subtraction is true for rational numbers. That is, $a(b-c)=ab-ac$.
    (v) The mean of two rational numbers is rational and lies between them.

13. If $\frac{1}{4}$ of a *ragi adai* weighs 120 grams, what will be the weight of $\frac{2}{3}$ of the same *ragi adai*?

14. If $p+2q=18$ and $pq=40$, find $\frac{2}{p}+\frac{1}{q}$.

15. Find x if $5\frac{x}{5}\times 3\frac{3}{4}=21$.

16. By how much does $\frac{1}{\left(\frac{10}{11}\right)}$ exceed $\frac{\left(\frac{1}{10}\right)}{11}$?

17. A group of 1536 cadets wanted to have a parade forming a square design. Is it possible? If it is not possible, how many more cadets would be required?

18. Evaluate: $\sqrt{286225}$ and use it to compute $\sqrt{2862.25}+\sqrt{28.6225}$

19. Simplify: $(3.769\times 10^5)+(4.21\times 10^5)$

20. Order the following from the least to the greatest: 16^{25}, 8^{100}, 3^{500}, 4^{400}, 2^{600}

SUMMARY

- A number that can be expressed in the form $\frac{a}{b}$ where a and b are integers and $b\neq 0$ is called a rational number.
- All natural numbers, whole numbers, integers and fractions are rational numbers.
- Every rational number can be represented on a number line.
- 0 is neither a positive nor a negative rational number.
- A rational number $\frac{a}{b}$ is said to be in the standard form if its denominator b is a positive integer and HCF $(a,b)=1$

- There are unlimited numbers of rational numbers between two rational numbers.
- Subtracting two rational numbers is the same as adding the additive inverse of the second number to the first rational number.
- Multiplying two rational numbers is the same as multiplying their numerators and denominators separately and then writing the product in the standard form.
- Dividing a rational number by another rational number is the same as multiplying the first rational number by the reciprocal of the second rational number.
- The following table is about the properties of rational numbers($\mathbb{Q}$).

$\mathbb{Q}$	Closure	Commutative	Associative	Multiplication is distributive over +/−
+	✓	✓	✓	✓
−	✓	×	×	✓
×	✓	✓	✓	-
÷	×	×	×	-

- 0 and 1 are respectively the additive and the multiplicative identities of rational numbers.
- The additive inverse for $\frac{a}{b}$ is $\frac{-a}{b}$ and vice – versa.
- The reciprocal or the multiplicative inverse of a rational number $\frac{a}{b}$ is $\frac{b}{a}$ since $\frac{a}{b}\times\frac{b}{a}=1$.
- A natural number n is called a square number, if we can find another natural number m such that $n = m^2$.
- The square root of a number n, written as $\sqrt{n}$ (or) $n^{\frac{1}{2}}$, is the number that gives n when multiplied by itself.
- The number of times a prime factor occurs in the square is equal to twice the number of times it occurs in the prime factorization of the number.
- For any two positive numbers a and b. we have

  (i) $\sqrt{ab} = \sqrt{a}\times\sqrt{b}$ and (ii) $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}\ (b\neq 0)$
- If you multiply a number by itself and then by itself again, the result is a cube number.
- The cube root of a number is the value that when cubed gives the original number.
- An expression that represents repeated multiplication of the same factor is called a power.
- The exponent corresponds to the number of times the base is used as a factor.
- Laws of Exponents: (i) $a^m \times a^n = a^{m+n}$ (ii) $\frac{a^m}{a^n} = a^{m-n}$ (iii) $(a^m)^n = a^{mn}$
- Other results: (i) $a^0 = 1$ (ii) $a^{-m} = \frac{1}{a^m}$ (iii) $a^m \times b^m = (ab)^m$ (iv) $\frac{a^m}{b^m} = \left(\frac{a}{b}\right)^m$
- To write in scientific notation, we follow the form $S \times 10^a$ where S is a number (integer or integer with decimal) between 1 and 10, but not 10 itself, and a is a positive or negative integer.

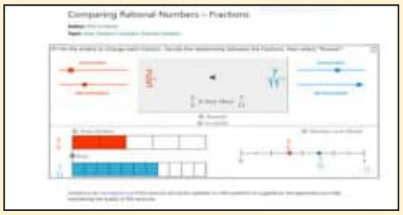
ICT CORNER

Step-1 Open the Browser and type www.Geogebra.com (or) scan the QR CODE given below.

Step-2 Type Rational Numbers on the search column

Step-3 Move the numerator and denominator slide to show the different rational numbers and click the number line models to show the number line.

Through this activity you will know about the rational numbers, operations on them and study their properties as well.



4VPX3

Step 1

Step 2

Web URL rational numbers (rational number links)
Web URL https://www.geogebra.org/m/n92AKzBF#material/ca5D7VbZ
*Pictures are indicatives only.
*If browser requires, allow Flash Player or Java Script to load the page.

ICT CORNER

Step-1 Open the Browser type the URL Link given below (or) Scan the QR Code. GeoGebra work sheet named "8th Standard III term" will open. Select the work sheet named "Square root_prime factors"

Step-2 Click on " NEW PROBLEM" and check the calculation.

Expected Outcome

7W8MC

Step 1

Step 2

Browse in the link

Numbers:

https://www.geogebra.org/m/xmm5kj9r or Scan the QR Code.

2 MEASUREMENTS

Learning Objectives

- To know the parts of a circle.
- To calculate the length of arc, area and circumference of a sector.
- To calculate the area and the perimeter of combined plane figures.
- To understand the representation of 3-D shapes in 2-D.
- To understand the representation of 3-D objects with cubes.

NK1H0

2.1 Introduction

An important aspect in everyone's day-to-day life is **'to measure'.** Measuring the length of a rope, the distance between two places, finding the perimeter and area of plots and lands, building the structures under specified measures etc., are a few of the many situations where the concept of 'measure' is used.

It is said that the biggest invention by man is the wheel. The wheel is otherwise called as the **'cradle of invention'.** What is the shape of a wheel? Circle, isn't it? In the things we use, apart from circles, we can also see different shapes like triangles, squares, rectangles etc.,

Measurements play a vital role in everyone's life. Not only the persons who learns proper mathematics in schools, but also the layman uses his logical thinking to apply the concept of measurements when he needs. For example, a carpenter who carves out the wooden wheels of a temple car wants to protect the outer surface by an iron strap. With all his experience, he says that a 22 feet strap will be required for a 7 feet high (diameter) wooden wheel.

MATHEMATICS ALIVE – AREA AND PERIMETER IN REAL LIFE

Sectors are used in manufacturing the dining table

Combined shapes are used in the construction of building

We have already learnt how to find the perimeter and the area of shapes like circles, triangles, squares, rectangles, trapeziums, parallelograms etc. In this chapter, we shall see the parts of a circle and how to find the perimeter and the area of a sector and some combined shapes.

Recap

The teacher asks the students to calculate the area of the circle of radius 7 *cm*. Many students have solved it as in Method 1, but a few of them have done it as in Method 2.

Method 1:	**Method 2:**
Area of the circle, A $= \pi r^2$ *sq.units*	Area of the circle, A $= \pi r^2$ *sq.units*
$= \frac{22}{7} \times 7 \times 7$	$= 3.14 \times 7 \times 7$
$= 154$ *sq.cm*	$= 153.86$ *sq.cm*

Then, they had the following conversation:

Sudha : Which of these two answers is correct, Teacher ?

Teacher : Both answers are correct but only approximate.

Sudha : We get two answers for the same question. How is it possible, teacher?

Teacher : We can infact solve this exactly as $\pi \times 7 \times 7 = 49\pi$ *sq.cm.*

Meena : When we find the area of a square, a rectangle etc., we all get unique answers, but why is the area of a circle not unique ?

Teacher : What is the value of π ?

Meena : The value of π is $\frac{22}{7}$ or 3.14

Teacher : Actually it is neither $\frac{22}{7}$ nor 3.14. They are only approximate values.

Meena : Then, what is the exact value of π?

Teacher : Though π is the constant, it is a non-terminating and non-recurring decimal number. So, we are not able to use its exact value and to make the calculations easy, $\frac{22}{7}$ or 3.14 is used as the approximate value of π.

Meena : Is the area of a circle always approximate, teacher?

Teacher : No, it is an exact answer unless we substitute the value of π as $\frac{22}{7}$ or 3.14.

Meena : Then, for the above problem, 49π *sq.cm* is an exact answer whereas 154 *sq.cm* and 153.86 *sq.cm.* are approximate answers. Am I right, teacher?

Teacher : Yes, you are right Meena.

Think

1. $\frac{22}{7}$ and 3.14 are rational numbers. Is 'π' a rational number? Why?
2. When is the 'π' day celebrated? Why?

DO YOU KNOW?

The circumference of a circle is $2\pi r$ *units*, which can also be written as πd *units*. As $\pi = 3.14$ (approximately) and it is slightly more than 3, for some quick guess, we shall say that a circle whose diameter is *d units* shall have its circumference slightly more than three times its diameter.

For example,

If a round table of diameter of 3*m* is to be decorated by flower strings around it, it will require a little more than 9*m* of flower strings.

2.2 Parts of a Circle

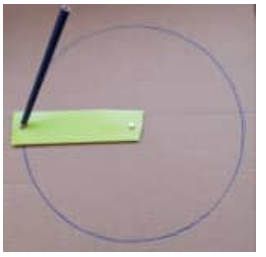


Fig. 2.1

A circle is the path traced by a moving point so that its distance from a fixed point is always constant. The fixed point of the circle is called its **'centre'** and the constant distance is called its **'radius'.**

Further, if any two points on a circle are joined by a line segment, then the line segment is called a **'chord'.** A chord divides a circle into two parts. A chord which passes through the centre of a circle is called as a **'diameter'.** A diameter of a circle divides it into two equal parts. It is also the 'longest chord' of a circle.

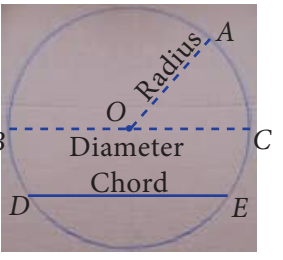


Fig. 2.2

Activity

1. Using a bangle, draw a circle on the paper and cut it. Then mark any two points A and B on it and fold the circle so that the fold has A and B on it. Now, this line segment represents a chord.
2. By paper folding, find two diameters and hence the centre of a circle.
3. Check whether the diameter of a circle is twice its radius.

2.2.1 Circular Arc and Circular Sector

Look at the glass bangle and the pizza given in Fig. 2.3 carefully.

Fig. 2.3

Though both are of circular shapes, what we understand is that, the bangle indicates the boundary of the circle, whereas the pizza indicates the plane enclosed within the boundary of the circle. Thus, it is clear that the bangle indicates the circumference of the circle whereas the pizza indicates the area of the circle.

From them, cut a part as shown in Fig. 2.4.

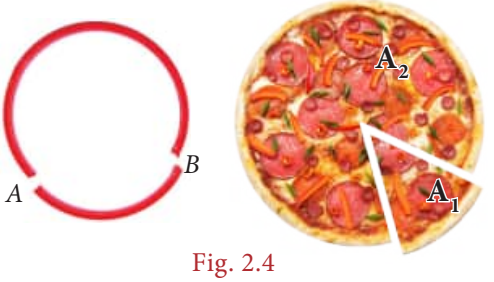


Fig. 2.4

Each of the glass bangle pieces represents the circular arcs. Here, arc AB $\left(\overset{\frown}{AB}\right)$ is the smaller one and

arc BA ($\overset{\frown}{BA}$) is the larger one. Each of the parts of the pizza represents the circular sectors. Here A_1 is the minor sector and A_2 is the major sector.

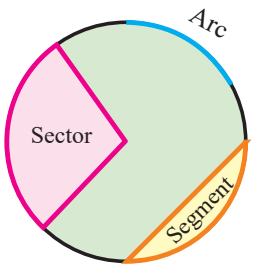
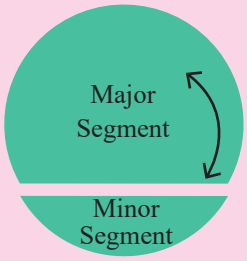


Fig. 2.5

- A part of the circumference of a circle is called a circular arc.
- The plane surface that is enclosed between two radii and the circular arc of a circle is called a sector.
- Each part of a circle which is divided by a chord is called a segment.

Note

The part which has a smaller arc is called as the **'minor segment'** and the part which has a larger arc is called as the **'major segment'.**



Think

The given circular figure is divided into six equal parts. Can we call the equal parts as sectors? Why?

Fig. 2.6

Central Angle

The angle formed by a sector of a circle at its centre is called the central angle. The vertex of the central angle of the sector is the centre of the circle. The two arms of it are the radii. In the Fig. 2.7, the shaded sector has the central angle, $\angle AOB = \theta^\circ$ (read as theta) and its two arms OA and OB are the radii of the circle.

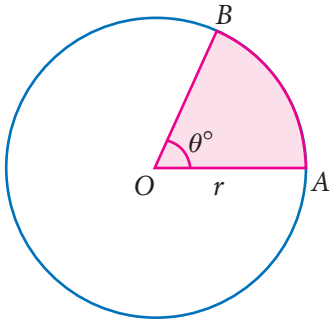


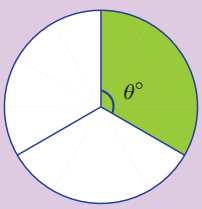
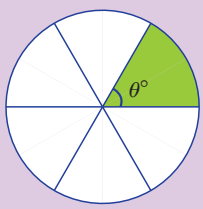
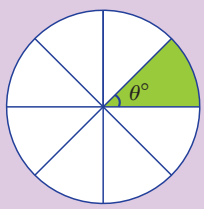
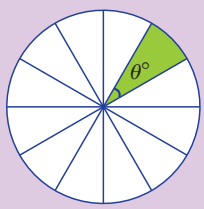
Fig. 2.7

The central angle of a circle is 360°. If a circle is divided into 'n' equal sectors, the central angle of each of the sectors is $\theta^\circ = \frac{360^\circ}{n}$.

For example, the central angle of a semicircle $= \frac{360^\circ}{2} = 180^\circ$ and the central angle of a quadrant of a circle $= \frac{360^\circ}{4} = 90^\circ$.

Try these

Fill the central angle of the shaded sector (each circle is divided into equal sectors)

Sectors				
Central angle $\theta^\circ \left(= \frac{360^\circ}{n}\right)$	$\theta^\circ = 120^\circ$			

2.2.2 Length of the arc and Area of the sector

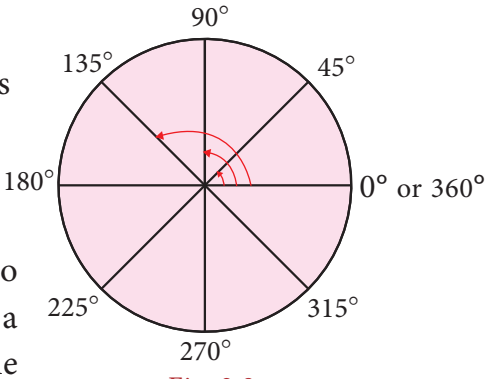


Fig. 2.8

We have already learnt that a circle of radius '*r*' *units* has

Central angle = 360°

Circumference of the circle = $2\pi r$ *units*

Area of the circle = πr^2 *Sq.units.*

First, let us take a circle . If a circle is divided into 2 equal sectors we will get 2 semi-circles. The length of a semi-circular arc is half of the circumference of the circle and the area of the semi-circle is half of the area of the circle.

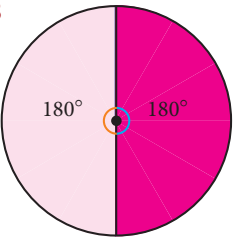


Fig. 2.9

Length of the semi-circular arc $= \frac{1}{2} \times 2\pi r = \frac{180^\circ}{360^\circ} \times 2\pi r$ *units.*

Area of the semi-circle $= \frac{1}{2} \times \pi r^2 = \frac{180^\circ}{360^\circ} \times \pi r^2$ *sq.units.*

Smilarly, if a circle is divided into 3 equal sectors,

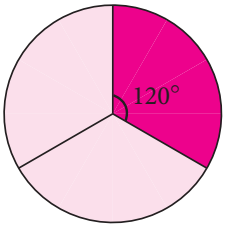


Fig. 2.10

Length of the arc of this sector $= \frac{1}{3} \times 2\pi r = \frac{120^\circ}{360^\circ} \times 2\pi r$ *units*

Area of the sector $= \frac{1}{3} \times \pi r^2 = \frac{120^\circ}{360^\circ} \times \pi r^2$ *sq. units.*

In the same way, if a circle is divided into 4 equal sectors,

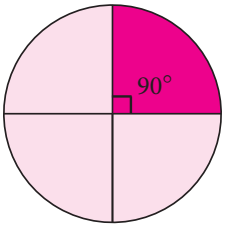


Fig. 2.11

Length of the arc of the circular quadrant $= \frac{1}{4} \times 2\pi r = \frac{90^\circ}{360^\circ} \times 2\pi r$ *units.*

Area of the quadrant $= \frac{1}{4} \times \pi r^2 = \frac{90^\circ}{360^\circ} \times \pi r^2$ *sq.units*

What do we know from this?

If the ratio of the central angle of a sector to the central angle of a circle is multiplied with the circumference and the area of the circle, we can find the length of the arc of that sector and its area respectively.

That is, if we assume that the central angle of a sector of radius '*r*' *units* as θ°, then, the ratio of the central angle θ° to 360° is $\frac{\theta^\circ}{360^\circ}$.

Length of the arc, $l = \frac{\theta^\circ}{360^\circ} \times 2\pi r$ *units*

Area of the sector, $A = \frac{\theta^\circ}{360^\circ} \times \pi r^2$ *sq.units.*

Think

Instead of multiplying by $\frac{1}{2}$, $\frac{1}{3}$ and $\frac{1}{4}$, we shall multiply by $\frac{180^\circ}{360^\circ}$, $\frac{120^\circ}{360^\circ}$ and $\frac{90^\circ}{360^\circ}$ respectively. Why?

Note

1. If a circle of radius r units divided into n equal sectors, then the

Length of the arc of each of the sectors $= \frac{1}{n} \times 2\pi r \ units$ and

Area of each of the sectors $= \frac{1}{n} \times \pi r^2 \ sq.units$

2. Also, the area of the sector is derived as $= \frac{\theta^\circ}{360^\circ} \times \pi r^2$

$$= \frac{1}{2}\left(\frac{\theta^\circ}{360^\circ} \times 2\pi r\right) \times r$$

$$= \left(\frac{1}{2} \times l\right) \times r = \frac{lr}{2} \ sq.units$$

2.2.3 Perimeter of a sector

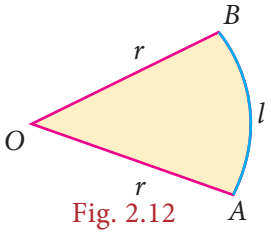


Fig. 2.12

We already know that the total length of the boundary of a closed part is its perimeter, Isn't it? What is the boundary of a sector? **Two radii** (OA and OB) and **an arc** $\left(\overset{\frown}{AB}\right)$.

So, the perimeter of a sector = length of the arc + length of two radii

$$P = l + 2r \ units$$

$\therefore$ **Perimeter of a sector,** $P = l + 2r \ units$

DO YOU KNOW?

Tamil people always have a prominent role in the history of Mathematics. They had recorded in the form of a song on how to find the area of a circle, a few thousand years before itself, in the book titled 'Kanakkathikaram'. (கணக்கதிகாரம்). The song is as follows:

"வட்டத்தரை கொண்டு விட்டத்தரை தாக்கச்
சட்டெனத் தோன்றும் குழி "

Meaning:

'**வட்டத்தரை**' represents half of the circumference and '**விட்டத்தரை**' represents half of the diameter which is the radius and '**குழி**' represents the area. In this song, the Tamil people had recorded that, if half the circumference is multiplied by half the diameter, the area of the circle can be calculated.

Area of the circle = **வட்டத்தரை** × **விட்டத்தரை**

$$= \frac{1}{2} \text{ of circumference} \times \frac{1}{2} \text{ of diameter} = \left(\frac{1}{2} \times 2\pi r\right) \times r$$

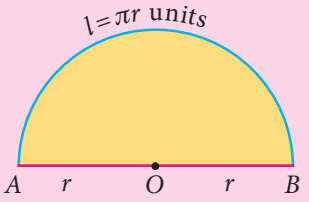
$\therefore$ Area of the circle, $A = \pi r^2 \ sq.units$

Note

1. The perimeter of a semi-circle

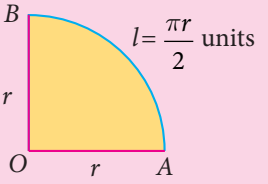
$P = l+2r$ *units*

$= \pi r + 2r = (\pi + 2)r$ *units*



2. The perimeter of a circular quadrant

$$P = l + 2r = \frac{\pi r}{2} + 2r = \left(\frac{\pi}{2} + 2\right) r \text{ units}$$



Example 2.1

The radius of a sector is 21*cm* and its central angle is 120°. Find (i) the length of the arc (ii) area of the sector (iii) perimeter of sector. $\left(\pi = \frac{22}{7}\right)$

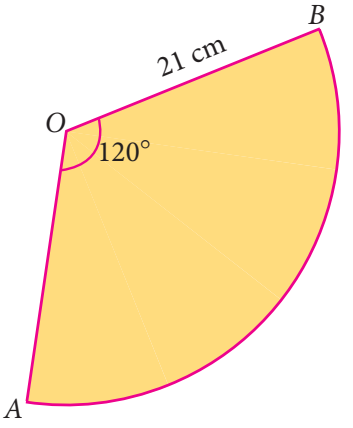


Fig. 2.13

Solution:

Radius, r = 21 *cm* and central angle, θ = 120°.

(i) Length of the arc, $l = \frac{\theta^\circ}{360^\circ} \times 2\pi r$ units

$= \frac{120^\circ}{360^\circ} \times 2 \times \frac{22}{7} \times 21$

= 44 cm (approximately)

(ii) Area of the sector, $A = \frac{\theta^\circ}{360^\circ} \times \pi r^2$ *sq.units.*

$= \frac{120^\circ}{360^\circ} \times \frac{22}{7} \times 21 \times 21$

A = 462 *sq.cm* (approximately)

Aliter:

Area of the sector

$A = \frac{lr}{2}$ *sq.units*

$= \frac{44 \times 21}{2} = 462\ cm^2$

(iii) Perimeter of the sector, $P = l + 2r$ *units*

= 44+2×21

= 44+42 = 86 *cm* (approximately)

Example 2.2

A circular shaped gymnasium ring of radius 35*cm* is divided into 5 equal arcs shaded with different colours. Find the length of each of the arcs.

Fig. 2.14

Solution:

Radius, r = 35 *cm* and n = 5.

Length of each of the arcs, $l = \frac{1}{n} \times 2\pi r$ *units*

$= \frac{1}{5} \times 2 \times \pi \times 35 = 14\pi$ *cm.*

Example 2.3

A spinner of radius 7.5 *cm* is divided into 6 equal sectors. Find the area of each of the sectors.

Fig. 2.15

Solution:

Radius, $r = 7.5\ cm$ and $n = 6$.

Area of each of the sectors, $A = \frac{1}{n} \times \pi r^2\ sq.\ units$

$$= \frac{1}{6} \times \pi \times 7.5 \times 7.5$$

$$= 9.375\pi\ sq.\ cm$$

Example 2.4

Kamalesh has a dining table, circular in shape of radius 70 *cm* whereas Tharun has a circular quadrant dining table of radius 140 *cm*. Whose dining table has a greater area? $\left(\pi = \frac{22}{7}\right)$

Fig. 2.16

Solution:

Area of the dining table with Kamalesh $= \pi r^2\ sq.\ units$

$$= \frac{22}{7} \times 70 \times 70$$

$A = 15400\ sq.cm$ (approximately.)

Fig. 2.17

Area of the circular quadrant dining table with Tharun

$$= \frac{1}{4}\pi r^2 = \frac{1}{4} \times \frac{22}{7} \times 140 \times 140$$

$A = 15400\ sq.cm$ (approximately.)

We find that, the area of the dining tables of both of them have the same area.

Think

If the radius of a circle is doubled, what will happen to the area of the new circle so formed?

Example 2.5

Four identical medals, each of diameter 7*cm* are placed as shown in Fig. 2.18. Find the area of the shaded region between the medals. $\left(\pi = \frac{22}{7}\right)$

Fig. 2.18

Solution:

Diameter, $d = 7\ cm$, therefore $r = \frac{7}{2}\ cm$.

Area of the shaded region = Area of the square – 4 × Area of the circular quadrant

$$= a^2 - 4 \times \frac{1}{4}\pi r^2$$

$$= (7 \times 7) - \left(4 \times \frac{1}{4} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2}\right)$$

$$= 49 - 38.5 = 10.5\ sq.cm.\ \text{(approximately)}$$

Exercise 2.1

1. **Fill in the blanks:**

   (i) The ratio between the circumference and diameter of any circle is _____.

   (ii) A line segment which joins any two points on a circle is a _____.

   (iii) The longest chord of a circle is _____.

   (iv) The radius of a circle of diameter 24 *cm* is _____.

   (v) A part of circumference of a circle is called as _____.

2. **Match the following:**

   (i) Area of a circle - (a) $\frac{1}{4}\pi r^2$

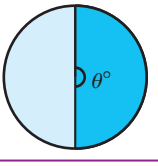
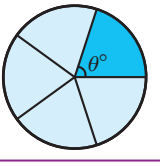
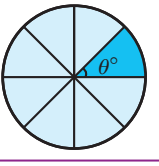
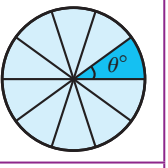
   (ii) Circumference of a circle - (b) $(\pi + 2)r$

   (iii) Area of the sector of a circle - (c) πr^2

   (iv) Circumference of a semicircle - (d) $2\pi r$

   (v) Area of a quadrant of a circle - (e) $\frac{\theta^\circ}{360^\circ} \times \pi r^2$

3. Find the central angle of the shaded sectors (each circle is divided into equal sectors).

Sectors				
Central angle of each sector (θ°)				

4. For the sectors with given measures, find the length of the arc, area and perimeter. (π=3.14)

   (i) central angle 45°, $r = 16\ cm$ (ii) central angle 120°, d =12.6 *cm*

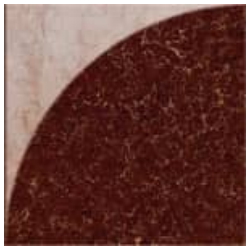
5. From the measures given below, find the area of the sectors.

   (i) length of the arc = 48 *m*, $r = 10\ m$ (ii) length of the arc = 50 *cm*, $r = 13.5\ cm$

6. Find the central angle of each of the sectors whose measures are given below. $\left(\pi = \frac{22}{7}\right)$

   (i) area = 462 cm^2, r = 21 cm (ii) length of the arc = 44 m, r = 35 m

7. A circle of radius 120 m is divided into 8 equal sectors. Find the length of the arc of each of the sectors.
8. A circle of radius 70 cm is divided into 5 equal sectors. Find the area of each of the sectors.



9. Dhamu fixes a square tile of 30 cm on the floor. The tile has a sector design on it as shown in the figure. Find the area of the sector. $(\pi = 3.14)$.

10. A circle is formed with 8 equal granite stones as shown in the figure each of radius 56 cm and whose central angle is 45°. Find the area of each of the granite stones. $\left(\pi = \frac{22}{7}\right)$

2.3 Combined shapes

In our day-to-day life, we use infinitely many things with shapes like circle, triangle, square, rectangle, rhombus etc., don't we? We use them separately as well as with two or three or more shapes combined together as shown in the following figures.

Glass window	Model house	Invitation card	Locker

What do we observe from the above figures?

The shape of a glass window is like a semi-circle placed over a rectangle whereas the front – facing wall of a model house looks like a triangle over a square.

Likewise, list the shapes used to make an invitation card and a locker.

Thus, two or more plane figures joined with the sides of same measure give rise to combined shapes.

2.3.1 Perimeter of combined shapes

The perimeter of a combined shape is the sum of all the lengths of the sides that form a closed boundary.

For example, observe the Fig. 2.19 which has a square of side *a units* and an equilateral triangle of side *a units* combined together.

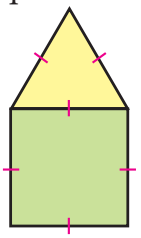


Fig. 2.19

Though a square has 4 sides and an equilateral triangle has 3 sides, when they are combined together, we will have a total of only 5 sides as the boundary of the combined shape and not 7 sides. Isn't it? So, the perimeter of the combined shape, here is 5*a units*.

2.3.2 Area of combined shapes

To find the area of a combined shape, split the combined shape into known simpler shapes, find their area separately and then add them up. That is, the area of combined shapes is nothing but the sum of all the areas of the simple shapes in it.

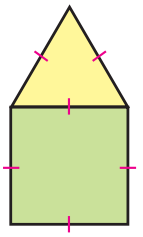


Fig. 2.20

To find the area of the Fig.2.20, find the area of the square and the area of the equilateral triangle separately and then add them up.

The combined shapes that we come across mostly in our day-to-day life are irregular polygons. To find the area of the given irregular polygon, we must identify the simple shapes in it, find their areas separately and then add them up.

For example, an irregular polygonal field given below can be split into known simpler shapes as under and then its area can be found.

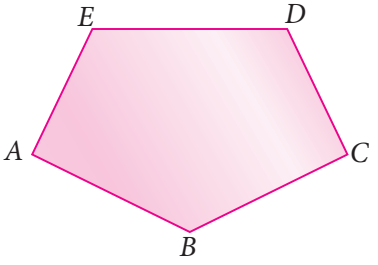


Fig. 2.21

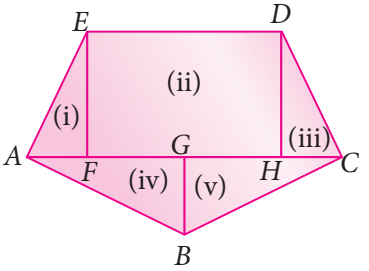


Fig. 2.22

A closed plane figure formed by three or more sides is called a 'polygon' .Based on the sides, some of the polygons are named as given below.

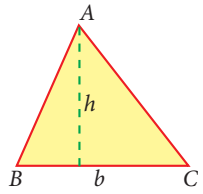
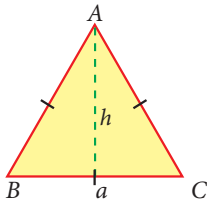
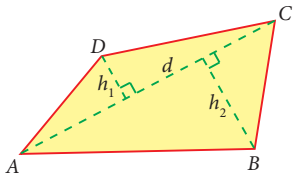
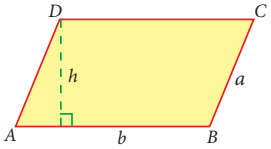
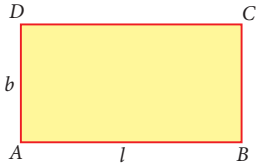
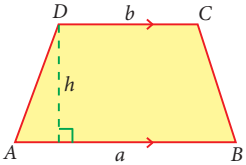
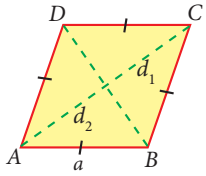
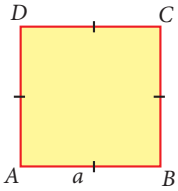
Number of sides	3	4	5	6	7	8	9	10
Name of the Polygon	Triangle	Quadrilateral	Pentagon	Hexagon	Heptagon	Octagon	Nonagon	Decagon

If all sides and all angles of a polygon are equal, then it is called as a regular polygon. Examples: equilateral triangle, square etc., Other polygons are irregular polygons. Examples: scalene triangle, rectangle etc.,

Think

All the sides of a rhombus are equal. Is it a regular polygon?

The formulae to find the area and the perimeter of some plane figures learnt in the earlier class are tabulated below and it will be helpful to find the area and the perimeter of combined figures.

S.No	Shape	Name	Area (*sq.units*)	Perimeter (*units*)
1		Triangle	$\frac{1}{2}\times b\times h$	Sum of all three sides
2		Equilateral triangle	$\frac{\sqrt{3}}{4}a^2\left(h=\frac{\sqrt{3}}{2}a\right)$	$3a$
3		Quadrilateral	$\frac{1}{2}\times d\times(h_1+h_2)$	Sum of all the four sides
4		Parallelogram	$b\times h$	$2(a+b)$
5		Rectangle	$l\times b$	$2(l+b)$
6		Trapezium	$\frac{1}{2}\times h\times(a+b)$	Sum of all the four sides
7		Rhombus	$\frac{1}{2}\times d_1\times d_2$	4a
8		Square	a^2	4a

Example 2.6

Find the perimeter and area of the given Fig.2.23. $\left(\pi = \frac{22}{7}\right)$

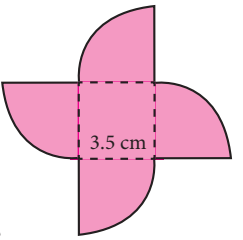


Fig. 2.23

Solution:

Radius of a circular quadrant, $r = 3.5\ cm$ and side of a square, $a = 3.5\ cm$.

The given figure is formed by the joining of 4 quadrants of a circle with each side of a square. The boundary of the given figure consists of 4 arcs and 4 radii.

(i) Perimeter of the given combined shape

$= 4 \times$ length of the arcs of the quadrant of a circle $+ 4 \times$ radius

$$= \left(4 \times \frac{1}{4} \times 2\pi r\right) + 4r$$

$$= \left(4 \times \frac{1}{4} \times 2 \times \frac{22}{7} \times 3.5\right) + (4 \times 3.5)$$

$= 22 + 14 = 36\ cm$ (approximately)

(ii) Area of the given combined shape

= area of the square + 4 × area of the quadrants of the circle

$$= a^2 + \left(4 \times \frac{1}{4}\pi r^2\right)$$

$$= (3.5 \times 3.5) + \left(\frac{22}{7} \times 3.5 \times 3.5\right)$$

$A = 12.25 + 38.5 = 50.75\ cm^2$ (approximately)

Example 2.7

Nishanth has a key-chain which is in the form of an equilateral triangle and a semicircle attached to a square of side 5 *cm* as shown in the Fig. 2.24. Find its area. $\left(\pi = 3.14, \sqrt{3} = 1.732\right)$

Fig. 2.24

Solution:

Side of the square $= 5\ cm$

Diameter of the semi-circle $= 5\ cm \Rightarrow$ Radius $= 2.5\ cm$

Side of the equilateral triangle $= 5\ cm$

$\therefore$ Area of the key-chain = area of the semi circle + area of the square + area of the equilateral triangle

$$= \frac{1}{2}\pi r^2 + a^2 + \frac{\sqrt{3}}{4}a^2$$

$$= \left(\frac{1}{2} \times 3.14 \times 2.5 \times 2.5\right) + (5 \times 5) + \left(\frac{\sqrt{3}}{4} \times 5 \times 5\right)$$

$$= 9.81 + 25 + 10.83$$

$= 45.64\ cm^2$ (approximately)

Example 2.8

A 3-fold invitation card is given with measures as in the Fig. 2.25. Find its area.

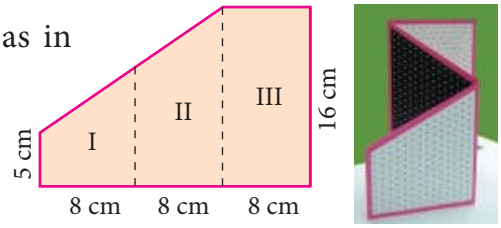


Fig. 2.25

Solution:

Figures I and II are trapeziums separately as well as combinedly.

The parallel sides of the combined trapezium (I and II) are 5 *cm* and 16 *cm* and its height, $h = 8 + 8 = 16\,cm$, length of the rectangle (III) = 16 *cm* and its breadth = 8 *cm*

$\therefore$ Area of the combined invitation card

= area of the combined trapezium + area of the rectangle

$$= \frac{1}{2} \times h \times (a + b) + l \times b$$

$$= \frac{1}{2} \times 16 \times (5 + 16) + 16 \times 8$$

$$= 168 + 128 = 296 \ cm^2$$

Aliter:

Area of the invitation card

= area of the outer rectangle –
area of the right angled triangle

$$= l \times b - \frac{1}{2} \times b \times h$$

$$= 24 \times 16 - \frac{1}{2} \times 11 \times 16$$

$$= 384 - 88 = 296 \ cm^2$$

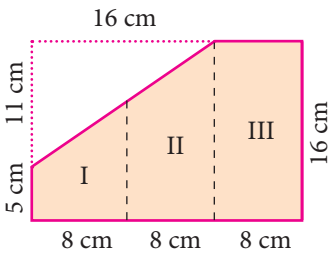


Fig. 2.26

Example 2.9

Seenu wants to buy a floor mat for his kitchen at home as given in Fig. 2.27. If the cost of the mat is ₹ 20 per square foot, what will be the cost of the entire mat?

Fig. 2.27

Solution:

The mat given in the figure can be split into two rectangles as follows:

∴ Area of the entire mat

= area of the I rectangle + area of the II rectangle

$= l_1 \times b_1 + l_2 \times b_2$

$= 5 \times 2 + 9 \times 2 = 10 + 18 = 28$ *sq.feet*

Cost per sq. foot = ₹ 20

∴ The total cost of the entire mat = 28 × ₹ 20 = ₹ 560.

Fig. 2.28

Try these

In the above example split the given mat into two trapeziums and verify your answer.

Example 2.10

Find the area of the shaded region in the square of side 10 *cm* as given in the Fig. 2.29. $\left(\pi = \frac{22}{7}\right)$

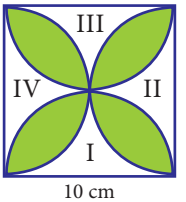


Fig. 2.29

Solution:

Mark the unshaded parts of the given figure as I, II, III and IV

Area of the I and III parts = Area of the square – Area of 2 semicircles

$$= a^2 - \left(2 \times \frac{1}{2}\pi r^2\right)$$

$$= 10 \times 10 - \frac{22}{7} \times 5 \times 5 = 100 - 78.57 = 21.43\ cm^2.$$

Similarly, the area of the II and IV parts = 21.43 cm^2 .

∴ Area of the unshaded parts (I, II, III and IV)

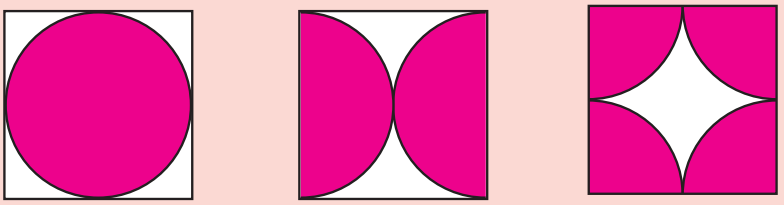
= 21.43 ×2 = 42.86 cm^2 (approximately)

∴ Area of the shaded part = area of the square – area of the unshaded parts

= 100 – 42.86 = 57.14 cm^2 (approximately)

DO YOU KNOW?

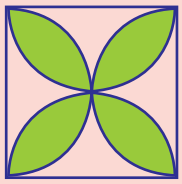
1. The area of the unshaded regions in each of the squares of side *a* units are the same in all the cases given below.



2. If the biggest circle is cut from a square of side '*a*' *units*, then the remaining area in the square is approximately $\frac{3}{14}a^2$ *sq.units*. $\left(\pi = \frac{22}{7}\right)$

3. The area of the biggest circle cut out from the square of 'a' units = $\frac{11}{14}a^2$ *sq. units* (approximately)

4. In the given figure if $\pi = \frac{22}{7}$, the area of the unshaded part of a square of side *a* units is approximately $\frac{3}{7}a^2$ *sq.units* and that of the shaded part is approximately $\frac{4}{7}a^2$ *sq.units*.



Example 2.11

Find the area of an irregular polygon field whose measures are as given in the Fig. 2.30.

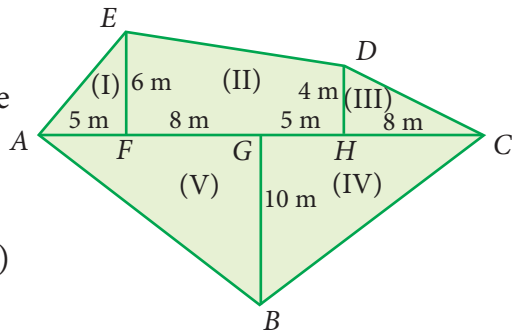


Fig. 2.30

Solution:

The given field has four triangles (I, III, IV and V) and a trapezium (II).

Area of the triangle (I) $= \frac{1}{2} \times b \times h = \frac{1}{2} \times 5 \times 6 = 15 \ m^2$

Area of the trapezium (II) $= \frac{1}{2} h(a+b) = \frac{1}{2} \times 13 \times (6+4) = 65 \ m^2$

Area of the triangle (III) $= \frac{1}{2} \times b \times h = \frac{1}{2} \times 8 \times 4 = \frac{32}{2} = 16 \ m^2$

Area of the triangle (IV) $= \frac{1}{2} \times b \times h = \frac{1}{2} \times 13 \times 10 = 65 \ m^2$

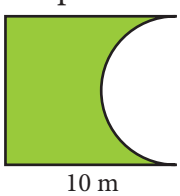
Area of the triangle (V) $= \frac{1}{2} \times b \times h = \frac{1}{2} \times 13 \times 10 = 65 \ m^2$

$\therefore$ The total area of the field $= 15 + 65 + 16 + 65 + 65 = 226 \ m^2$

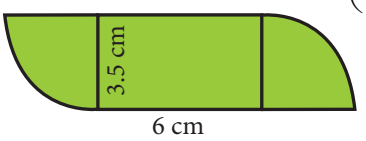
Exercise 2.2

1. Find the perimeter and area of the figures given below. $\left(\pi = \frac{22}{7}\right)$

(i)

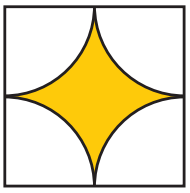


(ii)

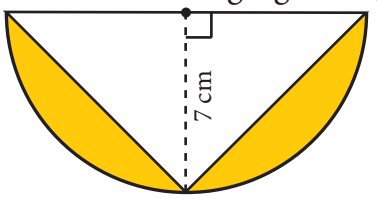


2. Find the area of the shaded part in the following figures. ($\pi = 3.14$)

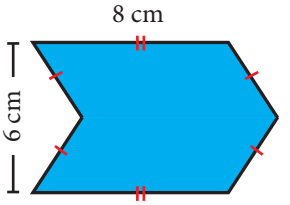
(i)



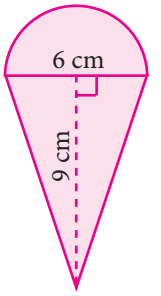
(ii)



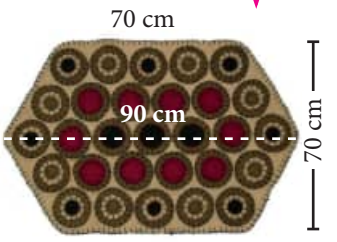
3. Find the area of the combined figure given which is got by the joining of two parallelograms.



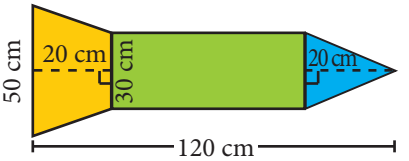
4. Find the area of the combined figure given, formed by joining a semicircle of diameter 6 *cm* with a triangle of base 6 *cm* and height 9 *cm*. ($\pi = 3.14$)



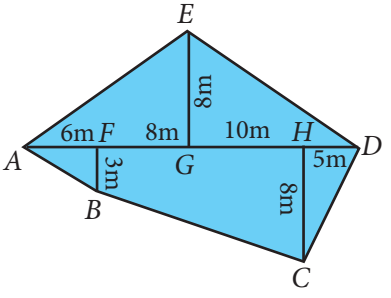
5. The door mat which is hexagonal in shape has the following measures as given in the figure. Find its area.



6. A rocket drawing has the measures as given in the figure. Find its area.



7. Find the area of the irregular polygon shaped fields given below.



2.4 Three dimensional (3-D) shapes

Trace the outline of a ₹ 2 coin, ₹ 10 note and a square shaped biscuit on a paper.

Fig. 2.31

What shapes have you traced? A **circle**, a **rectangle** and a **square**. Isn't it? These shapes represents the plane figures. Also, these plane figures have two dimensions namely length and breadth. Now, you place some two rupee coins, some ten rupee notes and some square shaped biscuits respectively on the drawn shapes as shown in the figure.

Fig. 2.32

What do you get now? A **cylinder, a cuboid and a cube.** Isn't it? These shapes do not lie completely on the plane and they occupy some space also. That is, they have the third dimension namely the height along with the dimensions length and breadth. Thus, **the shapes which have three dimensions namely length, breadth and height (depth) are called three dimensional shapes, simply called as 3-D shapes.** Some examples of 3-D shapes are

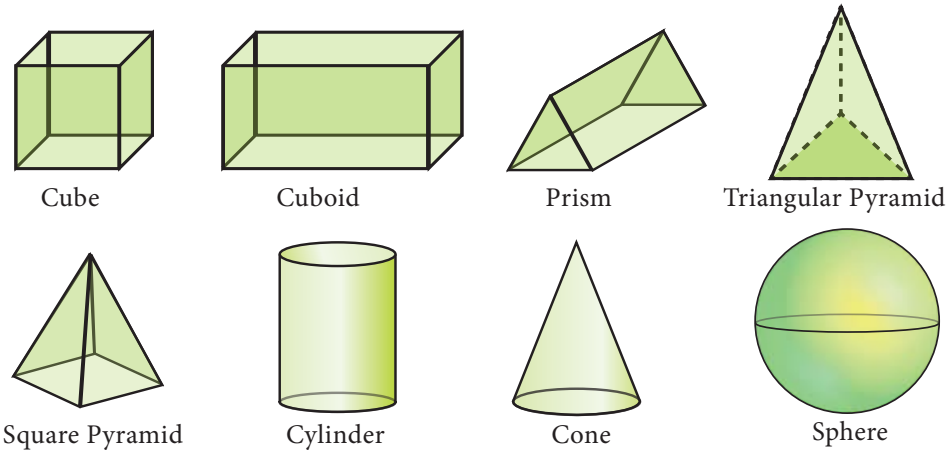


Fig. 2.33

2.4.1 Faces, Edges and Vertices

Observe the following shape. What is it? A cube. A cube is made of six square shaped planes. These 6 square shaped planes of the cube are known as its **faces.**

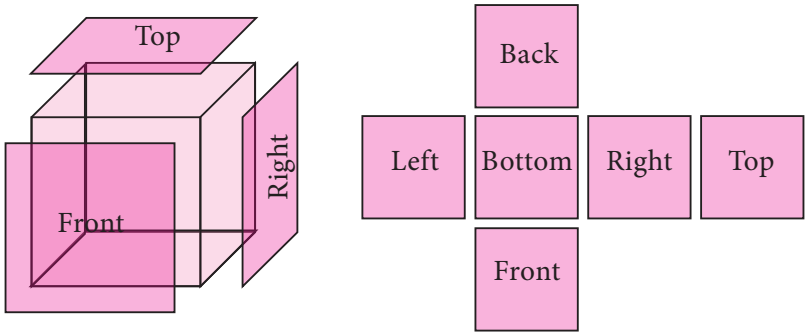
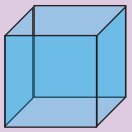
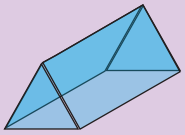
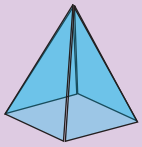
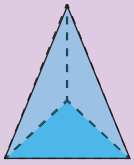


Fig. 2.34

A line segment which connects any two faces of a cube is called as **Edge** and each corner point where three edges meet is called as **Vertex**. So, **a cube has 6 faces, 12 edges and 8 vertices.**

Try these

Tabulate the number of faces (F), vertices (V) and edges (E) for the following polyhedrons. Also find F+V–E

Solid	Name	F	V	E	F+V–E
	Cube	6	8	12	
	Cuboid				
	Triangular Prism				
	Square Pyramid				
	Triangular Pyramid				

What do you observe from the above table? We observe that, F+V–E = 2 in all the cases. This is true for any polyhedron and this relation F+V–E = 2 is known as **Euler's formula**.

2.4.2 Nets for building three dimensional (3-D) shapes

When we buy sweets, a shop keeper picks a flat shaped card which has some flips and makes a rectangular shaped box (cuboid) by folding it as shown in the figure. Then, he arranges the sweets in the box and gives it to us.

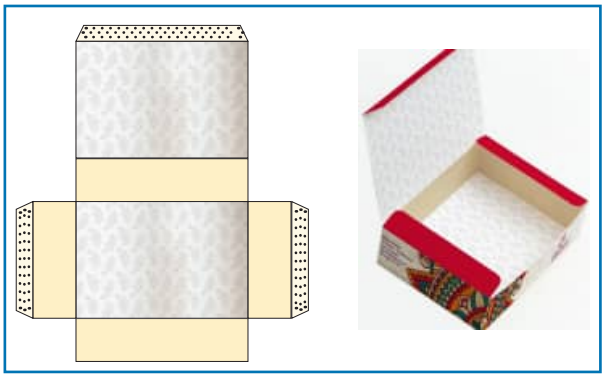
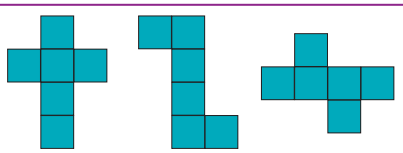


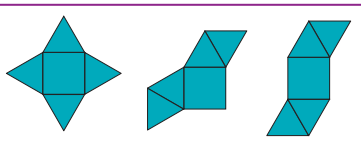
Fig. 2.35

The flat shaped card already designed for making the box excluding flaps (dotted lines) is known as a net.

For example, from the following nets we can build cubes and square pyramids.



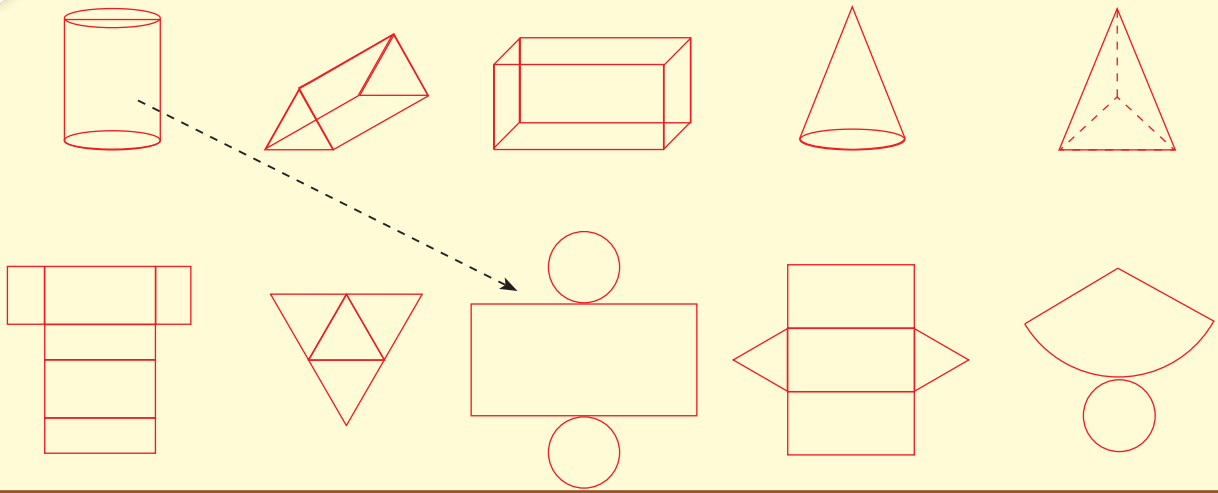
Nets for cubes



Nets for square pyramids

Activity

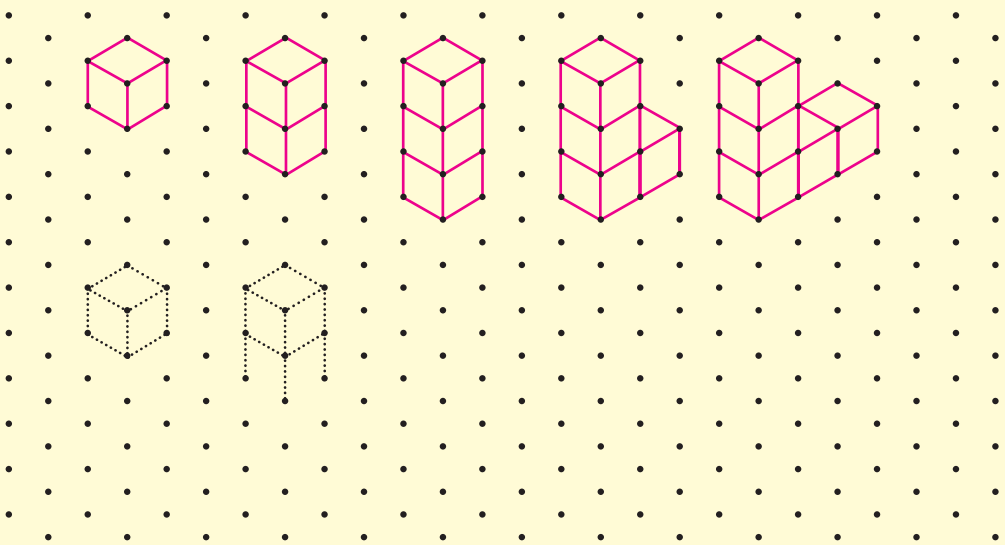
Draw a line to match the following shapes to their relevant nets.



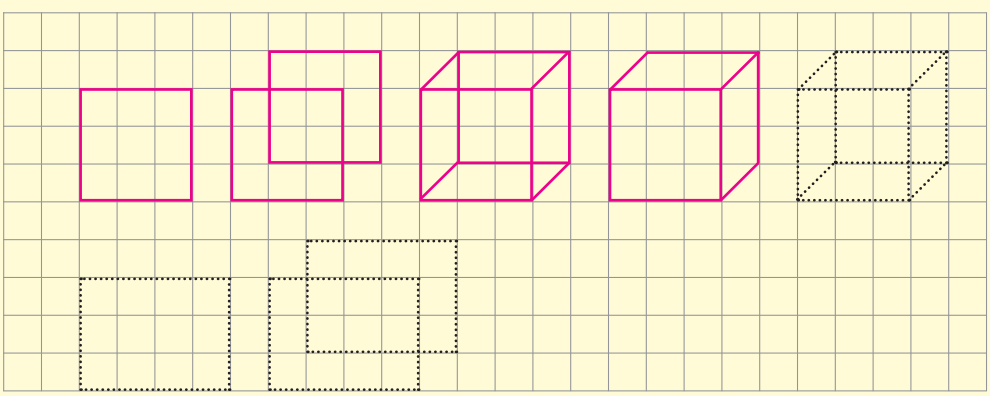
2.4.3 Drawing 3-D shapes using isometric dot sheets and grid sheets

Activity

1. Draw each of the given solid figures on an isometric dot sheet



2. Draw each of the given solid figures on a grid paper



2.4.4 Cross section of solid shapes

When we cut the vegetables in cross section for cooking purpose, we see some plane figures in it. For example, the cross section of a carrot and a plantain stem is a circle.

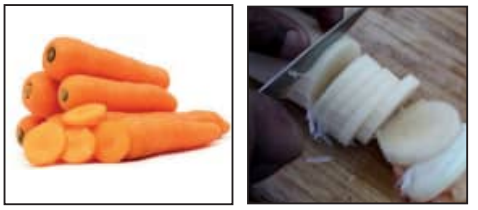


Fig. 2.36

In the same way, we can see squares and rectangles in the cross section of a bread loaf and bricks etc.,

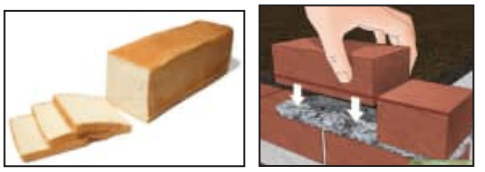
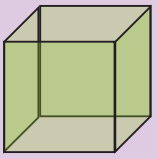
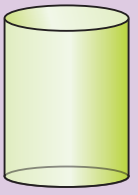
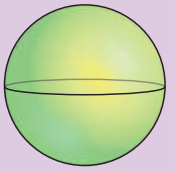
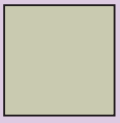


Fig. 2.37

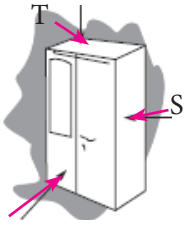
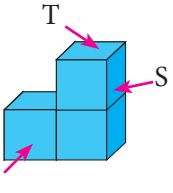
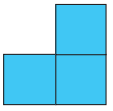
Activity

Draw and name the two dimensional shapes (2-D) which you get in the cross section of the following solid shapes.

Solids			
2-D Shape			
Name	Square		

2.4.5 3-D shapes in different views

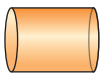
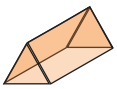
A 3-D object may look different from different positions. View of a 3D shape is what you see while observing the object from different positions. Some of the views are front view, top view and side view. The different views of some of the objects are as shown below.

Object	Front View	Top View	Side View
			
			

Exercise 2.3

1. **Fill in the blanks:**
   (i) The three dimensions of a cuboid are _____, _____ and _____.
   (ii) The meeting point of more than two edges in a polyhedron is called as _____.
   (iii) A cube has _____ faces.
   (iv) The cross section of a solid cylinder is _____.
   (v) If a net of a 3-D shape has six plane squares, then it is called _____.

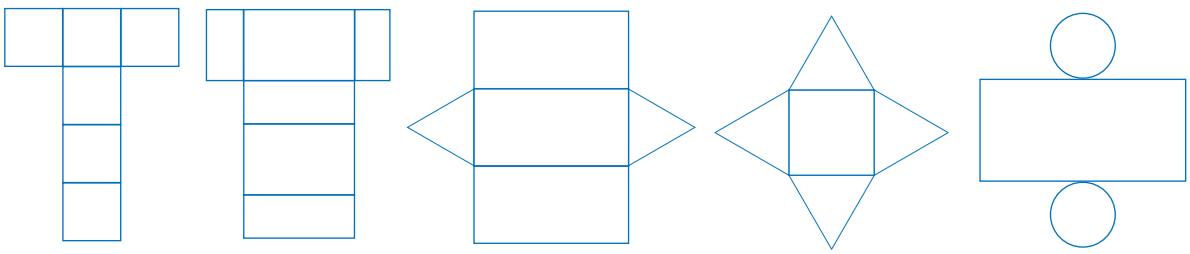
2. **Match the following:**

(i)		-	(a) Cylinder
(ii)		-	(b) Cuboid
(iii)		-	(c) Triangular Prism
(iv)		-	(d) Square Pyramid

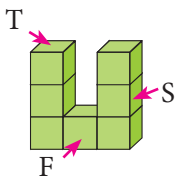
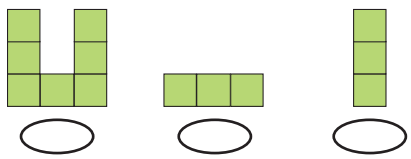
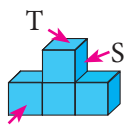
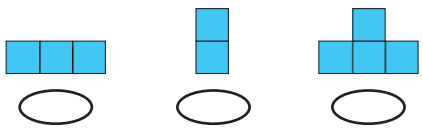
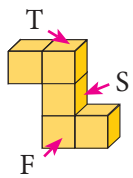
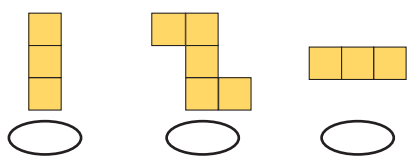
E64KN

3. Which 3-D shapes do the following nets represent? Draw them.

(i) (ii) (iii) (iv) (v)



4. For each solid, three views are given. Identify for each solid, the corresponding Top, Front and Side (T, F and S) views.

Solid	Three views
	
	
	

5. Verify Euler's formula for the table given below.

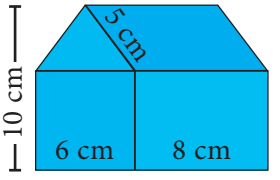
S.No.	Faces	Vertices	Edges
(i)	4	4	6
(ii)	10	6	12
(iii)	12	20	30
(iv)	20	13	30
(v)	32	60	90

Exercise 2.4

Miscellaneous Practice Problems

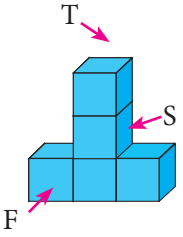
1. Two gates are fitted at the entrance of a library. To open the gates easily, a wheel is fixed at 6 feet distance from the wall to which the gate is fixed. If one of the gates is opened to 90°, find the distance moved by the wheel $(\pi = 3.14)$.

2. With his usual speed, if a person covers a circular track of radius 150 *m* in 9 minutes, find the distance that he covers in 3 minutes $(\pi = 3.14)$.
3. Find the area of the house drawing given in the figure.

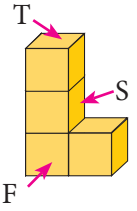


4. Draw the top, front and side view of the following solid shapes

(i)

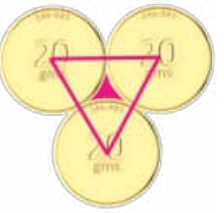


(ii)



Challenging problems

5. Guna has fixed a single door of width 3 *feet* in his room where as Nathan has fixed a double door, each of width $1\frac{1}{2}$ *feet* in his room. From the closed position, if each of the single and double doors can open up to 120°, whose door takes a minimum area?
6. In a rectangular field which measures 15 $m \times 8m$, cows are tied with a rope of length $3m$ at four corners of the field and also at the centre. Find the area of the field where no cows can graze. $(\pi = 3.14)$
7. Three identical coins each of diameter 6 *cm* are placed as shown. Find the area of the shaded region between the coins. $(\pi = 3.14)\ (\sqrt{3} = 1.732)$



8. Using Euler's formula, find the unknowns.

S.No.	Faces	Vertices	Edges
(i)	?	6	14
(ii)	8	?	10
(iii)	20	10	?

SUMMARY

- If any two points on a circle are joined by a line segment, then the line segment is called a 'chord'.
- A diameter of a circle divides it into two equal parts. It is the longest chord of a circle.
- A part of the circumference of a circle is called the circular arc.
- The plane surface that is enclosed between two radii and the circular arc of a circle is called a 'sector'.
- The angle made by the sector with the centre of the circle is called the 'central angle'.
- The perimeter of a combined shape is the sum of all the lengths of the sides that form a closed boundary.
- The area of combined shapes is nothing but the sum of all areas of the simpler shapes in it.
- The shapes which have three dimensions namely length, breadth and height are called three dimensional shapes (or) simply called as 3-D shapes.
- A cube has 6 faces, 12 edges and 8 vertices.

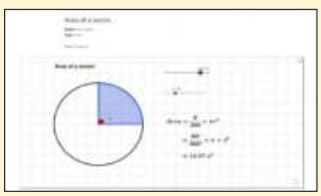
ICT CORNER

"Through this activity you will know about how to find the perimeter and area of the sectors."

Step-1 Open the Browser and type www.Geogebra.com (or)scan the QR CODE given below.

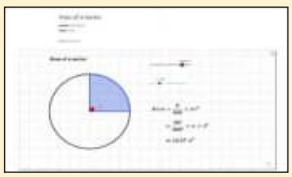
Step-2 Type Area of sector on the search column

Step-3 Move the radius slide and degree slide. You will get various results.



Step 1

Step 2

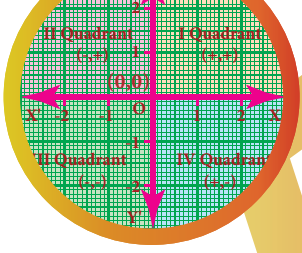


Step 3

Go through the remaining worksheets given for this chapter

Web URL Measurement (measurements links)
https://www.geogebra.org/m/FSqNDNxN
*Pictures are indicatives only.
*If browser requires, allow Flash Player or Java Script to load the page.

3 ALGEBRA



Learning Objectives

YUTO6

- To recall addition and subtraction of expressions.
- To know how to multiply algebraic expressions with integer co-efficients
- To know how to divide algebraic expressions by monomials.
- To recall the identities $(a+b)^2,(a-b)^2,(a^2-b^2)$ and $(x+a)(x+b)$ and able to apply them in problems.
- To understand the identities $(a+b)^3,(a-b)^3,(x+a)(x+b)(x+c)$ and apply them in problems.
- To recognize expressions that are factorizable of the type $(a+b)^3$ and $(a-b)^3$.
- To solve word problems that involve linear equations.
- To know how to plot the points in the graph.
- To draw graphs of simple linear equations.

Recap

In our earlier classes, we have learnt about constants, variables, like terms, unlike terms, co-efficients, numerical and algebraic expressions. Later, we have done some basic operations like addition and subtraction on algebraic expressions. Now, we shall recollect them and extend the learning.

Further, we are going to learn about multiplication and division of algebraic expressions and algebraic identities.

Answer the following questions :

1. Write the number of terms in the following expressions

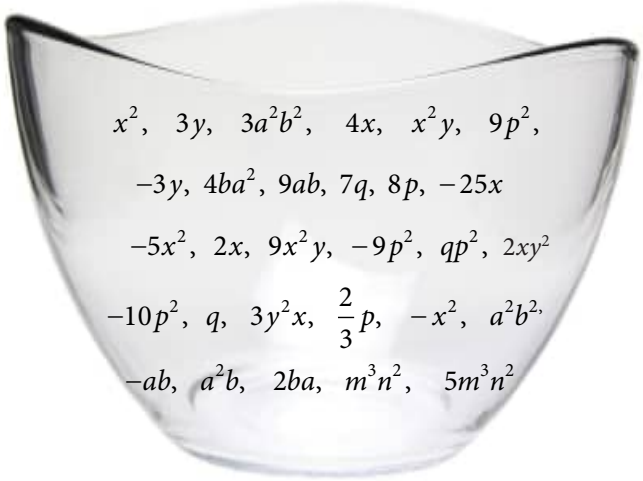
   (i) $x+y+z-xyz$ (ii) m^2n^2c

   (iii) $a^2b^2c-ab^2c^2+a^2bc^2+3abc$ (iv) $8x^2-4xy+7xy^2$

2. Identify the numerical co-efficient of each term in the following expressions.

   (i) $2x^2-5xy+6y^2+7x-10y+9$ (ii) $\frac{x}{3}+\frac{2y}{5}-xy+7$

3. Pick out the like terms from the following:



Like Terms

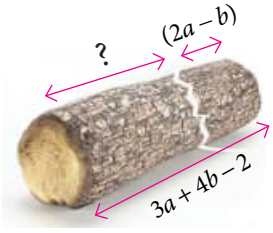
The variables of the terms along with their respective exponents must be same

Examples : $x^2, 4x^2$

$a^2b^2, -5a^2b^2$

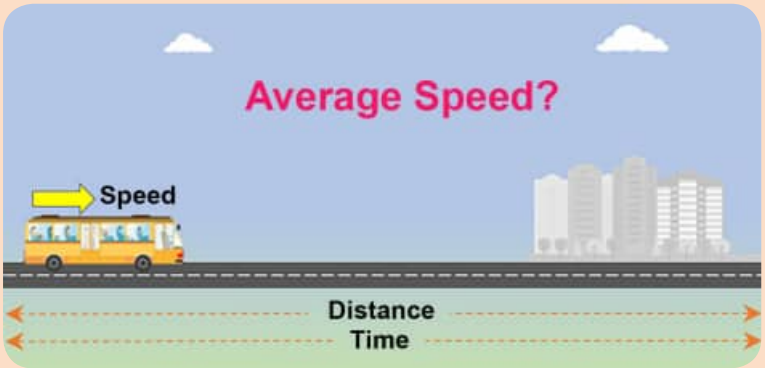
$2m, -7m$

4. Add : $2x$, $6y$, $9x$, $-2y$
5. Simplify : $(5x^3y^3 - 3x^2y^2 + xy + 7) + (2xy + x^3y^3 - 5 + 2x^2y^2)$
6. The sides of a triangle are $2x - 5y + 9$, $3y + 6x - 7$ and $-4x + y + 10$. Find the perimeter of the triangle .
7. Subtract $-2mn$ from $6mn$.
8. Subtract $6a^2 - 5ab + 3b^2$ from $4a^2 - 3ab + b^2$.
9. The length of a log is $3a + 4b - 2$ and a piece $(2a - b)$ is removed from it. What is the length of the remaining log?

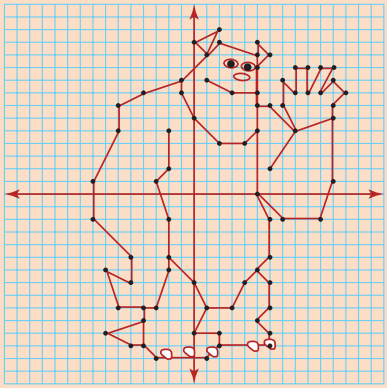


10. A tin had *x litres* of oil. Another tin had $(3x^2 + 6x - 5)$ *litres* of oil. The shopkeeper added $(x+7)$ *litres* more to the second tin. Later, he sold (x^2+6) *litres* of oil from the second tin. How much oil was left in the second tin?

MATHEMATICS ALIVE – ALGEBRA IN REAL LIFE



Linear equations are used for speed, distance, time and average speed



Graphs are used for drawing

3.1 Introduction

Let us consider the given situation that Ganesh planted saplings in his garden. He planted 10 rows each with 5 saplings. Can you say how many saplings were planted?

Yes, we know that, the total number of saplings is the product of number of rows and number of saplings in each row.

Hence, the total number of saplings = 10 rows $\times$ 5 saplings in each row $= 10 \times 5 = 50$ saplings

Likewise, David planted some saplings. Not knowing the total number of rows and saplings in each row, how will you express the total number of saplings?

For the unknown quantities, we call them as 'x' and 'y'. Therefore, the total number of saplings = 'x' rows $\times$ 'y' sapling in each row

= '$x \times y$' = xy saplings

Let us extend this situation, Rahim planted saplings where the number of rows are $(2x^2 + 5x - 7)$ and each row contains $3y^2$ saplings. Now the above idea will help us to find the total number of saplings planted by Rahim.

The total number of saplings = $(2x^2 + 5x - 7)$ rows $\times\ 3y^2$ saplings in each row.

$$= 3y^2 \times (2x^2 + 5x - 7)$$

How do we find the product of the above algebraic expression?

Now,we will learn to find the product of algebraic expressions.

Note

Polynomial	A polynomial is an expression containing two or more algebraic terms. In a polynomial all variables are raised to only whole number powers. $a^2 + 2ab + b^2$ $\quad$ $4x^2 + 3x - 7$ A polynomial cannot contain : 1) Division by a variable. Eg. $4x^2 - \frac{5}{1+x}$ is not a polynomial. 2) Negative exponents. Eg. $7x^{-2} + 5x - 6$ is not a polynomial. 3) Fractional exponents. Eg. $3x^3 + 4x^{\frac{1}{2}} + 5$ is not a polynomial.
Monomial	An expression which contains only one term is called a monomial. Examples: $4x$, $3x^2y$, $-2y^2$.
Binomial	An expression which contains only two terms is called a binomial. Examples: $2x+3$, $5y^2+9y$, a^2b^2+2b .
Trinomial	An expression which contains only three terms is called a trinomial. Examples : $2a^2b-8ab+b^2$, m^2-n^2+3 .

3.2 Multiplication of Algebraic Expressions

While doing the product of algebraic expressions, we should follow the steps given below.

Step 1: Multiply the signs of the terms. That is, the product of two like signs are positive and the product of two unlike signs are negative.

Like signs	$(+)\times(+)=+$	$(-)\times(-)=+$
Unlike signs	$(+)\times(-)=-$	$(-)\times(+)=-$

Step 2: Multiply the corresponding co-efficients of the terms.

Step 3: Multiply the variable factors by using laws of exponents.

If 'x' is a variable and m, n are positive integers then,

$$x^m \times x^n = x^{m+n}$$

For example, $x^3 \times x^4 = x^{3+4} = x^7$

Think

Every algebraic expression is a polynomial. Is this statement true? Why?

DO YOU KNOW?

A polynomial is a special kind of algebraic expression. The difference between an algebraic expression and a polynomial is,

Algebraic Expression

May contains whole numbers, fractions, negative numbers as the power of their variables.

Example: $4x^{3/2} - 3x + 9$

$2y^2 + \frac{5}{y} - 3$, $3x^2 - 4x + 1$

Polynomial

contains only whole numbers as the power of their variables.

Example: $4x^2 - 3x + 9$

$2y^6 + 5y^3 - 3$

Note

Product of two terms is represented by the symbols ( ), dot (.) or $\times$.

For example,

multiplying $4x^2$ and xy can be written in any one of the following ways.

$(4x^2)(xy)$	$4x^2 \times xy$	$4x^2(xy)$	$(4x^2)\times xy$	$4x^2 \cdot xy$

3.2.1 Multiplication of two or more monomials

Consider that, Geetha buys 3 pens each @ ₹5, how much she has to pay to the shopkeeper?

Geetha has to pay to the shopkeeper $= 3 \times ₹5$

$= ₹15$

If there are 'x' pens and the cost of each pen is ₹ 'y', then the cost of $(3x^2)$ pens bought by Geetha @ ₹ $5y$

@ - at the rate of

$$= (3x^2) \times 5y$$
$$= (3 \times 5)(x^2 \times y)$$
$$= ₹\ 15x^2y$$

Example 3.1

If the length and breadth of a rectangular painting are $4xy^3$ and $3x^2y$. Find its area.

Solution:

Area of the rectangular painting, $A = (l \times b)$ *sq.units*

$$= (4xy^3) \times (3x^2y)$$
$$= (4 \times 3)(x \times x^2)(y^3 \times y)$$
$$A = 12x^3y^4 \text{ sq.units}$$

Example 3.2

Find the product of $2x^2y^2$, $3y^2z$ and $-z^2x^3$

Solution:

We have, $(2x^2y^2) \times (3y^2z) \times (-z^2x^3)$

$$= (+) \times (+) \times (-)(2 \times 3 \times 1)(x^2 \times x^3)(y^2 \times y^2)(z \times z^2)$$
$$= -6x^5y^4z^3$$

Try these

Find the product of

(i) $3ab^2, -2a^2b^3$

(ii) $4xy, 5y^2x, (-x^2)$

(iii) $2m, -5n, -3p$

3.2.2 Multiplication of a polynomial by a monomial

If there are 'a' shops and each shop has 'x' apples in 8 baskets and 'y' oranges in 3 baskets and 'z' bananas in 5 baskets, then the total number of apples, oranges and bananas are

$$= a \times (8x + 3y + 5z)$$
$$= a(8x) + a(3y) + a(5z)$$

(using distributive law).

$$= 8ax + 3ay + 5az$$

DO YOU KNOW?

monomial × monomial	= monomial
binomial × monomial	= binomial
binomial × binomial	= binomial/polynomial
polynomial × monomial	= polynomial

Note

Distributive law

If a is a constant, x and y are variables then $a(x + y) = ax + ay$

For example, $5(x + y) = 5x + 5y$

Example 3.3

Multiply $3x^2y$ and $(2x^3y^3 - 5x^2y + 9xy)$

Think

Why $3+(4x-7y) \neq 12x-21y$?

Solution:

Now, $(3x^2y) \times (2x^3y^3 - 5x^2y + 9xy)$

$= 3x^2y(2x^3y^3) - 3x^2y(5x^2y) + 3x^2y(9xy)$

multiplying each term of the polynomial by the monomial

$= (3 \times 2)(x^2 \times x^3)(y \times y^3) - (3 \times 5)(x^2 \times x^2)(y \times y) + (3 \times 9)(x^2 \times x)(y \times y)$

$= 6x^5y^4 - 15x^4y^2 + 27x^3y^2$

Example 3.4

Ram deposited 'x' number of ₹2000 notes, 'y' number of ₹500 notes, 'z' number of ₹100 notes in a bank and Velan deposited '$3xy$' times of amount of what Ram had deposited. How much amount did Velan deposit in the bank?

Solution:

Amount deposited by Ram

$= (x \times ₹2000 + y \times ₹500 + z \times ₹100)$

$= ₹(2000x + 500y + 100z)$

Amount deposited by Velan $= 3xy$ times $\times$ Amount deposited by Ram

$= ₹\, 3xy \times (2000x + 500y + 100z)$

$= (3 \times 2000)(x \times x \times y) + (3 \times 500)(x \times y \times y) + (3 \times 100)(x \times y \times z)$

$= ₹(6000x^2y + 1500xy^2 + 300xyz)$

Try these

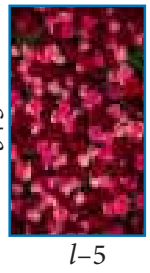
Multiply

(i) $(5x^2 + 7x - 3)$ by $-4x^2$ (ii) $(10x - 7y + 5z)$ by $6xyz$

(ii) $(ab+3bc-5ca)$ by $3a^2bc$ (iv) $(4m^2 - 3m + 7)$ by $-5m^3$

3.2.3 Multiplication of two binomials

Consider that a rectangular flower bed whose length is decreased by 5 *units* from the original length and whose breadth is increased by 3 *units* to the original breadth. What is the area of the rectangular flower bed?



Area of the rectangle $= l \times b$

Here, area of the rectangular flower bed $A = (l-5) \times (b+3)$ *sq.units*

How do we multiply this?

Now, let us learn how to multiply two binomials.

If $(x+y)$ and $(p+q)$ are two binomials, we can find their product as given below,

(i) Horizontal distributive approach

$$(x+y)(p+q) = x(p+q) + y(p+q)$$
$$= xp + xq + yp + yq$$

(ii) Vertical distributive approach

$$\begin{array}{r} x + y \\ p + q \\ \hline xq + yq \\ xp + yp \\ \hline = xp + xq + yp + yq \end{array}$$

(iii) Grid approach

×	x	y
p	xp	yp
q	xq	yq

$= xp + xq + yp + yq$

(iv) FOIL approach

We can do it directly like, $(x+y)(p+q) = x(p) + x(q) + y(p) + y(q)$

Outer, Last, Firsts, Inner

So, the above area of the rectangle $= (l-5)\times(b+3)$

(By horizontal distributive approach) $= l(b+3) - 5(b+3)$

$$A = (lb + 3l - 5b - 15)\ sq.u$$

Aliter

By grid approach

×	l	-5
b	lb	$-5b$
3	$3l$	-15

$= (lb + 3l - 5b - 15)$

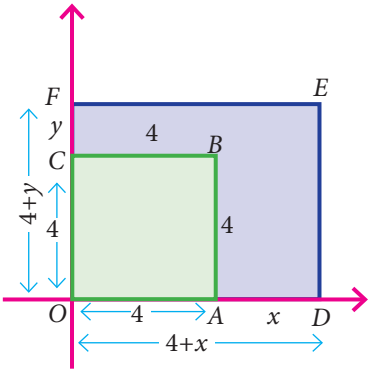
Let us consider one more example. Consider the given figure , In the square $OABC$,

$OA = 4$ *units* ; $OC = 4$ *units*

The area of the square $OABC = 4 \times 4$

$$A = 16\ sq.units$$

If the sides of the square are increased by 'x' *units* and 'y' *units* respectively,then we get the rectangle $ODEF$ whose sides are $OD=(4+x)$ *units* and $OF=(4+y)$ *units*.



Now, the area of the rectangle $ODEF = (4+x)(4+y)$

(by FOIL approach) $A = 16 + 4y + 4x + xy$ *sq. units.*

(all are unlike terms andso, we can't add)

Example 3.5

Multiply $(2x+5y)$ and $(3x-4y)$

Solution:

By horizontal distributive approach,

$$(2x+5y)(3x-4y) = 2x(3x-4y) + 5y(3x-4y)$$
$$= 6x^2 - 8xy + 15xy - 20y^2$$
$$= 6x^2 + 7xy - 20y^2 \text{ (simplify the like terms)}$$

Aliter

By grid approach

×	$2x$	$5y$
$3x$	$6x^2$	$15xy$
$-4y$	$-8xy$	$-20y^2$

$= 6x^2 - 8xy + 15xy - 20y^2$

$= 6x^2 + 7xy - 20y^2$

Try these

Multiply

(i) $(a-5)$ and $(a+4)$

(ii) $(a+b)$ and $(a-b)$

(iii) (m^4+n^4) and $(m-n)$

(iv) $(2x+3)(x+4)$

(v) $(3x+7)(x-5)$

(vi) $(x-2)(6x-3)$

Think

(i) In $3x^2(x^4-7x^3+2)$, what is the highest power in the expression?

(ii) Is $-5y^2+2y-6 = -(5y^2+2y-6)$? If not, correct the mistake.

Exercise 3.1

1. Complete the table.

×	$2x^2$	$-2xy$	x^4y^3	$2xyz$	$(\_\_)xz^2$
x^4					
(___)			$4x^5y^4$		
$-x^2y$					
$2y^2z$					$-10xy^2z^3$
$-3xyz$					
(___)				$-14xyz^2$	

2. Find the product of the terms.

   (i) $-2mn, (2m)^2, -3mn$ (ii) $3x^2y, -3xy^3, x^2y^2$

3. If $l=4pq^2$, $b=-3p^2q, h=2p^3q^3$ then, find the value of $l\times b\times h$.

4. Expand

   (i) $5x(2y-3)$ (ii) $-2p(5p^2-3p+7)$

   (iii) $3mn(m^3n^3-5m^2n+7mn^2)$ (iv) $x^2(x+y+z)+y^2(x+y+z)+z^2(x-y-z)$

5. Find the product of

   (i) $(2x+3)(2x-4)$ (ii) $(y^2-4)(2y^2+3y)$

   (iii) $(m^2-n)(5m^2n^2-n^2)$ (iv) $3(x-5)\times 2(x-1)$

6. Find the missing term.

   (i) $6xy\times$ _____ $=-12x^3y$ (ii) _____ $\times(-15m^2n^3p)=45m^3n^3p^2$

   (iii) $2y(5x^2y-$ ___ $+3$ ___ $)=10x^2y^2-2xy+6y^3$

7. Match the following.

a)	$4y^2 \times -3y$	(i)	$20x^2y - 20x$
b)	$-2xy(5x^2 - 3)$	(ii)	$5x^3 - 5xy^2 + 5x^2y$
c)	$5x(x^2 - y^2 + xy)$	(iii)	$4x^2 - 9$
d)	$(2x + 3)(2x - 3)$	(iv)	$-12y^3$
e)	$5x(4xy - 4)$	(v)	$-10x^3y + 6xy$

A) iv, v, ii, i, iii B) v, iv, iii, ii, i C) iv, v, ii, iii, i D) iv, v, iii, ii, i

8. A car moves at a uniform speed of $(x + 30)$ km/hr. Find the distance covered by the car in $(y + 2)$ hours. (Hint: distance = speed × time).

Objective Type Questions

9. The product of $7p^3$ and $(2p^2)^2$ is

(A) $14p^{12}$ (B) $28p^7$ (C) $9p^7$ (D) $11p^{12}$

10. The missing terms in the product $-3m^3n \times 9(\_\_) = \_\_\_\_\_ m^4n^3$ are

(A) $mn^2, 27$ (B) $m^2n, 27$ (C) $m^2n^2, -27$ (D) $mn^2, -27$

11. If the area of a square is $36x^4y^2$ then, its side is _____

(A) $6x^4y^2$ (B) $8x^2y^2$ (C) $6x^2y$ (D) $-6x^2y$

12. If the area of a rectangle is $48m^2n^3$ and whose length is $8mn^2$ then, its breadth is__.

(A) 6 mn (B) $8m^2n$ (C) $7m^2n^2$ (D) $6m^2n^2$

13. If the area of a rectangular land is $(a^2 - b^2)$ *sq.units* whose breadth is $(a - b)$ then, its length is_____

(A) $a - b$ (B) $a + b$ (C) $a^2 - b$ (D) $(a + b)^2$

3.3 Division of Algebraic Expressions

In the previous sessions, we have learnt how to add, subtract and multiply algebraic expressions. Now, we are going to learn about another basic operation **'division'** on algebraic expressions. We know that the division is the reverse operation of multiplication.

Now, the cost of 10 balls at the rate of ₹5 each $= 10 \times 5$

$= ₹50$

whereas if we have ₹50 and we want to buy 10 balls then,

the cost of each ball is $= \frac{50}{10}$

$= ₹5$

What we have seen above is division on numbers. But how will you divide an algebraic expression by another algebraic expression?

Of course, the same procedure has to be followed for the algebraic expressions with the help of laws of exponents.

If x is a variable and m, n are constants, then $x^m \div x^n = x^{m-n}$ where $m > n$.

3.3.1 Division of a monomial by another monomial

Dividing a monomial $10p^4$ by another monomial $2p^3$, we get

$10p^4 \div 2p^3$

$$\frac{10p^4}{2p^3} = \frac{\cancel{10}^{5} \times p \times \cancel{p} \times \cancel{p} \times \cancel{p}}{\cancel{2} \times \cancel{p} \times \cancel{p} \times \cancel{p}} \text{ (expansion of power)}$$

$= 5p$

However, to divide we can also follow laws of exponents as,

$$\frac{\cancel{10}^{5} p^4}{\cancel{2} p^3} = 5p^{4-3}$$

$= 5p$

$$\frac{x^m}{x^n} = x^{m-n}$$

Think

Are the following correct?

(i) $\frac{x^3}{x^8} = x^{8-3} = x^5$

(ii) $\frac{10m^4}{10m^4} = 0$

(iii) When a monomial is divided by itself, we will get 1?

Example 3.6

Velu pastes '$4xy$' pictures in one page of his scrap book. How many pages will he need to paste $100x^2y^3$ pictures? (x, y are positive integers)

Solution:

Total number of pictures $= 100x^2y^3$

Pictures in one page $= 4xy$

Total number of pages needed $= \frac{\text{Total number of pictures}}{\text{pictures in one page}}$

$$= \frac{\cancel{100}^{25} x^2 y^3}{\cancel{4}xy} = 25x^{2-1}y^{3-1}$$

$= 25xy^2$ pages

Try these

Divide

(i) $12x^3y^2$ by x^2y

(ii) $-20a^5b^2$ by $2a^3b^7$

(iii) $28a^4c^2$ by $21ca^2$

(iv) $(3x^2y)^3$ by $6x^2y^3$

(v) $64m^4(n^2)^3$ by $4m^2n^2$

(vi) $(8x^2y^2)^3$ by $(8x^2y^2)^2$

(vii) $81p^2q^4$ by $\sqrt{81p^2q^4}$

(vii) $(4x^2y^3)^0$ by $\frac{(x^3)^2}{x^6}$

3.3.2 Division of an algebraic expression (polynomial) by a monomial

To divide a polynomial by a monomial, divide each term of the polynomial by the monomial.

Example 3.7

Divide : $(5y^3 - 25y^2 + 8y)$ by $5y$

Solution:

We have, $(5y^3 - 25y^2 + 8y) \div 5y$

$$= \frac{5y^3 - 25y^2 + 8y}{5y}$$

$$= \frac{\cancel{5}y^3}{\cancel{5}y} - \frac{\overset{5}{\cancel{25}}y^2}{\cancel{5}y} + \frac{8y}{5y}$$

$$= y^{3-1} - 5y^{2-1} + \frac{8}{5}$$

$$= y^2 - 5y + \frac{8}{5}$$

Think

Are the following divisions correct?

(i) $\frac{4y+3}{4} = y + 3$ (ii) $\frac{5m^2+9}{9} = 5m^2$

(iii) $\frac{2x^2+8}{4} = 2x^2 + 2$. If not , correct it.

Try these

(i) $(16y^5 - 8y^2) \div 4y$

(ii) $(p^5q^2 + 24p^3q - 128q^3) \div 6q$

(iii) $(4m^2n + 9n^2m + 3mn) \div 4mn$

3.4 Avoid Some Common Errors

S.No	Error	Correct	Reason
1.	$2xx = 2x$	$2xx = 2 \times x^1 \times x^1 = 2x^2$	Product of variables
2.	$-3x - 4x = -1x$	$-3x - 4x = -7x$	Same sign factors should be added and put the same sign.
3.	$4y + 3y + y = 7y$	$4y + 3y + y = 8y$	y is same as $1y$,co-efficient 1 of a term is usually not written.
4.	$5x + 3x = 8x^2$	$5x + 3x = 8x$	When we add or subtract like terms, add or subtract only the co-efficient of the like terms, keep the variable as it is.
5.	$9x + 1 = 10x$	$9x + 1 = 9x + 1$	Unlike terms cannot be added or subtracted
6.	$3x + 4y = 7xy$	$3x + 4y = 3x + 4y$	Unlike terms cannot be added
7.	$3(4x + 9)$ $= 12x + 9$	$3(4x + 9) = 12x + 27$	3 is common factor multiply both the terms.
8.	$5 + (3y - 4)$ $= 15y - 20$	$5 + (3y - 4)$ $= 5 + 3y - 4$	Addition symbol is in between the terms, not multiplication

9.	$(-7x^2+2x+3)$ $=-(7x^2+2x+3)$	$-7x^2+2x+3$ $=-(7x^2-2x-3)$	Taking (-1) as common, we will have change in sign for all the terms.
10.	$(2x)^2=2x^2$	$(2x)^2=2^2\times x^2=4x^2$	Power is common for all the basic factors within the bracket
11.	$(2x-5)(3x-4)$ $=6x^2+20$	$(2x-5)(3x-4)$ $=2x(3x-4)-5(3x-4)$ $=6x^2-8x-15x+20$ $=6x^2-23x+20$	Distributive law should be followed
12.	$(x-9)^2=x^2-9^2$	$(x-9)^2=(x-9)(x-9)$ $=x^2-2(x)(9)+9^2$ $=x^2-18x+81$	Product of binomials $(a+b)^2$, $(a-b)^2$ use identities
13.	$\frac{p^2}{p^5}=p^{5-2}=p^3$	$\frac{p^2}{p^5}=\frac{1}{p^{5-2}}=\frac{1}{p^3}$	Law of exponents $x^m\div x^n=x^{m-n}$ when $m>n$
14.	$\frac{x^2+5}{5}=x^2$	$\frac{x^2+5}{5}=\frac{x^2}{5}+\frac{5}{5}$ $=\frac{x^2}{5}+1$	Divide each term of the expressions by the denominator
15.	$\frac{5\not{m}^2}{5\not{m}^2}=0$	$\frac{5m^2}{5m^2}=1$	a term divided by itself gives 1

Exercise 3.2

1. **Fill in the blanks:**

(i) $\frac{18m^4(\_\_)}{2m^3n^3}=\_\_\_\_\_\,mn^5$ (ii) $\frac{l^4m^5n^{(\_)}}{2lm^{(\_)}n^6}=\frac{l^3m^2n}{(\_\_)}$ (iii) $\frac{42a^4b^5(\_\_)}{6a^4b^2}=(\_\_)b^{(\_)}c^2$

2. Say True or False

(i) $8x^3y\div 4x^2=2xy$

(ii) $7ab^3\div 14ab=2b^2$

3. Divide

(i) $27y^3$ by $3y$ (ii) x^3y^2 by x^2y

(iii) $45x^3y^2z^4$ by $(-15xyz)$ (iv) $(3xy)^2$ by $9xy$

4. Simplify

(i) $\frac{3m^2}{m}+\frac{2m^4}{m^3}$ (ii) $\frac{14p^5q^3}{2p^2q}-\frac{12p^3q^4}{3q^2}$

5. Divide:

(i) $(32y^2-8yz)$ by $2y$ (ii) $(4m^2n^3+16m^4n^2-mn)$ by $2mn$

(iii) $5xy^2-18x^2y^3+6xy$ by $6xy$ (iv) $81(p^4q^2r^3+2p^3q^3r^2-5p^2q^2r^2)$ by $(3pqr)^2$

6. Identify the errors and correct them.

(i) $7y^2 - y^2 + 3y^2 = 10y^2$ (ii) $6xy + 3xy = 9x^2y^2$

(iii) $m(4m-3) = 4m^2 - 3$ (iv) $(4n)^2 - 2n + 3 = 4n^2 - 2n + 3$

(v) $(x-2)(x+3) = x^2 - 6$ (vi) $-3p^2 + 4p - 7 = -(3p^2 + 4p - 7)$

7. Statement A: If $24p^2q$ is divided by $3pq$, then the quotient is $8p$.

Statement B: Simplification of $\frac{(5x+5)}{5}$ is $5x$.

(i) Both A and B are true (ii) A is true but B is false

(iii) A is false but B is true (iv) Both A and B are false

8. Statement A: $4x^2 + 3x - 2 = 2(2x^2 + \frac{3x}{2} - 1)$

Statement B: $(2m-5)-(5-2m) = (2m-5) + (2m-5)$

(i) Both A and B are true (ii) A is true but B is false

(iii) A is false but B is true (iv) Both A and B are false

3.5 Identities

We have studied in the previous class about standard algebraic identities. An identity is an equation satisfied by any value that replaces its variable(s). Now, we shall recollect four known identities, which are,

$$(a+b)^2 \equiv a^2 + 2ab + b^2 \qquad (a-b)^2 \equiv a^2 - 2ab + b^2$$

$$(a^2 - b^2) \equiv (a+b)(a-b) \qquad (x+a)(x+b) \equiv x^2 + (a+b)x + ab$$

Instead of the symbol ≡, we use = to represent an identity without any confusion.

Try these

Expand the following

(i) $(p+2)^2 =$ ..... (ii) $(3-a)^2 =$ .....

(iii) $(6^2 - x^2) =$ ..... (iv) $(a+b)^2 - (a-b)^2 =$ .....

(v) $(a+b)^2 = (a+b) \times$ ..... (vi) $(m+n)(.....) = m^2 - n^2$

(vii) $(m+...)^2 = m^2 + 14m + 49$ (viii) $(k^2 - 49) = (k+\ldots)(k-\ldots)$

(ix) $m^2 - 6m + 9 =$ ..... (x) $(m-10)(m+5) =$ .....

Note

x=1 is the only solution for $7x + 3 = 10$ whereas any value of x satisfies $(x+2)^2 = x^2 + 4x + 4$. So $7x + 3 = 10$ is an equation $(x+2)^2 = x^2 + 4x + 4$ is an identity. An identity is an equation but vice versa is not true.

3.5.1 Application of Identities

The identities give an alternative method of solving problems on multiplication of algebraic expressions and also of numbers.

Example 3.8

Find the value of $(3a+4c)^2$ by using $(a+b)^2$ identity.

Solution:

Comparing $(3a+4c)^2$ with $(a+b)^2$, we have $a=3a, b=4c$

Now $(a+b)^2 = a^2+2ab+b^2$

$\therefore \quad (3a+4c)^2 = (3a)^2 + 2(3a)(4c) + (4c)^2$ (replacing a and b values)

$= 3^2a^2 + (2\times3\times4)(a\times c) + 4^2c^2$

$(3a+4c)^2 = 9a^2 + 24ac + 16c^2$

Example 3.9

Find the value of 998^2 by using $(a-b)^2$ identity.

Solution:

We know, 998 can be expressed as $(1000-2)$

$\therefore \ (998)^2 = (1000-2)^2$

This is in the form of $(a-b)^2$, we get $a=1000, \ b=2$

Now $(a-b)^2 = a^2-2ab+b^2$

$(1000-2)^2 = (1000)^2 - 2(1000)(2) + (2)^2$

$(998)^2 = 1000000 - 4000 + 4 = 996004$

Think

Which is corrcet?
$(3a)^2$ is equal to
(i) $3a^2$ (ii) 3^2a
(iii) $6a^2$ (iv) $9a^2$

Example 3.10

Simplify $(3x+5y)(3x-5y)$ by using $(a+b)\ (a-b)$ identity.

Solution:

We have $(3x+5y)(3x-5y)$

Comparing it with $(a+b)(a-b)$ we get $a=3x \quad b=5y$

Now $(a+b)(a-b) = a^2-b^2$

$(3x+5y)(3x-5y) = (3x)^2 - (5y)^2$ (replacing a and b values)

$= 3^2x^2 - 5^2y^2$

$(3x+5y)(3x-5y) = 9x^2 - 25y^2$

Example 3.11

Expand y^2-16 by using a^2-b^2 identity

Solution:

y^2-16 can be written as y^2-4^2

Comparing it with a^2-b^2, we get $a=y, b=4$

Now $a^2-b^2 = (a+b)(a-b)$

$y^2-4^2 = (y+4)(y-4)$

$y^2-16 = (y+4)(y-4)$

Example 3.12

Simplify $(5x+3)(5x+4)$ by using $(x+a)\,(x+b)$ identity.

Solution:

We have $(5x+3)(5x+4)$

Comparing it with $(x+a)(x+b)$, we get $x=5x$ and $a=3, b=4$

We know $(x+a)(x+b)=x^2+(a+b)x+ab$ (replacing x, a and b values)

$$(5x+3)(5x+4) = (5x)^2 + (3+4)\,(5x) + (3)\,(4)$$
$$= 5^2x^2 + (7)(5x) + 12$$
$$(5x+3)(5x+4) = 25x^2 + 35x + 12$$

Try these

Expand using appropriate identities.

(i) $(3p+2q)^2$ (ii) $(105)^2$ (iii) $(2x-5d)^2$

(iv) $(98)^2$ (v) $(y-5)(y+5)$ (vi) $(3x)^2-5^2$

(vii) $(2m+n)(2m+p)$ (viii) 203×197

(ix) Find the area of the square whose side is $(x-2)$ units.

(x) Find the area of the rectangle whose length and breadth are $(y+4)$ units and $(y-3)$ units.

3.6 Cubic Identities

I. $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

We shall prove it now,

$$LHS = (a+b)^3$$
$$= \big[(a+b)(a+b)\big](a+b) \text{ (expanded form)}$$
$$= (a+b)^2(a+b)$$
$$= (a^2+2ab+b^2)(a+b) \text{ (using identity)}$$
$$= a(a^2+2ab+b^2) + b(a^2+2ab+b^2) \text{ (using distributive law)}$$
$$= a^3 + 2a^2b + ab^2 + ba^2 + 2ab^2 + b^3$$
$$= a^3 + (2a^2b + ba^2) + (ab^2 + 2ab^2) + b^3 \text{ (grouping 'like' terms)}$$
$$= a^3 + 3a^2b + 3ab^2 + b^3$$
$$= RHS$$

$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

Hence, we proved the cubic identity by direct multiplication.

Aliter

$\times$	a^2	$2ab$	b^2
a	a^3	$2a^2b$	ab^2
b	a^2b	$2ab^2$	b^3

$= a^3 + 3a^2b + 3ab^2 + b^3$

Activity

You can visualize the geometrical proof of $(a+b)^3$ with the help of your teacher.

II. $(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$

We can prove this identity by direct multiplication

We have $(a-b)^3 = (a-b)(a-b)(a-b)$

$= (a-b)^2 \times (a-b)$

$= (a^2 - 2ab + b^2)(a-b)$

$= a(a^2 - 2ab + b^2) - b(a^2 - 2ab + b^2)$

$= a^3 - 2a^2b + ab^2 - ba^2 + 2ab^2 - b^3$

$= a^3 - 2a^2b - ba^2 + ab^2 + 2ab^2 - b^3$

$= a^3 - 3a^2b + 3ab^2 - b^3$

$= RHS$

$(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$

Hence, we proved.

Aliter

$\times$	a^2	$-2ab$	b^2
a	a^3	$-2a^2b$	ab^2
$-b$	$-a^2b$	$2ab^2$	$-b^3$

$= a^3 - 3a^2b + 3ab^2 - b^3$

$a^2b = ba^2$
Multiplication is commutative

III. $(x+a)(x+b)(x+c) = x^3 + (a+b+c)x^2 + (ab+bc+ca)x + abc$

We know that the identity $(x+a)(x+b) = x^2 + (a+b)x + ab$. Let us multiply this by a binomial $(x+c)$. Then we get.

$(x+a)(x+b)(x+c) = \big[(x+a)(x+b)\big](x+c)$

$= (x^2 + (a+b)x + ab) \times (x+c)$

$= x\big[x^2 + (a+b)x + ab\big] + c\big[x^2 + (a+b)x + ab\big]$ (distributive law)

$= x^3 + (a+b)x^2 + abx + cx^2 + (a+b)xc + abc$

$= x^3 + ax^2 + bx^2 + abx + cx^2 + acx + bcx + abc$

$= x^3 + (a+b+c)x^2 + (ab+bc+ca)x + abc$ (Combine x^2, x terms)

$(x+a)(x+b)(x+c) = x^3 + (a+b+c)x^2 + (ab+bc+ca)x + abc$

Thus, we summarise the cubic identities as :

- $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$
- $(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$
- $(x+a)(x+b)(x+c) = x^3 + (a+b+c)x^2 + (ab+bc+ca)x + abc$

Deductions:

The above identities give the following deductions:

(i) $(a+b)^3 = a^3 + b^3 + 3ab(a+b)$

(ii) $a^3 + b^3 = (a+b)\left(a^2 - ab + b^2\right)$

(iii) $(a-b)^3 = a^3 - b^3 - 3ab(a-b)$

(iv) $a^3 - b^3 = (a-b)\left(a^2 + ab + b^2\right)$

How? Try it!

3.6.1 Application of Cubic Identities

I. Using the identity $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

Example 3.13

Expand $(x+4)^3$

Solution:

Comparing $(x+4)^3$ with $(a+b)^3$, we get $a = x, b = 4$

We know $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

$$(x+4)^3 = (x)^3 + 3(x)^2(4) + 3(x)(4)^2 + (4)^3 \quad \text{(replacing } a, b \text{ values)}$$

$$= (x)^3 + 3x^2(4) + 3(x)(16) + 64$$

$$(x+4)^3 = x^3 + 12x^2 + 48x + 64$$

$(4)^2 = 4 \times 4 = 16$

$(4)^3 = 4 \times 4 \times 4 = 64$

Try to expand this by using

$(a+b)^3 = a^3 + b^3 + 3ab(a+b)$

Example 3.14

Find the value of $(103)^3$

Solution:

Now, $(103)^3 = (100+3)^3$

Comparing this with $(a+b)^3$, we get $a = 100, b = 3$

$$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3 \quad \text{replacing } a, b \text{ values,}$$

$$(100+3)^3 = (100)^3 + 3(100)^2(3) + 3(100)(3)^2 + (3)^3$$

$$= 1000000 + 3(10000)(3) + 3(100)(9) + 27$$

$$= 1000000 + 90000 + 2700 + 27$$

$$(103)^3 = 1092727$$

II. Using the identity $(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$

Example 3.15

Expand: $(y-5)^3$

Solution:

Comparing $(y-5)^3$ with $(a-b)^3$, we get $a = y, b = 5$

$$(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$$

$$(y-5)^3 = (y)^3 - 3(y)^2(5) + 3(y)(5)^2 - (5)^3$$

$$= (y)^3 - 3y^2(5) + 3(y)(25) - 125$$

$$(y-5)^3 = y^3 - 15y^2 + 75y - 125$$

Try to expand this by using

$(a-b)^3 = a^3 - b^3 - 3ab(a-b)$

Example 3.16

Find the value of $(98)^3$

Solution:

Now, $(98)^3 = (100-2)^3$

Comparing this with $(a-b)^3$, we get $a=100, b=2$

$$(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$$

$$(100-2)^3 = (100)^3 - 3(100)^2(2) + 3(100)(2)^2 - (2)^3$$

$$= 1000000 - 3(10000)(2) + 3(100)(4) - 8$$

$$= 1000000 - 60000 + 1200 - 8$$

$$(98)^3 = 941192$$

III. Using the identity $(x+a)(x+b)(x+c) = x^3 + (a+b+c)x^2 + (ab+bc+ca)x + abc$

Example 3.17

Expand: $(x+3)(x+5)(x+2)$

Solution:

Given $(x+3)(x+5)(x+2)$

Comparing this with $(x+a)(x+b)(x+c)$, we get $x=x, a=3, b=5, c=2$

$$(x+a)(x+b)(x+c) = x^3 + (a+b+c)x^2 + (ab+bc+ca)x + abc$$

$$(x+3)(x+5)(x+2) = (x)^3 + (3+5+2)(x)^2 + (3\times5 + 5\times2 + 2\times3)x + (3)(5)(2)$$

$$= x^3 + 10x^2 + (15+10+6)x + 30$$

$$(x+3)(x+5)(x+2) = x^3 + 10x^2 + 31x + 30$$

Try these

Expand : (i) $(x+5)^3$ (ii) $(y-2)^3$ (iii) $(x+1)(x+4)(x+6)$

Exercise 3.3

1. Expand

   (i) $(3m+5)^2$ (ii) $(5p-1)^2$ (iii) $(2n-1)(2n+3)$ (iv) $4p^2 - 25q^2$

2. Expand

   (i) $(3+m)^3$ (ii) $(2a+5)^3$ (iii) $(3p+4q)^3$ (iv) $(52)^3$ (v) $(104)^3$

3. Expand

   (i) $(5-x)^3$ (ii) $(2x-4y)^3$ (iii) $(ab-c)^3$ (iv) $(48)^3$ (v) $(97xy)^3$

4. Simplify $(p-2)(p+1)(p-4)$

5. Find the volume of the cube whose side is $(x+1)$ *cm*

6. Find the volume of the cuboid whose dimensions are $(x+2), (x-1)$ and $(x-3)$

7. If $x^2-y^2 = 16$ and $(x+y) = 8$ then $(x-y)$ is _____

(A) 8 (B) 3 (C) 2 (D) 1

8. $\frac{(a+b)(a^3-b^3)}{(a^2-b^2)}$ = _____

(A) a^2-ab+b^2 (B) a^2+ab+b^2 (C) $a^2+2ab+b^2$ (D) $a^2-2ab+b^2$

9. $(p+q)(p^2-pq+q^2)$ is equal to _____

(A) p^3+q^3 (B) $(p+q)^3$ (C) p^3-q^3 (D) $(p-q)^3$

10. $(a-b)=3$ and $ab=5$ then a^3-b^3 = _____

(A) 15 (B) 18 (C) 62 (D) 72

11. $a^3+b^3 = (a+b)^3$ – _____

(A) $3a(a+b)$ (B) $3ab(a-b)$ (C) $-3ab(a+b)$ (D) $3ab(a+b)$

3.7 Factorisation

Expressing any number as the product of two or more numbers is called as **factorisation.** The number 12 can be expressed as the product of prime factors like $12 = 2 \times 2 \times 3$. This is called prime factorisation. How will you factorise an algebraic expression? Yes. Expressing an algebraic expression as the product of two or more expressions is called the Factorisation.

Note

A number which is divisible by 1 and itself (or) A number which has only 2 factors are called prime numbers. Example: 2, 3, 5, 7, 11, ...

A number which has more than 2 factors are called composite numbers.Example: 4, 6, 8, 9, 10, 12, ...

For example, (i) $a^2-b^2 = (a+b)(a-b)$ Here, $(a+b)$ and $(a-b)$ are the two factors of a^2-b^2

(ii) $5y+30 = 5(y+6)$, Here 5 and $(y+6)$ are the factors of $5y+30$

Any expression can be Factorised as $(1) \times$ (expression)

For example, a^2-b^2 can also be factorised as

$$(1)\times\left(a^2-b^2\right) \text{ or } (-1)\times\left(b^2-a^2\right)$$

because '1' is a factor for all numbers and expressions

So, when we factorise the expressions, follow the suitable type of factorisation given below to get two or more factors other than 1. Stop doing the factorisation process once you have taken out all the common factors from the expression and then list out the factors.

Type 1: Factorisation by taking out the common factor from each term.

Example 3.18

Factorise: $4x^2y+8xy$

Solution:

We have, $4x^2y+8xy$ This can be written as,

$= (2\times2\times x\times x\times y) + (2\times2\times2\times x\times y)$

Taking out the common factor $2, 2, x, y$, we get

$$= 2 \times 2 \times x \times y(x+2)$$
$$= 4xy(x+2)$$

Type 2 : Factorisation by taking out the common binomial factor from each term

Example 3.19

(i) Factorise: $(2x+5)(x-y)+(4y)(x-y)$

Solution:

We have $(2x+5)(x-y)+(4y)(x-y)$

Taking out the common binomial factor $(x-y)$

we get, $(x-y)(2x+5+4y)$

(ii) Factorise $3n(p-2)+4(2-p)$

Solution:

We have $3n(p-2)+4(2-p)$ (taking – as common)

$3n(p-2)-4(p-2)$

Taking out the common binomial factor $(p-2)$

we get, $(p-2)(3n-4)$

Type 3 : Factorisation by grouping

Sometimes, the terms of a given expression are grouped suitably in such a way that they have a common factor so that the factorisation is easy to take out common factor from those terms.

Example 3.20

Factorise : $x^2 + yz + xy + xz$

Solution:

We have, $x^2 + yz + xy + xz$

Group the terms suitably as, $= (x^2 + xy) + (yz + xz)$

$= x(x+y) + z(y+x)$

$= x(x+y) + z(x+y)$ (addition is commutative)

$= (x+y)(x+z)$ [taking out the common factor $(x+y)$]

Type 4 : Factorisation using identities

(i) $(a+b)^2 = a^2 + 2ab + b^2$ (ii) $(a-b)^2 = a^2 - 2ab + b^2$ (iii) $a^2 - b^2 = (a+b)(a-b)$

Example 3.21

Factorise : $x^2 + 8x + 16$

Solution:

Now, $x^2 + 8x + 16$ can be written as $x^2 + 8x + 4^2$

Comparing this with $a^2 + 2ab + b^2 = (a+b)^2$ we get $a = x$; $b = 4$

$(x^2) + 2(x)(4) + (4)^2 = (x+4)^2$

$x^2 + 8x + 16 = (x+4)^2$

$(x+4)$, $(x+4)$ are the two factors.

Example 3.22

Factorise $49x^2 - 84xy + 36y^2$

Solution:

Now, $49x^2 - 84xy + 36y^2$

$7^2x^2 - 84xy + 6^2y^2 = (7x)^2 - 84xy + (6y)^2$

Comparing this with $a^2 - 2ab + b^2 = (a-b)^2$ we get $a = 7x$, $b = 6y$

$(7x)^2 - 2(7x)(6y) + (6y)^2 = (7x-6y)^2$

$\therefore\ 49x^2 - 84xy + 36y^2 = (7x-6y)^2$

$(7x–6y)$, $(7x–6y)$ are the two factors.

Try these

Find the factors

factor 1	factor 2	product	sum
		35	12
		-40	-3
		60	-17
		-51	+14
		-32	-4

Example 3.23

Factorise : $49x^2 - 64y^2$

Solution:

Now, $49x^2 - 64y^2$

$7^2x^2 - 8^2y^2 = (7x)^2 - (8y)^2$

Comparing this with $a^2 - b^2 = (a+b)(a-b)$ we get $a = 7x$, $b = 8y$

$(7x)^2 - (8y)^2 = (7x+8y)(7x-8y)$

$(7x+8y)$, $(7x–8y)$ are the two factors.

Type 5 : Factorisation of the expression ($ax^2 + bx + c$)

Example 3.24

Factorise $x^2 + 8x + 15$

Product = 15	Sum = 8
1×15 = 15	1+15 = 16
3 × 5 = 15	**3+5 = 8** ✓

Solution:

Given $x^2 + 8x + 15$

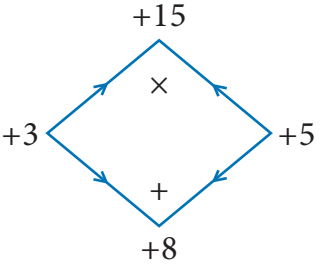
This is in the form of $ax^2 + bx + c$

We get $a = 1, b = 8, c = 15$

Now, the product $= a \times c$ and sum $= b$

$= 1 \times 15$ $\quad b = 8$

$= 15$



$= x^2 + 8x + 15$

$= x^2 + 3x + 5x + 15$ (the middle term $8x$ can be written as $3x+5x$)

$= (x^2 + 3x) + (5x + 15)$

$= x(x+3) + 5(x+3)$ taking out the common factor $x+3$)

$x^2 + 8x + 15 = (x+3)(x+5)$

Therefore, $(x+3)$, $(x+5)$ are the two factors.

Think

$x^2–4(x–2) = (x^2–4)(x–2)$

Is this correct? If not correct it.

Example 3.25

Factorise $7c^2 + 2c - 5$

Solution:

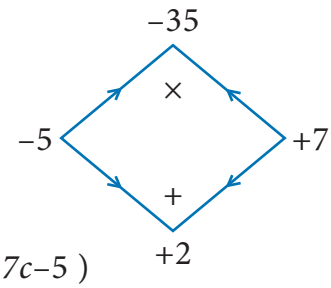
Given $7c^2 + 2c - 5$

This is in the form of $ax^2 + bx + c$

We get $a = 7,\ b = 2,\ c = -5$

Product = –35	Sum = 2
1×(–35) = –35	1–35 = –34
–1 × 35 = –35	–1+35 = 34
5 × (–7) = –35	5–7 = –2
–5 × 7 = –35	**–5+7 = 2** ✓

Now, the product $= a \times c = 7 \times (-5) = -35$ and sum $b = 2$



$= 7c^2 + 2c - 5$

$= 7c^2 - 5c + 7c - 5$ (the middle term $2c$ can be written as $-5c+7c$)

$= (7c^2 - 5c) + (7c - 5)$

$= c(7c - 5) + 1(7c - 5)$ (taking out the common factor $7c-5$)

$= (7c - 5)(c + 1)$

Therefore, $(7c-5)$, $(c+1)$ are the two factors.

Try these

Factorise the following :

1) $3y + 6$ 2) $10x^2 + 15y^2$ 3) $7m(m-5) + 1(5-m)$ 4) $64 - x^2$ 5) x^2-3x+2

6) $y^2-4y-32$ 7) $p^2+2p-15$ 8) $m^2+14m+48$ 9) x^2-x-90 10) $9x^2-6x-8$

3.7.1 Factorisation using cubic identities

The cubic identities are

(i) $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

(ii) $(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$

Note

$8a^3 = 2 \times 2 \times 2 \times a^3$

$= 2^3 a^3 = (2a)^3$

I. Factorise using the identity $(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

Example 3.26

Factorise: $x^3 + 15x^2 + 75x + 125$

Solution:

Given $x^3 + 15x^2 + 75x + 125$

This can be written as $x^3 + 15x^2 + 75x + 5^3$

Comparing with $a^3 + 3a^2b + 3ab^2 + b^3 = (a+b)^3$

we get $a = x$, $b = 5$

The given expression can be expressed as

$(x)^3 + 3(x)^2(5) + 3(x)(5)^2 + (5)^3 = (x + 5)^3$

$= (x + 5)(x + 5)(x + 5)$ are the three factors.

Note

Perfect cube numbers

A number which can be written in the form of $x \times x \times x$ is called perfect cube number.

Examples:

$8 = 2 \times 2 \times 2 = 2^3$

$27 = 3 \times 3 \times 3 = 3^3$

$125 = 5 \times 5 \times 5 = 5^3$

Here 8, 27, 125 are some of perfect cube numbers

II. Factorise using the identity $(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$

Example 3.27

Factorise: $8p^3 - 12p^2q + 6pq^2 - q^3$

Solution:

Given $8p^3 - 12p^2q + 6pq^2 - q^3$

This can be written as $(2p)^3 - 12p^2q + 6pq^2 - (q)^3$

Comparing this with $a^3 - 3a^2b + 3ab^2 - b^3 = (a-b)^3$ we get $a = 2p, b = q$

The given expression can be expressed as

$(2p)^3 - 3(2p)^2(q) + 3(2p)(q)^2 - (q)^3 = (2p - q)^3$

$= (2p - q), (2p - q), (2p - q)$ are the three factors.

Exercise 3.4

1. Factorise the following by taking out the common factor

(i) $18xy - 12yz$ (ii) $9x^5y^3 + 6x^3y^2 - 18x^2y$ (iii) $x(b-2c) + y(b-2c)$

(iv) $(ax + ay) + (bx + by)$ (v) $2x^2(4x-1) - 4x + 1$ (vi) $3y(x-2)^2 - 2(2-x)$

(vii) $6xy - 4y^2 + 12xy - 2yzx$ (viii) $a^3 - 3a^2 + a - 3$ (ix) $3y^3 - 48y$ (x) $ab^2 - bc^2 - ab + c^2$

2. Factorise the following expressions

(i) $x^2 + 14x + 49$ (ii) $y^2 - 10y + 25$ (iii) $c^2 - 4c - 12$ (iv) $m^2 + m - 72$ (v) $4x^2 - 8x + 3$

3. Factorise the following expressions using $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$ identity

(i) $64x^3 + 144x^2 + 108x + 27$ (ii) $27p^3 + 54p^2q + 36pq^2 + 8q^3$

4. Factorise the following expressions using $(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$ identity

(i) $y^3 - 18y^2 + 108y - 216$ (ii) $8m^3 - 60m^2n + 150mn^2 - 125n^3$

Objective type Questions

5. Factors of $9x^2 + 6xy$ are

(A) $3y, (x+2)$ (B) $3x, (3x+3y)$ (C) $6x, (3x+2y)$ (D) $3x, (3x+2y)$

6. Factors of $4 - m^2$ are

(A) $(2+m)(2+m)$ (B) $(2-m)(2-m)$ (C) $(2+m)(2-m)$ (D) $(4+m)(4-m)$

7. $(x+4)$ and $(x-5)$ are the factors of _____

(A) $x^2 - x + 20$ (B) $x^2 - 9x - 20$ (C) $x^2 + x - 20$ (D) $x^2 - x - 20$

8. The factors of $x^2 - 5x + 6$ are $(x-2)(x-p)$ then the value of p is _____

(A) -3 (B) 3 (C) 2 (D) -2

9. The factors of $1 - m^3$

(A) $(1+m), (1+m+m^2)$ (B) $(1-m), (1-m-m^2)$

(C) $(1-m), (1+m+m^2)$ (D) $(1+m), (1-m+m^2)$

10. One factor of $x^3 + y^3$ is

(A) $(x - y)$ (B) $(x + y)$ (C) $(x + y)^3$ (D) $(x - y)^3$

Exercise 3.5

Miscellaneous Practice Problems

1. Subtract: $-2(xy)^2(y^3+7x^2y+5)$ from $5y^2(x^2y^3-2x^4y+10x^2)$
2. Multiply $\left(4x^2+9\right)$ and $\left(3x-2\right)$
3. Find the simple interest on Rs. $5a^2b^2$ for $4ab$ years at $7b$% per annum.
4. The cost of a note book is Rs. $10ab$. If Babu has Rs. $\left(5a^2b+20ab^2+40ab\right)$. Then how many note books can he buy?
5. Factorise : $\left(7y^2-19y-6\right)$

Challenging problems

6. A contractor uses the expression $4x^2+11x+6$ to determine the amount of wire to order when wiring a house. If the expression comes from multiplying the number of rooms times the number of outlets and he knows the number of rooms to be $\left(x+2\right)$, find the number of outlets in terms of 'x'. [Hint : factorise $4x^2+11x+6$]
7. A mason uses the expression x^2+6x+8 to represent the area of the floor of a room. If the decides that the length of the room will be represented by $\left(x+4\right)$, what will the width of the room be in terms of x?
8. Find the missing term: $y^2+\left(\ldots\right)x+56=\left(y+7\right)\left(y+\ldots\right)$
9. Factorise : $16p^4-1$
10. Factorise : $3x^3 - 45x^2y + 225xy^2 - 375y^3$

3.8 Linear Equation in One Variable

3.8.1 Introduction

We shall recall some earlier ideas in algebra.

What is the formula to find the perimeter of a rectangle? If we denote the length by l and breadth by b, the perimeter P is given as $P = 2(l + b)$. In this formula, 2 is a fixed number whereas the literal numbers P, l and b are not fixed because they depend upon the size of the rectangle and hence P, l and b are **variables.** For rectangles of different sizes, their values go on changing. 2 is a **constant** (which does not change whatever may be the size of the rectangle).

An algebraic **expression** is a mathematical phrase having one or more algebraic terms including variables, constants and operating symbols (such as plus and minus signs).

Example: $4x^2 + 5x + 7xy + 100$ is an algebraic expression; note that the first term $4x^2$ consists of constant 4 and variable x^2. What is the constant in the term $7xy$? Is there a variable in the last term of the expression?

The 'number parts' of the terms with variables are **coefficients**. In $4x^2 + 5x + 7xy + 100$, the coefficient of the first term is 4. What is the coefficient of the second term? It is 5. The coefficient of the xy term is 7.

3.8.2 Forming algebraic expressions

We now to translate a few statements into an algebraic language and recall how to frame expressions. Here are some examples:

Statement	Expression	Comment
The sum of 8 and 7	$8 + 7$	When simplified, we get a single number. This is a numerical expression
The sum of x and 7	$x + 7$	We get an algebraic expression $x + 7$, since x is a variable.
16 divided by y	$\frac{16}{y}$	Here, y is a variable.
One more than thrice a number p	$3p+1$	Here p is a variable; 3 is the coefficient of p.
The product of a number and the same number less 3	$x(x - 3)$	Note that here x stands for the same number throughout in the expression. (We use brackets to indicate multiplication).

3.8.3 Equations

An **equation** is a statement that asserts the **equality** of two expressions; the expressions are written one on each side of an "equal to" sign.

For example: $2x + 7 = 17$ is an equation (where x is a variable). $2x + 7$ forms the Left Hand Side (LHS) of the equation and 17 is its Right Hand Side (RHS).

Linear equations

An equation containing only one variable with its highest power as one is called a linear equation. Examples: $3x - 7 = 10$.

Linear equations in one or more variables:

An equation is formed when a statement is put in the form of mathematical terms. Here are some examples:

(i) A number is added to 5 to get 25

This statement can be written as $x + 5 = 25$.

This equation $x + 5 = 25$ is formed by one variable (x) whose highest power is 1. So it is called a linear equation in one variable.

Therefore, an equation containing only one variable with its highest power as one is called a linear equation in one variable.

Examples: $5x - 2 = 8$, $3y + 24 = 0$

This linear equation in one variable is also known as **simple equation.**

(ii) Sum of two numbers is 45

This statement can be written as $x + y = 45$.

This equation $x + y = 45$ is formed by two variables x and y whose highest power is 1. Hence, we call it as a linear equation in two variables.

Now, in this class we shall learn to solve linear equations in one variable only. You will learn to solve other type of equations in higher classes.

Note

The equations so formed with power more than 1 of its variables, (2,3....etc.) are called as quadratic, cubic equations and so on.

Examples: (i) $x^2 + 4x + 7 = 0$ is a quadratic equation.

(ii) $5x^3 - x^2 + 3x = 10$ is a cubic equation.

Try these

Identify which among the following are linear equations.

(i) $2 + x = 19$ (ii) $7x^2 - 5 = 3$ (iii) $4p^3 = 12$ (iv) $6m+2$ (v) $n=10$

(vi) $7k - 12 = 0$ (vii) $\frac{6x}{8} + y = 1$ (viii) $5 + y = 3x$ (ix) $10p+2q=3$ (x) x^2-2x-4

Convert the following statements into linear equations:

Example 3.28

7 is added to a given number to give 19.

Solution:

Let the number be n.

When 7 is added to this number we get $n + 7$.

This result is to give 19.

Therefore, the equation is $n + 7 = 19$.

Example 3.29

The sum of 4 times a number and 18 is 28.

Solution:

Let the number be x.

4 times the number is $4x$.

Adding 18 now, we get $18 + 4x$.

Now the result should be 28.

Thus, the equation has to be $18 + 4x = 28$.

Think

(i) Is $t(t-5)=10$ a linear equation? Why?

(ii) Is $x^2 = 2x$, a linear equation? Why?

Try these

Convert the following statements into linear equations:

1. On subtracting 8 from the product of 5 and a number, I get 32.
2. The sum of three consecutive integers is 78.
3. Peter had a Two hundred rupee note. After buying 7 copies of a book he was left with ₹60.
4. The base angles of an isosceles triangle are equal and the vertex angle measures 80°.
5. In a triangle ABC, ∠A is 10° more than ∠B. Also ∠C is three times ∠A. Express the equation in terms of angle B.

3.8.4 Solution of a linear equation

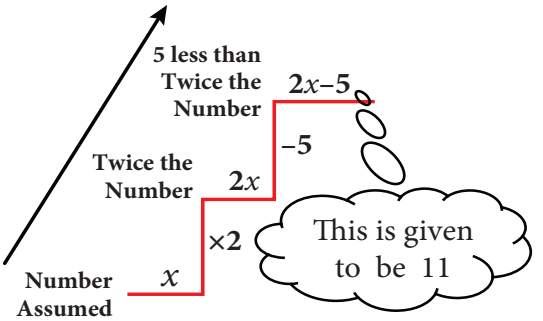
The value which replaces a variable in an equation so as to make the two sides of the equation equal is called a solution or root of the equation.

Example : $2x = 10$

We find that the equation is "satisfied" with the value $x = 5$. That is, if we put $x = 5$, in the equation, the value of the LHS will be equal to the RHS. Thus $x = 5$ is **a solution** of the equation. Note that no other value for x satisfies the equation. Thus one can say $x = 5$ is "**the**" solution of the equation.

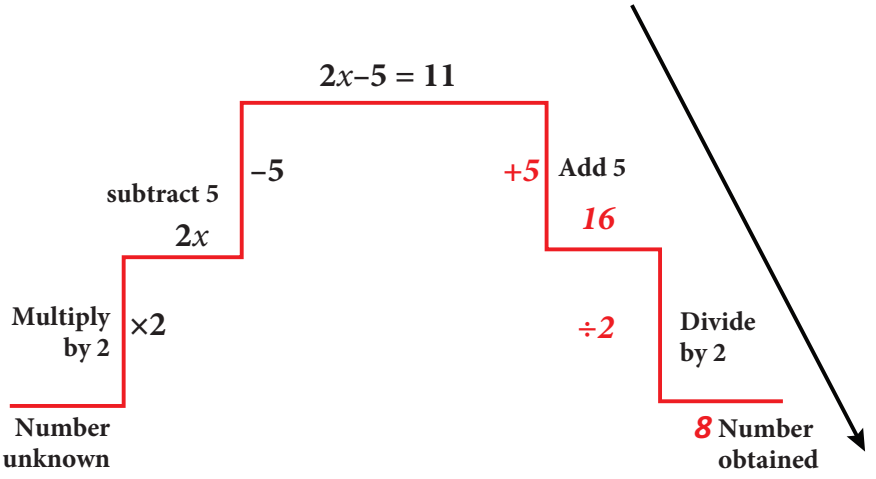
(i) The DO-UNDO Method:

Statement (given)	**You** **Think**
5 less than twice a number is 11.	The number needed is unknown. Let it be x. Twice the number gives $2x$. 5 less is $2x - 5$. This result is given to be 11.



This formation of equation can be visualized as follows:

From the number x, we reached $2x - 5$ by performing operations like subtraction, multiplication etc. So when $2x - 5 = 11$ is given, to get back to the value of x, we have to *'undo'* all that we did! Thus, we *'do'* to form the equation and *'undo'* to get the solution.



Example 3.30

(a) Solve the equation: $x - 7 = 6$

Solution:

$x - 7 = 6$ (Given)

$x - 7 + 7 = 6 + 7$ (add 7 on both sides)

$x = 13$

(b) Solve the equation: $3x = 51$

Solution:

$3x = 51$ (Given)

$3 \times x = 51$

$\frac{3 \times x}{3} = \frac{51}{3}$ (÷ 3 on both sides)

$x = 17$

(ii) Transposition method

The shifting of a number from one side of an equation to other is called transposition.

For the above example, (a) doing addition of 7 on both sides is the same as changing the number –7 on the left hand side to its additive inverse +7 and add it on the right hand side.

$x - 7 = 6$

$x = 6 + 7$

$x = 13$

Think

Can you get more than one solution for a linear equation?

likewise, (b) doing division by 3 on both sides is the same as changing the number 3 on the LHS to its reciprocal $\frac{1}{3}$ and multiply it on the RHS and vice-versa.

For Example (i) $6x = 12$

$x = \frac{12}{6}$

$x = 2$

(ii) $\frac{y}{7} = 10$

$y = 10 \times 7$

$y = 70$

Note

While rearranging the given linear equation, group the like terms on one side of the equality sign, and then do the basic arithmetic operations according to the signs that occur in the expression.

Example 3.31

Solve $2x + 5 = 9$

Solution:

$2x + 5 = 9$

$2x = 9 - 5$

$2x = 4$

$x = \frac{\cancel{4}^{2}}{\cancel{2}} = 2$

Example 3.32

Solve $\frac{4y}{3} - 7 = \frac{2y}{5}$

Solution:

(Rearranging the like terms)

$\frac{4y}{3} - \frac{2y}{5} = 7$

Try these

1. Solve for 'x' and 'y'

(i) $2x = 10$ (ii) $3 + x = 5$

(iii) $x - 6 = 10$ (iv) $3x + 5 = 2$

(v) $\frac{2x}{7} = 3$ (vi) $-2 = 4m - 6$

(vii) $4(3x - 1) = 80$ (viii) $3x - 8 = 7 - 2x$

(ix) $7 - y = 3(5 - y)$ (x) $4(1 - 2y) - 2(3 - y) = 0$

$$\frac{20y-6y}{15}=7$$

$$14y=7\times15$$

$$y=\frac{\cancel{7}\times15}{\cancel{14}_{2}}$$

$$y=\frac{15}{2}$$

Think

1. "An equation is multiplied or divided by a non zero number on either side." Will there be any change in the solution?
2. "An equation is multiplied or divided by two different numbers on either side". What will happen to the equation?

Exercise 3.6

1. **Fill in the blanks:**

   (i) The value of x in the equation $x+5=12$ is ————.

   (ii) The value of y in the equation $y-9=(-5)+7$ is ————.

   (iii) The value of m in the equation $8m=56$ is————.

   (iv) The value of p in the equation $\frac{2p}{3}=10$ is ————.

   (v) The linear equation in one variable has ————solution.

2. **Say True or False.**

   (i) The shifting of a number from one side of an equation to other is called transposition.

   (ii) Linear equation in one variable has only one variable with power 2.

3. **Match the following :**

(a) $\frac{x}{2}=10$	(i) $x=4$
(b) $20=6x-4$	(ii) $x=1$
(c) $2x-5=3-x$	(iii) $x=20$
(d) $7x-4-8x=20$	(iv) $x=\frac{8}{3}$
(e) $\frac{4}{11}-x=\frac{-7}{11}$	(v) $x=-24$

(A) (i), (ii), (iv), (iii), (v)
(B) (iii), (iv), (i), (ii), (v)
(C) (iii), (i), (iv), (v), (ii)
(D) (iii), (i), (v), (iv), (ii)

4. Find x and p (i) $\frac{2x}{3}-4=\frac{10}{3}$ (ii) $y+\frac{1}{6}-3y=\frac{2}{3}$ (iii) $\frac{1}{3}-\frac{x}{3}=\frac{7x}{12}+\frac{5}{4}$

5. Find x (i) $-3(4x+9)=21$ (ii) $20-2(5-p)=8$ (iii) $(7x-5)-4(2+5x)=10(2-x)$

6. Find x and m (i) $\frac{3x-2}{4}-\frac{(x-3)}{5}=-1$ (ii) $\frac{m+9}{3m+15}=\frac{5}{3}$

3.8.5 Word problems that involve linear equations

The challenging part of solving word problems is translating the statements into equations. Collect as many such problems and attempt to solve them.

Example 3.33

The sum of two numbers is 36 and one number exceeds another by 8. Find the numbers.

Solution:

Let the smaller number be x and the greater number be $x+8$

Given: the sum of two numbers = 36

$$\begin{aligned} x + (x+8) &= 36 \\ 2x + 8 &= 36 \\ 2x &= 36 - 8 \\ 2x &= 28 \\ x &= \frac{28}{2} = 14 \end{aligned}$$

Hence,

(i) The smaller number, $x = 14$

(ii) The greater number, $x + 8 = 14 + 8 = 22$

Example 3.34

A bus is carrying 56 passengers with some people having ₹8 tickets and the remaining having ₹10 tickets. If the total money received from these passengers is ₹500, find the number of passengers with each type of tickets.

Solution:

Let the number of passengers having ₹8 tickets be y. Then, the number of passengers with ₹10 tickets is (56–y).

Total money received from the passengers = ₹500

That is,
$$\begin{aligned} y \times 8 + (56 - y) \times 10 &= 500 \\ 8y + 560 - 10y &= 500 \\ 8y - 10y &= 500 - 560 \\ -2y &= -60 \\ y &= \frac{60}{2} \\ y &= 30 \end{aligned}$$

Hence, the number of passengers having,

(i) ₹8 tickets = 30

(ii) ₹10 tickets = 56 – 30 = 26

Example 3.35

The length of a rectangular field exceeds its breadth by 9 metres. If the perimeter of the field is 154m, find the length and breadth of the field.

Solution:

Let the breadth of the field be 'x' metres; then its length (x+9) metres.

Perimeter of the P = 2(length + breadth) = $2(x + 9 + x) = 2(2x + 9)$

Given that, $2(2x + 9) = 154$.

$$\begin{aligned} 4x + 18 &= 154 \\ 4x &= 154 - 18 \\ 4x &= 136 \\ x &= 34 \end{aligned}$$

Hence,

(i) Thus, breadth of the rectangular field = 34m

(ii) length of the rectangular field = $x+9 = 34+9 = 43m$

Example 3.36

There is a wooden piece of length 2*m*. A carpenter wants to cut it into two pieces such that the first piece is 40 *cm* smaller than twice the other piece. What is the length of the smaller piece ?

Solution:

Let us assume that the length of the first piece is x *cm*.

Then the length of the second piece is (200*cm* – x *cm*) i.e., (200 – x) *cm*.

According to the given statement (change *m* to *cm*),

First piece = 40 less than twice the second piece.

$$x = 2\times (200 - x) - 40$$

$$x = 400 - 2x - 40$$

$$x + 2x = 360$$

$$3x = 360$$

$$x = \frac{360}{3}$$

$$x = 120$$

Think

Suppose we take the second piece to be x and the first piece to be (200 – x), how will the steps vary? Will the answer be different?

Hence,

(i) Thus the length of the first piece is 120 cm

(ii) The length of second piece is 200cm – 120cm = 80cm, which happens to be the smaller.

Example 3.37

Mother is five times as old as her daughter. After 2 years, the mother will be four times as old as her daughter. What are their present ages?

Solution:

Age / Person	Now	After 2 years
Daughter	x	$x+2$
Mother	$5x$	$5x+2$

Given condition: After two years, Mother's age = 4 times of Daughter's age

$$5x + 2 = 4(x + 2)$$

$$5x + 2 = 4x + 8$$

$$5x - 4x = 8 - 2$$

$$x = 6$$

Hence daughter's present age = 6 years;

and mother's present age = $5x = 5 \times 6 = 30$ years

Example 3.38

The denominator of a fraction is 3 more than its numerator. If 2 is added to the numerator and 9 is added to the denominator, the fraction becomes $\frac{5}{6}$. Find the original fraction.

Solution:

Let the original fraction be $\frac{x}{y}$.

Given that $y = x + 3$. (Denominator = Numerator + 3).

Therefore, the fraction can be written as $\frac{x}{x+3}$. As per the given condition, $\frac{x+2}{(x+3)+9} = \frac{5}{6}$

By cross multiplication, $6(x+2) = 5(x+3+9)$

$$6x+12 = 5(x+12)$$
$$6x+12 = 5x+60$$
$$6x - 5x = 60 - 12$$
$$x = 60 - 12$$
$$x = 48.$$

Therefore, the original fraction is $\frac{x}{x+3} = \frac{48}{48+3} = \frac{48}{51}$.

Example 3.39

The sum of the digits of a two-digit number is 8. If 18 is added to the value of the number, its digits get reversed. Find the number.

Solution:

Let the two digit number be xy (i.e., ten's digit is x, ones digit is y)

Its value can be expressed as $10x+y$.

Given, $x+y = 8$ which gives $y = 8 - x$

Therefore its value is $10x+y$

$$= 10x + 8 - x$$
$$= 9x + 8.$$

The new number is yx with value is $10y + x$

$$= 10(8 - x) + x$$
$$= 80 - 9x$$

Given, when 18 is added to the given number (xy) gives new number (yx)

$$(9x + 8) + 18 = 80 - 9x$$
$$\text{This simplifies to } 9x + 9x = 80 - 8 - 18$$
$$18x = 54$$
$$x = 3 \Rightarrow y = 8 - 3 = 5$$
$$\text{The two digit number is } xy = 10x+y \Rightarrow 10(3)+5 = 30+5 = 35$$

Example 3.40

From home, Rajan rides on his motorbike at 35 km/hr and reaches his office 5 minutes late. If he had ridden at 50 km/hr, he would have reached his office 4 minutes earlier. How far is his office from his home?

Solution:

Let the distance be 'x' km. (Recall that, time = $\frac{\text{Distance}}{\text{Speed}}$)

Speed 1 = 35 km/hr
Speed 2 = 50 km/hr

Time taken to cover 'x' km at 35 km/hr: $T_1 = \frac{x}{35}$ hr

Time taken to cover 'x' km at 50 km/hr: $T_2 = \frac{x}{50}$ hr

According to the problem, the difference between two timings

$$= 4-(-5)$$
$$= 4+5 = 9 \text{ minutes}$$
$$= \frac{9}{60} \text{ hour (changing minutes to hour)}$$

Given, $T_1 - T_2 = \frac{9}{60}$

$$\frac{x}{35} - \frac{x}{50} = \frac{9}{60}$$
$$\frac{10x - 7x}{350} = \frac{9}{60}$$
$$\frac{3x}{350} = \frac{9}{60}$$
$$x = \frac{9}{60} \times \frac{350}{3}$$

The distance to his office $x = 17\frac{1}{2}$ km.

Exercise 3.7

1. Fill in the blanks:

(i) The solution of the equation $ax+b=0$ is_____.

(ii) If a and b are positive integers then the solution of the equation $ax=b$ has to be always_____.

(iii) One-sixth of a number when subtracted from the number itself gives 25. The number is_____.

(iv) If the angles of a triangle are in the ratio 2:3:4 then the difference between the greatest and the smallest angle is _____.

(v) In an equation $a + b = 23$. The value of a is 14 then the value of b is_____.

2. Say True or False

(i) "Sum of a number and two times that number is 48" can be written as $y+2y = 48$

(ii) $5(3x+2) = 3(5x-7)$ is a linear equation in one variable.

(iii) $x = 25$ is the solution of one third of a number is 10 less than the original number.

3. One number is seven times another. If their difference is 18, find the numbers.

4. The sum of three consecutive odd numbers is 75. Which is the largest among them?

5. The length of a rectangle is $\frac{1}{3}$ of its breadth. If its perimeter is 64m, then find the length and breadth of the rectangle.

6. A total of 90 currency notes, consisting only of ₹5 and ₹10 denominations, amount to ₹500. Find the number of notes in each denomination.

7. At present, Thenmozhi's age is 5 years more than that of Murali's age. Five years ago, the ratio of Thenmozhi's age to Murali's age was 3:2. Find their present ages.

8. A number consists of two digits whose sum is 9. If 27 is subtracted from the original number, its digits are interchanged. Find the original number.
9. The denominator of a fraction exceeds its numerator by 8. If the numerator is increased by 17 and the denominator is decreased by 1, we get $\frac{3}{2}$. Find the original fraction.
10. If a train runs at 60 km/hr it reaches its destination late by 15 minutes. But, if it runs at 85 kmph it is late by only 4 minutes . Find the distance covered by the train.

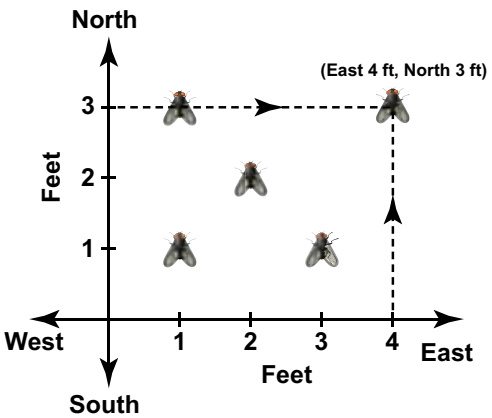
Objective Type Questions

11. Sum of a number and its half is 30 then the number is_____.
    (A) 15 (B) 20 (C) 25 (D) 40
12. The exterior angle of a triangle is 120° and one of its interior opposite angle 58°, then the other opposite interior angle is_____.
    (A) 62° (B) 72° (C) 78° (D) 68°
13. What sum of money will earn ₹500 as simple interest in 1 year at 5% per annum?
    (A) 50000 (B) 30000 (C) 10000 (D) 5000
14. The product of LCM and HCF of two numbers is 24. If one of the number is 6, then the other number is _____.
    (A) 6 (B) 2 (C) 4 (D) 8
15. The largest number of the three consecutive numbers is $x+1$, then the smallest number is
    (A) x (B) $x+1$ (C) $x+2$ (D) $x-1$

3.9 Graph

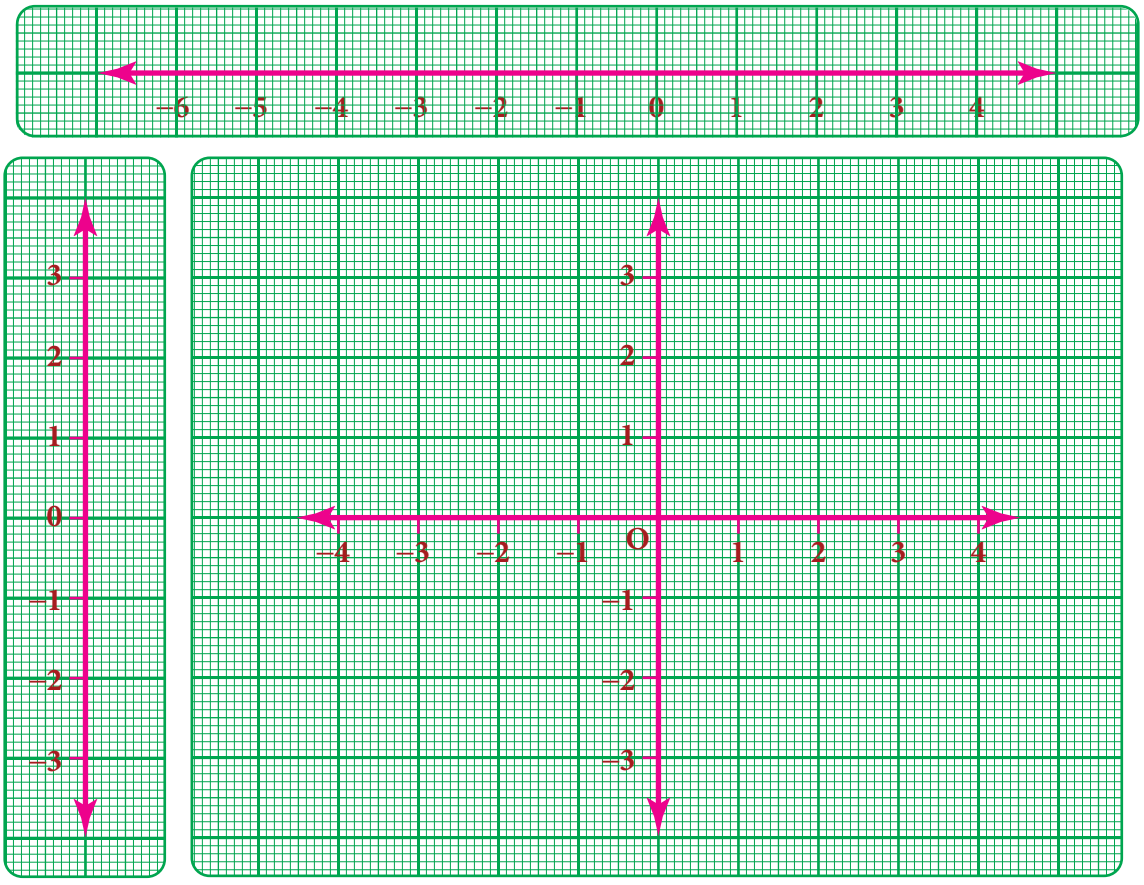
3.9.1 Introduction

There was an instance in the 17th century when **Rene Descartes**, a famous mathematician became ill and from his bed, noticed an insect hovering over a corner and sitting at various places on the ceiling. He wanted to identify all the places where the insect sat on the ceiling. Immediately, he drew the top plane of the room in a paper, creating the horizontal and vertical lines. Based on these perpendicular lines, he used the directions and understood that the places of the insect can be spotted by the movement of the insect in the east, west, north and south directions. He called that place as (x,y) in the plane which indicates two values, one (x) in the horizontal direction and the other (y) in the vertical direction (say east and north in this case). This is how the concept of graphs came into existence.



3.9.2 Graph sheets

Graph is just a visual method for showing relationships between numbers. In the previous class, we studied how to represent integers on a number line horizontally. Now take one more number line – vertically. We take the graph sheet keeping both number lines mutually perpendicular to each other at '0'zero as given in the figure. The number lines and the marking integers should be placed along the dark lines of the graph sheet.



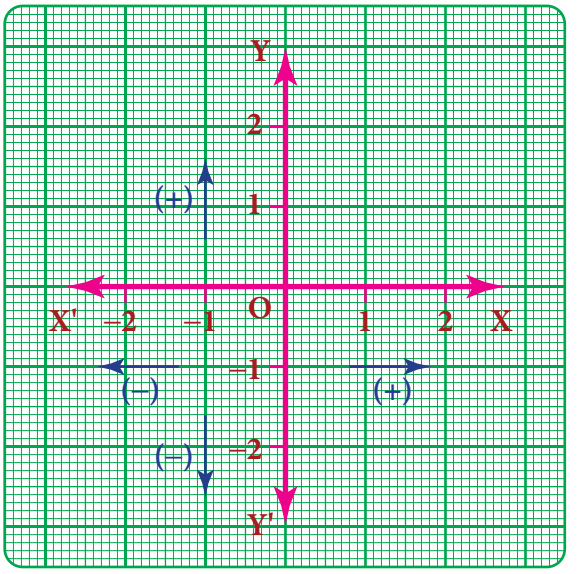
The intersecting point of the perpendicular lines 'O' represent the **origin (0, 0)**.

Cartesian system

Rene Descartes system of fixing a point with the help of two measurements, horizontal and vertical, is named as **Cartesian system,** in his honour. The horizontal line is named as XOX', called the X-axis. The vertical line is named as YOY', called the Y- axis. Both the axes are called **coordinate axes**. The plane containing the X axis and the Y axis is known as the coordinate plane or the **Cartesian plane.**

3.9.3 Signs in the graphs

1. X-coordinate of a point is positive along OX and negative along OX'
2. Y-coordinate of a point is positive along OY and negative along OY'



3.9.4 Ordered pairs

A point represents a position in a plane. A point is denoted by a pair (a,b) of two numbers 'a' and 'b' listed in a specific order in which 'a' represents the distance along the X-axis and 'b' represents the distance along the Y-axis. It is called an **ordered pair (*a*,*b*)**. It helps us to locate *precisely* a point in the plane. Each point can be exactly identified by a pair of numbers. It is also clear that the point (*b*,*a*)

is not same as (a,b) as they both indicate different orders.

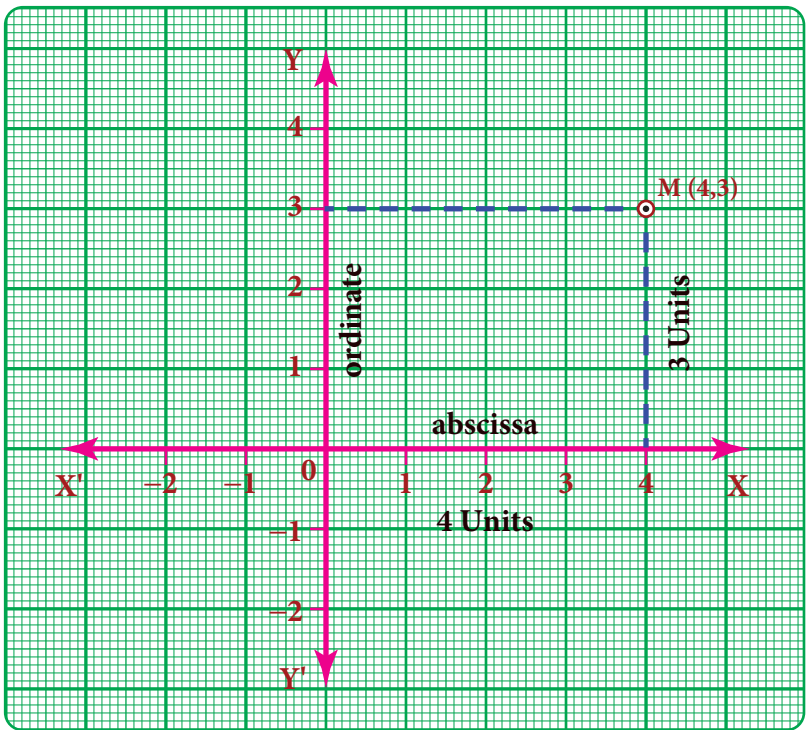
We have, XOX' and YOY' as the co-ordinate axes and let 'M' be (4,3) in the plane.To locate 'M'

(i) you (always) start at O, a *fixed* point (which we have, agreed to call as *origin*),

(ii) *first*, move 4 units along the horizontal direction (that is, the direction of x-axis)

(iii) *and then* trek along the y- direction by 3 units.

To understand how we have travelled to reach M, we denote by (4,3).



4 is called the x-coordinate of M and 3 is called the y-coordinate of M.

It is also habitual to name the **x-coordinate as abscissa** and the **y-coordinate as ordinate**. (4,3) is as an *ordered pair*.

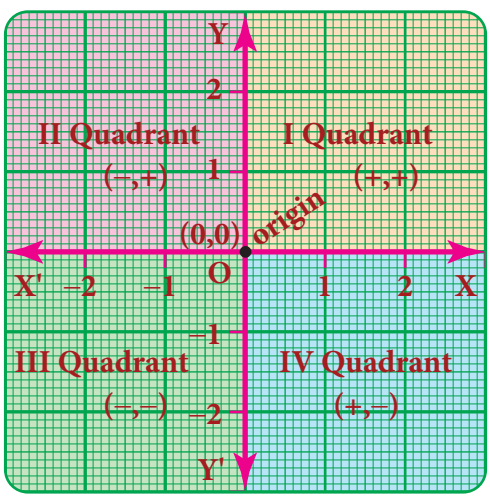
Think

If instead of (4,3), we write (3,4) and try to mark it, will it represent 'M' again?

3.9.5 Quadrants

The coordinate axes divide the plane of the graph into four regions called quadrants. It is a convention that the quadrants are named in the anti clock wise sense starting from the positive side of the X axis.

Quadrant	Sign
I the region XOY	$x>0$, $y>0$, then the coordinates are (+,+) Examples: (5,7) (2,9) (10,15)
II the region X'OY	$x<0$, $y>0$ then the coordinates are (−,+) Examples (−2,8) (−1,10) (−5,3)
III the region X'OY'	$x<0$, $y<0$ then the coordinates are (−,−) Examples: (−2,−3) (−7,−1) (−5,−7)
IV the region XOY'	$x>0$, $y<0$ then the coordinates are (+,−) Examples : (1,−7) (4,−2) (9,−3)



Coordinate of a point on the axes:

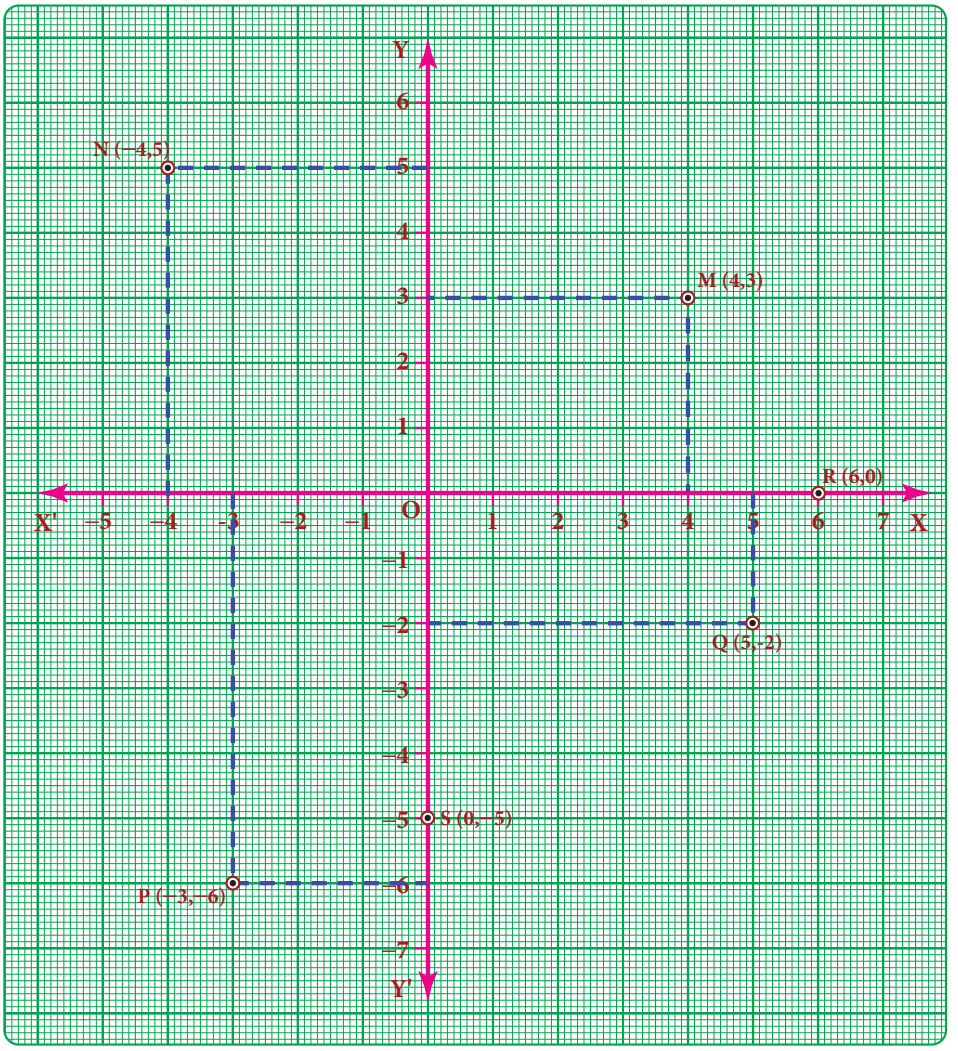
- If y=0 then the coordinate $(x, 0)$ lies on the 'x'-axis.
  For example (2, 0) (–5, 0) (7,0) are points on the 'x'-axis.
- If x=0 then the coordinate (0,y) lies on the 'y'-axis.
  For example (0, 3) (0,–4) (0, 9) are points on the 'y'-axis.

3.9.6 Plotting the given points on a graph

Consider the following points (4,3), (–4,5), (–3,–6), (5,–2), (6,0), (0,–5)

(i) To locate (4,3).

Start from origin O, move 4 units along OX and from 4, move 3 units parallel to OY to reach M(4,3).



(ii) To locate (–4,5)

From the origin, move 4 units along OX' and from –4, move 5 units parallel to OY to reach N(–4,5).

(iii) To locate (– 3, – 6)

From the origin move 3units along OX' and from –3, move 6 units parallel to OY' to reach P(-3,-6).

(iv) To locate (5,–2)

From the origin move 5 points along OX and from 5, move 2 units parallel to OY' to reach Q(5,–2).

(v) To locate (6,0) and (0, – 5)

In the given point (6, 0), X-coordinate is 6 and Y-coordinate is zero. So the point lies on the x-axis. Move 6 units on OX from the origin to reach R(6, 0).

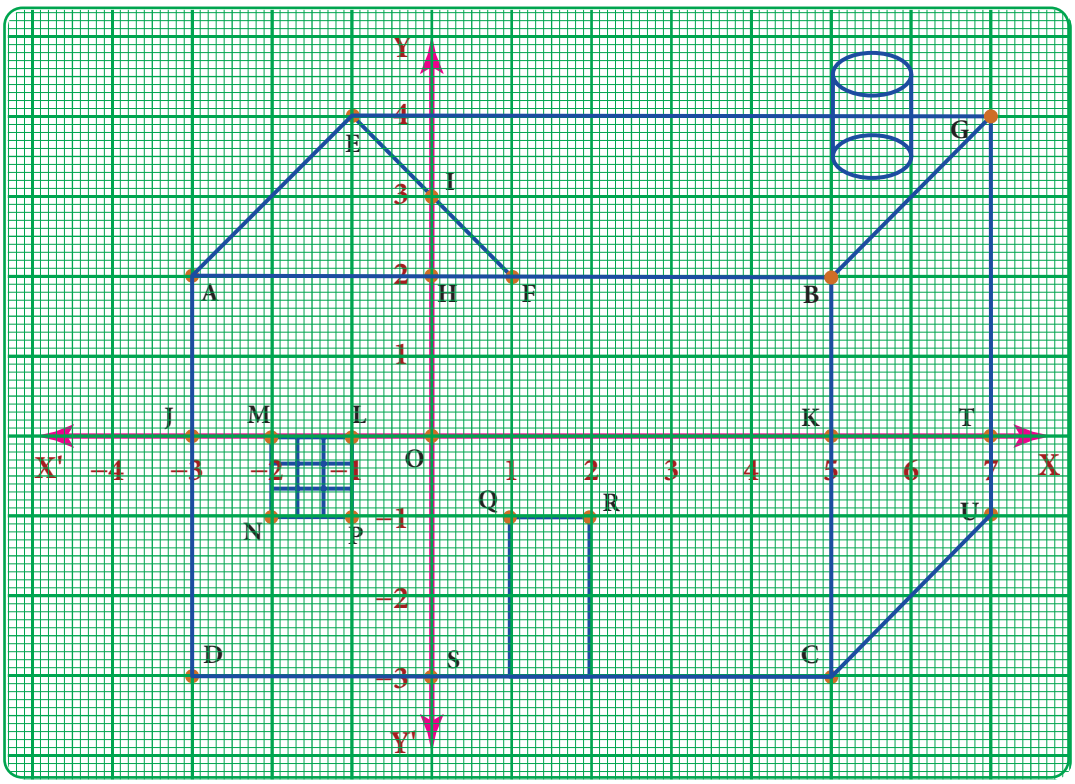
In the given point (0,–5) X-coordinate is zero and Y- coordinate is (–5). So, the point lies on Y-axis. Move 5 units on OY' from the origin to reach S (0,–5).

Try these

1. Complete the table given below.

S.No	Point	Sign of X-coordinate	Sign of Y-coordinate	Quadrant
1	(−7,2)			
2	(10,−2)			
3	(−3,−7)			
4.	(3,1)			
5.	(7,0)			
6.	(0,−4)			

2. Write the coordinates of the points marked in the following figure



Exercise 3.8

1 Fill in the blanks:

(i) X- axis and Y-axis intersect at _____.

(ii) The coordinates of the point in third quadrant are always_____.

(iii) (0, −5) point lies on _____axis.

(iv) The x- coordinate is always _____on the y-axis.

(v) _____ coordinates are the same for a line parallel to Y-axis.

2. Say True or False:

(i) (−10,20) lies in the second quadrant.

(ii) (−9, 0) lies on the x-axis.

(iii) The coordinates of the origin are (1,1).

2BJOP

3 Find the quadrants without plotting the points on a graph sheet.

(3, −4), (5,7), (2,0), (−3, −5), (4, −3), (−7,2), (−8,0), (0,10), (−9,50).

4 Plot the following points in a graph sheet.

A(5,2), B(−7, −3), C(−2,4), D(−1,−1), E(0, −5), F(2,0), G(7, −4), H(−4,0), I(2,3), J(8,−4), K(0,7).

5 Use the graph to determine the coordinates where each figure is located.

a) Star _____

b) Bird _____

c) Red Circle _____

d) Diamond _____

e) Triangle _____

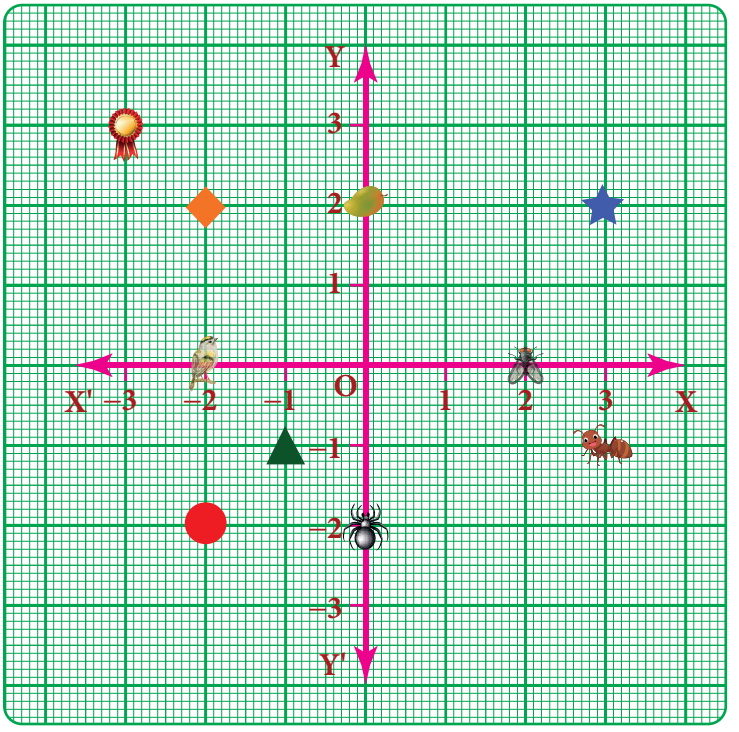
f) Ant _____

g) Mango _____

h) Housefly _____

i) Medal _____

j) Spider _____



3.9.7 To obtain a straight line

Now, we know how to plot the points on the graph. The points may lie on the graph in different order. If we join any two points we will get a straight line.

Example 3.41

Draw a straight line by joining the points A (−2,6) and B (4,−3)

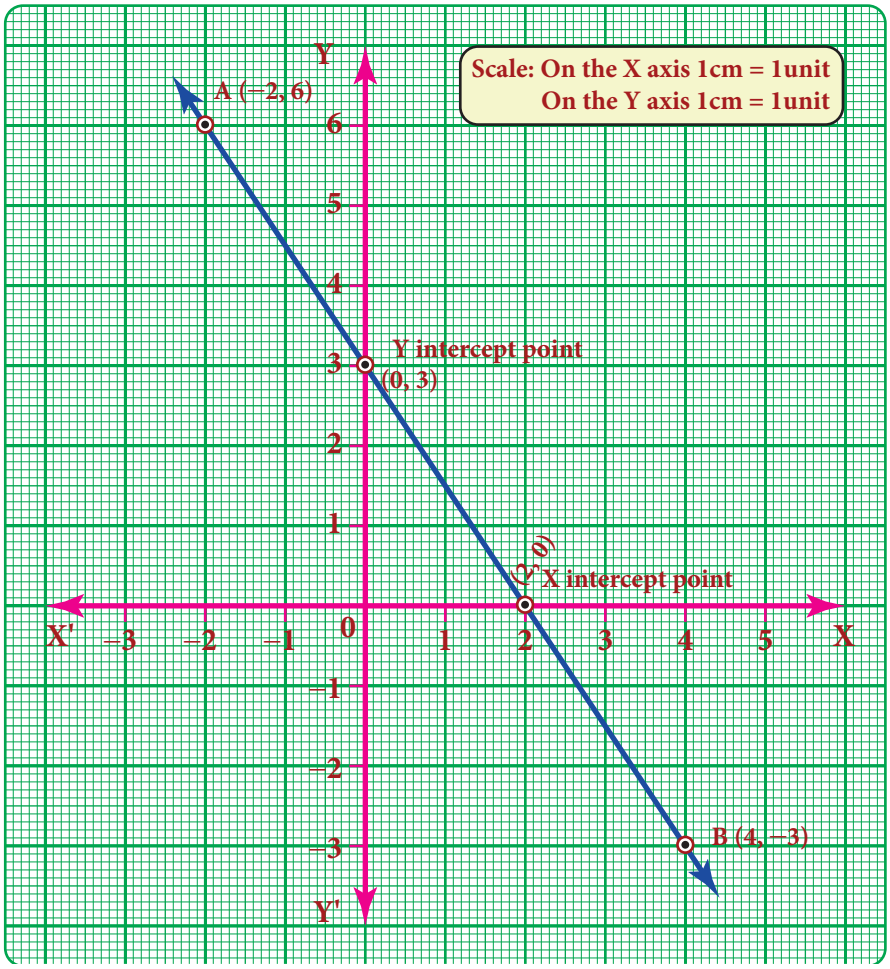
Solution:

The given first point A (−2,6) lies in the II quadrant and plot it. Second point B (4,−3) lies in the IV quadrant and plot it.

Now join the point A and point B using scale and extend it. We get a straight line.

Note:

The straight line intersects X axis at (2,0) and Y axis at (0,3).

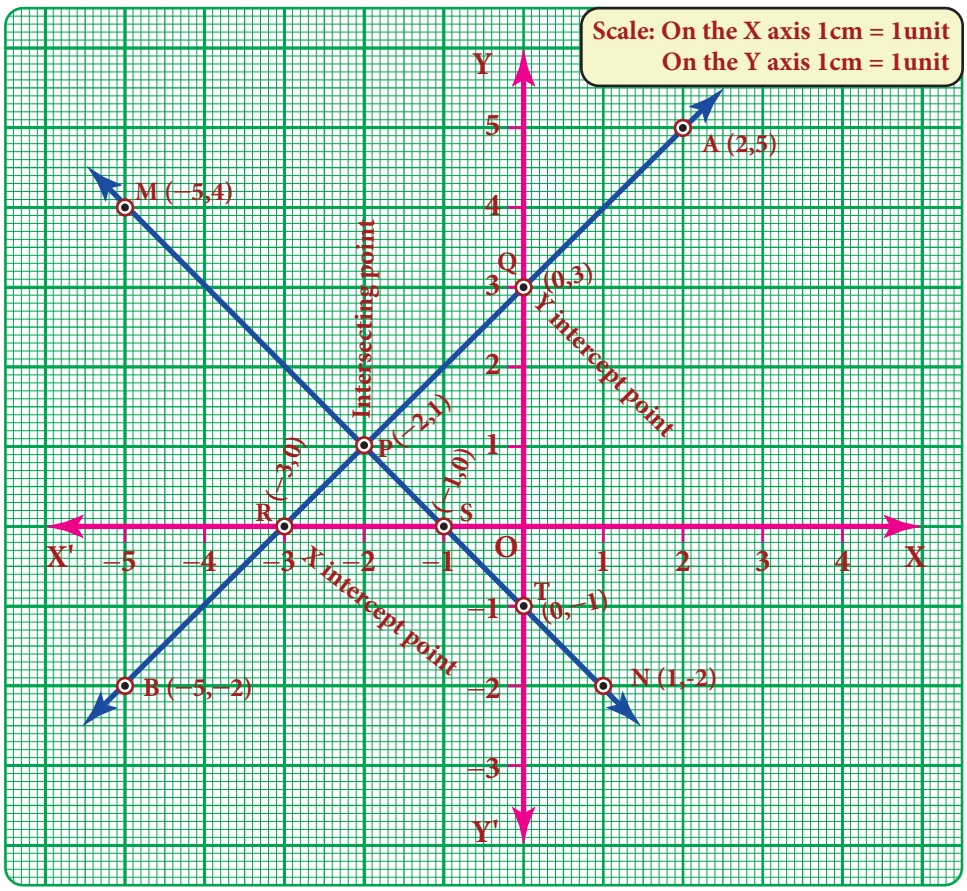


Example 3.42

Draw straight lines by joining the points A(2, 5) B(−5,−2) M(−5, 4) N(1,−2) also find the point of intersection

Solution:

Plot the first pair of points A and B in I and III quadrants. Join the points and extend it to get AB straight line. Plot the second pair of points M and N in II and IV quadrants. Join the points and extend it to get MN straight line.



Now, both lines are intersect at P(−2,1)

(i) The line AB intersect the coordinate axis, ie) x-axis at R(−3,0) and y-axis at Q(0,3)

(ii) The line MN intersect the coordinate axis, ie) x-axis at S(−1,0) and y-axis at T(0,−1)

3.9.8 Line parallel to the coordinate axes

- If a line is parallel to the X-axis, its distance from X axis is the same and it is represented as y = c .
- If a line is parallel to the Y-axis, its distance from Y axis is the same and it is represented as x = k. (Here c and k are constants)

Example 3.43

Draw the graph of $x = 5$

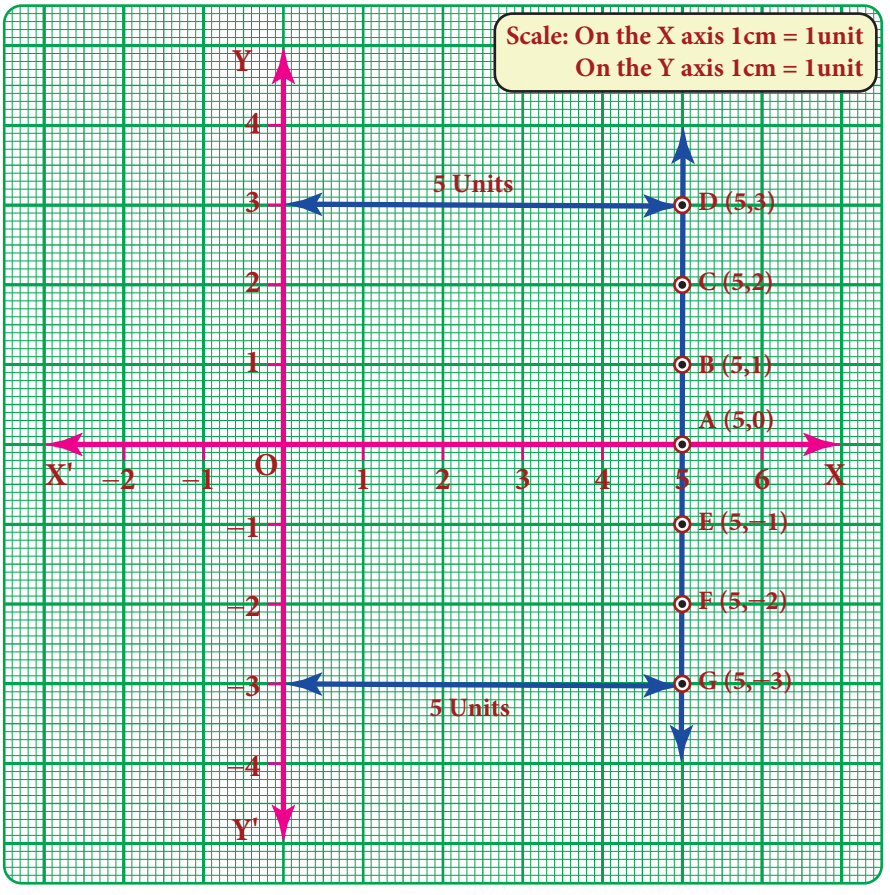
Solution:

x=5 means that x-coordinate is always 5 for whatever value of y-coordinate. So we may give any value for y-coordinate and this is tabulated as follows.

X	5	5	5	5	5
Y	−2	−1	0	2	3

x =5 is given (fixed)

Take any value for y (Why?)



Note

x =0 represents the Y axis

y =0 represents the X axis

Think

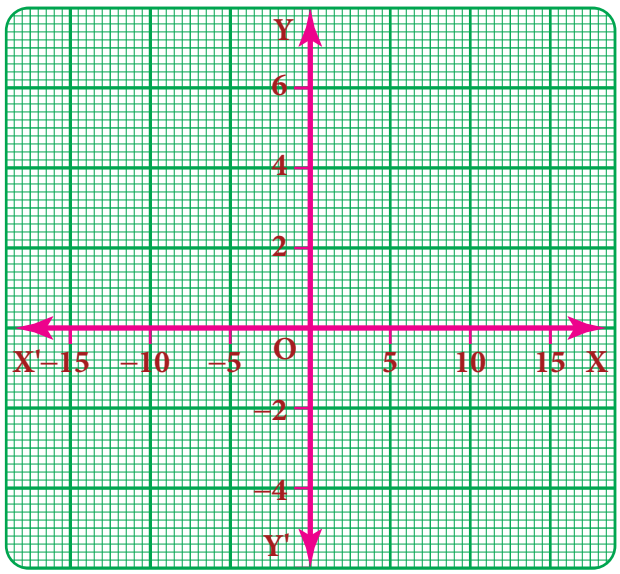
Which of the points (5, −10) (0, 5) (5, 20) lie on the straight line X = 5?

The points are (5,−2) (5,−2) (5,0) (5,2) (5,3). Plot the points in the graph and join them.We get a straight line parallel to Y axis at a distance of 5 units from the Y axis.

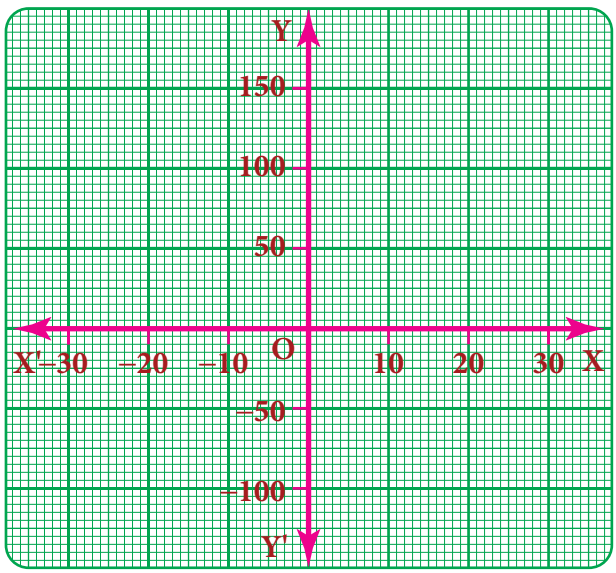
3.9.9 Scale in a graph

There will be situations in drawing a graph where 'y' is a big multiple of 'x' and the usual graph in units may not be enough to locate the 'y' coordinate and vice-versa. In this situation,

we use the concept of scale in both the axes as per the need. Represent a convenient scale at the right side corner of the graph. A few examples are given below.



Example 1
On the X axis 1 cm = 5 units.
On the Y axis 1 cm = 2 units.

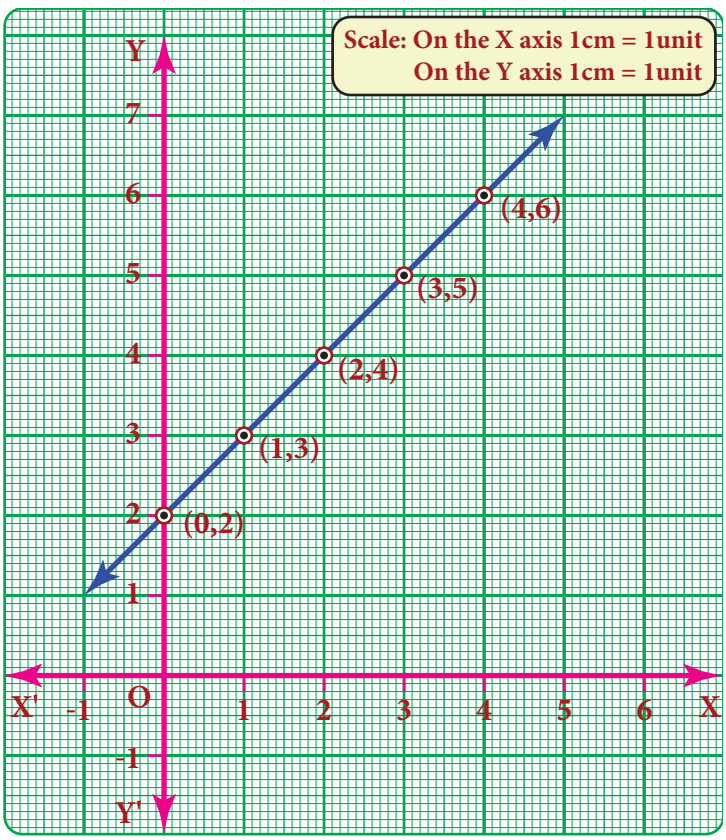


Example 2
On the X axis 1 cm = 10 units.
On the Y axis 1 cm = 50 units.

3.10 Linear Graph

3.10.1 Linear pattern

Plot the following points on a coordinate plane: (0,2), (1,3), (2,4), (3,5), (4, 6). What do you find? They all lie on a line! There is some pattern in them. Look at the y- coordinate in each ordered pair: 2 = 0+2; 3 = 1+2; 4 = 2+2; 5 = 3+2; 6 = 4+2 In each pair, the y- coordinate is 2 more than the x- coordinate. The coordinates of each point have the same relationship between them. All the points plotted lie on a line!



In such a case, when all the points plotted lie on a line, we say '*a linear pattern*' exists.

In this example, we found that in each ordered pair y value = x value +2. Therefore the linear pattern above can be denoted by the algebraic equation $y = x + 2$. Such an equation is called a *linear equation* and the line graph for linear equation is called a *linear graph*.

Linear equations use one (or more) variables where one variable is dependent on the other(s).

The longer the distance we travel by a taxi, the more we have to pay. The distance travelled is an example of an *independent variable*. Being dependent on the distance, the taxi fare is called the *dependent variable*.

The more one uses electricity, greater will be the amount of electricity bill. The amount of electricity consumed is an example for independent variable and the bill amount is naturally the dependent variable.

3.10.2 Graph of a linear function in two variables

We have talked about parallel lines, intersecting lines etc., in geometry but never actually looked at how far apart they were, or where they were .Drawing graphs helps us place lines. A linear equation is an equation with two variables (like x and y) whose graph is a line. To graph a linear equation, we need to have at least two points, but it is usually safe to use more than two points. (Why?) When choosing points, it would be nice to include both positive and negative values as well as zero. There is a **unique line** passing through any pair of points.

Example 3.44

Draw the graph of $y = 5x$

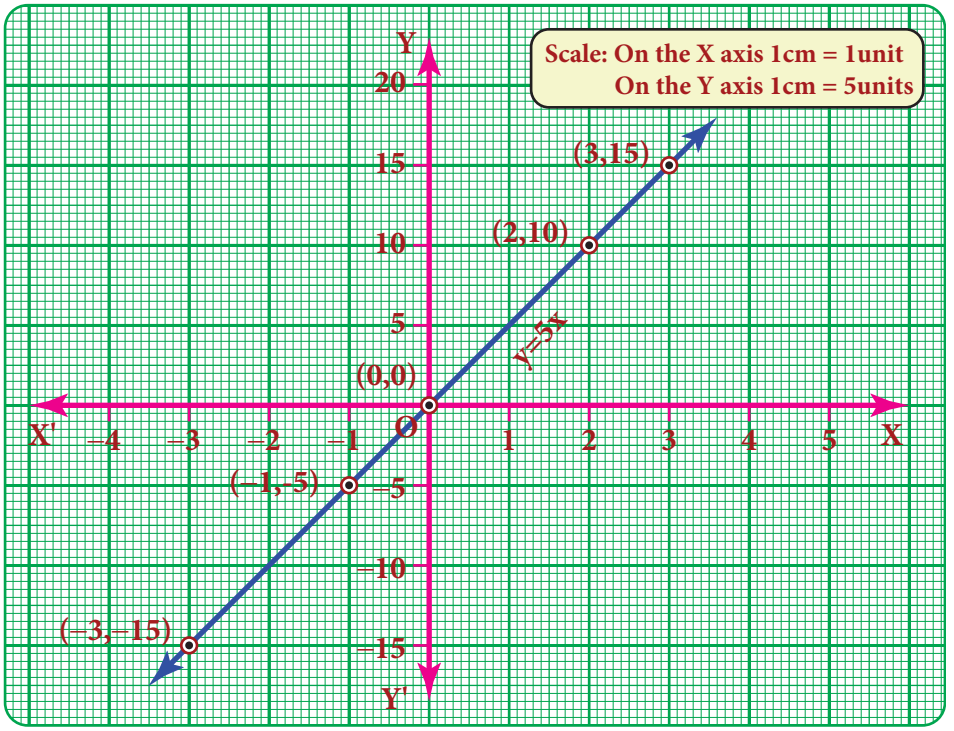
x	−3	−1	0	2	3
y	−15	−5	0	10	15

x	$y = 5x$
−3	$y = 5 \times (-3) = -15$
−1	$y = 5 \times (-1) = -5$
0	$y = 5 \times (0) = 0$
2	$y = 5 \times (2) = 10$
3	$y = 5 \times (3) = 15$

Solution:

The given equation $y = 5x$ means that for any value of x, y takes five times of x value.

Plot the point (−3,−15) (−1,−5) (0,0) (2,10) (3,15)



Example 3.45

Graph the equation $y = x + 1$.

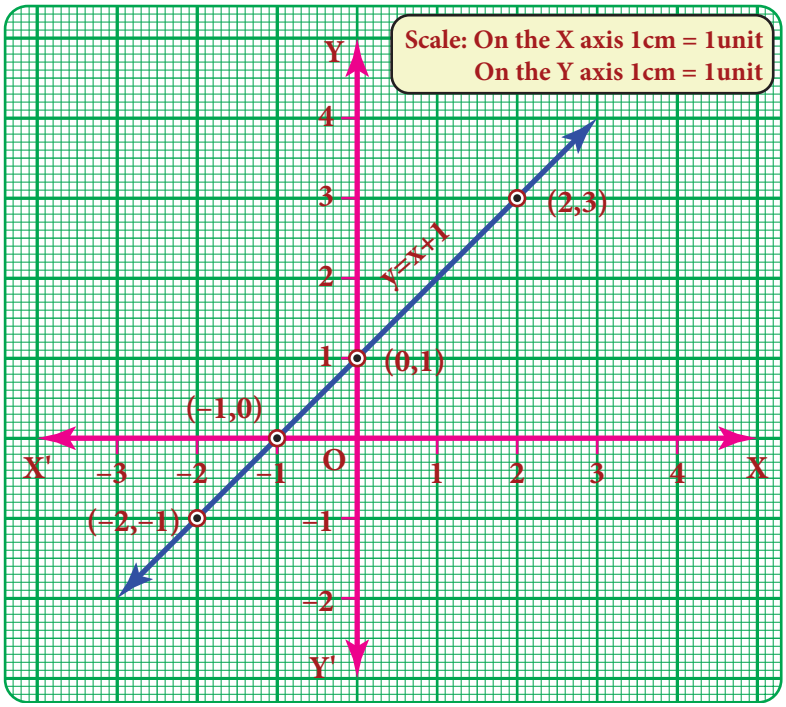
Begin by choosing a couple of values for x and y. It will firstly help to see

(i) what happens to y when x is zero and

(ii) what happens to x when y is zero.

After this we can go on to find one or two more values.

Let us find at least two more ordered pairs. For easy graphing, let us avoid fractional answers. We shall make suitable guesses.



x	−2	−1	0	1	2
y	−1	0	1	2	3

x	$y = x+1$
−2	$y = -2+1 = -1$
−1	$y = -1+1 = 0$
0	$y = 0+1 = 1$
1	$y = 1+1 = 2$
2	$y = 2+1 = 3$

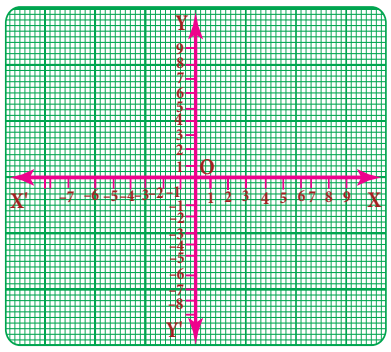
Note

The orientation of the graph will be different according to the scale chosen.

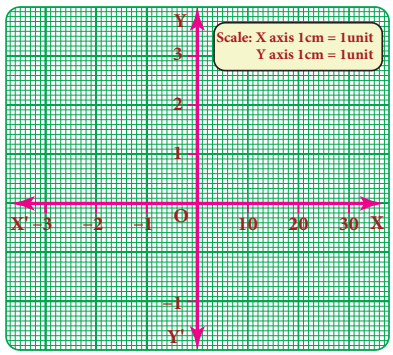
We have now five points of the graph: **(−2,−1) (−1, 0) (0, 1) (1, 2) and (2, 3).**

Try these

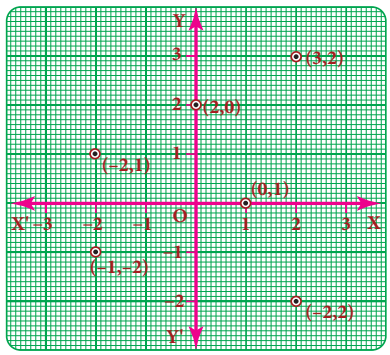
Identify and correct the errors



(i)



(ii)



(iii)

Exercise 3.9

1. Fill in the blanks:

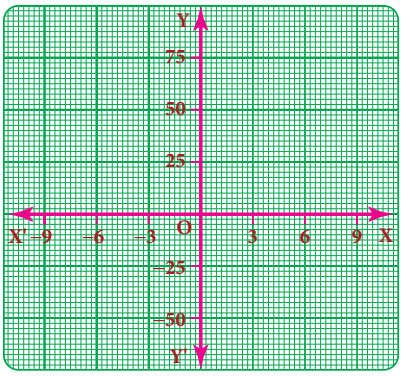
(i) $y = p\,x$ where $p \in z$ always passes through the_____.

(ii) The intersecting point of the line $x = 4$ and $y = -4$ is_____.

(iii) Scale for the given graph ,

On the x-axis 1 cm = ------- units

y-axis 1 cm = ------- units



2. Say True or False.

(i) The points (1,1) (2,2) (3,3) lie on a same straight line.

(ii) $y = -9x$ not passes through the origin.

3. Will a line pass through (2, 2) if it intersects the axes at (2, 0) and (0, 2).

4. A line passing through (4, −2) and intersects the Y-axis at (0, 2). Find a point on the line in the second quadrant.

5. If the points P(5, 3) Q(−3, 3) R(−3, −4) and S form a rectangle, then find the coordinate of S.

6. A line passes through (6, 0) and (0, 6) and an another line passes through (−3,0) and (0, −3). What are the points to be joined to get a trapezium?

7. Find the point of intersection of the line joining points (−3, 7) (2, −4) and (4,6) (−5,−7). Also find the point of intersection of these lines and also their intersection with the axis.

8. Draw the graph of the following equations, (i) $x = -7$ (ii) $y = 6$

9. Draw the graph of (i) $y = -3x$ ii) $y = x-4$ iii) $y = 2x+5$

10. Find the values.

a) $y = x + 3$

x	0		−2	
y		0		−3

b) $2x + y - 6 = 0$

x	0		−1	
y		0		−2

c) $y = 3x+1$

x	−1	0	1	2
y				

Exercise 3.10

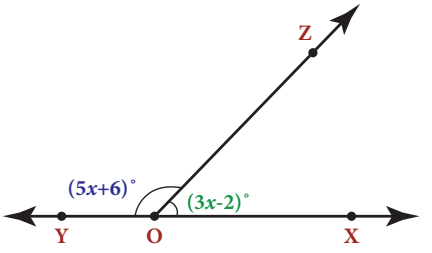
Miscellaneous Practice Problems

1. The sum of three numbers is 58. The second number is three times of two-fifth of the first number and the third number is 6 less than the first number. Find the three numbers.

2. In triangle ABC, the measure of ∠B is two-third of the measure of ∠A. The measure of ∠C is 20° more than the measure of ∠A. Find the measures of the three angles.

3. Two equal sides of an isosceles triangle are 5y−2 and 4y+9 units. The third side is 2y+5 units. Find 'y' and the perimeter of the triangle.

4. In the given figure, angle XOZ and angle ZOY form a linear pair. Find the value of x.



5. Draw a graph for the following data:

Side of a square (cm)	2	3	4	5	6
Area (cm^2)	4	9	16	25	36

Does the graph represent a linear relation?

Challenging Problems

6. Three consecutive integers, when taken in increasing order and multiplied by 2, 3 and 4 respectively, total up to 74. Find the three numbers.
7. 331 students went on a field trip. Six buses were filled to capacity and 7 students had to travel in a van. How many students were there in each bus?
8. A mobile vendor has 22 items, some which are pencils and others are ball pens. On a particular day, he is able to sell the pencils and ball pens. Pencils are sold for ₹15 each and ball pens are sold at ₹20 each. If the total sale amount with the vendor is ₹380, how many pencils did he sell?
9. Draw the graph of the lines $y = x$, $y = 2x$, $y = 3x$ and $y = 5x$ on the same graph sheet. Is there anything special that you find in these graphs?
10. Consider the number of angles of a convex polygon and the number of sides of that polygon. Tabulate as follows:

Name of Polygon	*No. of angles*	*No. of sides*

Use this to draw a graph illustrating the relationship between the number of angles and the number of sides of a polygon.

SUMMARY

- When the product of two algebraic expressions we follow,
  Multiply the signs of the terms,
  Multiply the corresponding co-efficients of the terms.
  Multiply the variable factors by using laws of exponents.
- For dividing a polynomial by a monomial, divide each term of the polynomial by a monomial.
- Identity: An identity is an equation is satisfied by any value that replaces its variables (s).

$(a+b)^2 = a^2 + 2ab + b^2$ $\quad$ $(x+a)(x+b) = x^2 + (a+b)x + ab$

$(a-b)^2 = a^2 - 2ab + b^2$ $\quad$ $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

$a^2 - b^2 = (a+b)(a-b)$ $\quad$ $(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$

$(x+b)(x+b)(x+c) = x^3 + (a+b+c)x^2 + (ab+bc+ca)x + abc$

- **Factorisation:** Expressing an algebraic expression as the product of two or more expression is called Factorisation.
- An equation containing only one variable with its highest power as one is called a linear equation in one variable.
- The value which replaces a variable in an equation so as to make the two sides of the equation equal is called a solution or root of the equation.
- Graphing is just a visual method for showing relationships between numbers.
- The horizontal line is named as XOX', called the X-axis. The vertical line is named as YOY', called the Y-axis. Both the axes are called **coordinate axes**. The plane containing the x axis and the y axis is known as the coordinate plane or the **Cartesian plane**.
- A point is denoted by a pair (a,b) of two numbers 'a' and 'b' listed in a specific order in which 'a' represents the distance along the X-axis and 'b' represents the distance along the Y axis. It is called an ordered pair (a,b).
- The coordinate axes divide the plane of the graph into four regions called quadrants.
- The line graph for the linear equation is called a **linear graph**.

ICT CORNER

Through this activity you will know about the Algebra, operations on them and study their properties as well.

Step-1 Open the Browser and type the URL given below.

Step-2 Click on any one of the link in the items to know about the basics in algebra, exponents, polynomial, quadric equation etc.

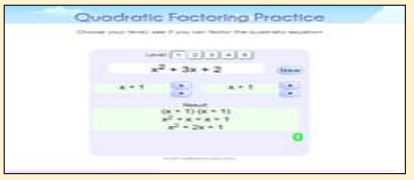
Step-3 For example, click on the "Balance While adding and subtracting", link under the Basic menu. A new tab will open in the browser where you can see the interactive game on adding and subtracting algebra.

Step-4 Likewise you can learn all the concepts in algebra.

Step 1

Step 2

Step 3



Step 4

AZR3S

Web URL **Algebra:**

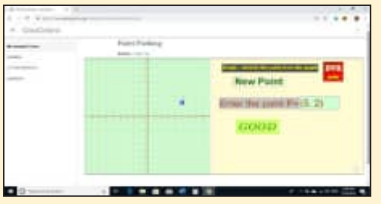
https://www.mathsisfun.com/algebra/index.html

*Pictures are indicatives only.

*If browser requires, allow Flash Player or Java Script to load the page.

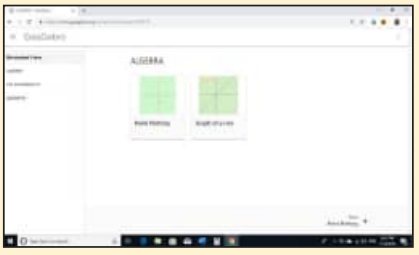
ICT CORNER

Expected Outcome

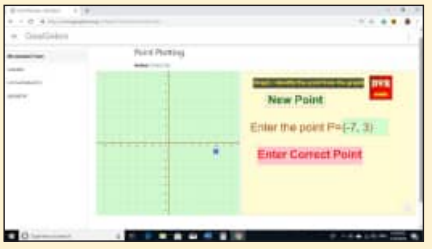


Step-1 Open the Browser type the URL Link given below (or) Scan the QR Code. GeoGebra work book named "ALGEBRA" will open. Click on the worksheet named "Point Plotting".

Step-2 In the given worksheet you can get new point by clicking on "New point".Enter the correct point in the input box and press enter.



Step 1



Step 2

OJYE7

Browse in the link

Algebra:

https://www.geogebra.org/m/fqxbd7rz#chapter/409574 or Scan the QR Code.

4 LIFE MATHEMATICS

Learning Objectives

- To recall the concepts of percentage, profit, loss and simple interest.
- To solve problems involving percentage, applications of percentage in profit, loss, overhead expenses, discount and GST.
- To know what compound interest is and to be able to find the compound interest through patterns and formula and use them in simple problems.
- To find the difference between compound interest and simple interest for 2 years and 3 years.
- To recall direct and inverse proportions.
- To know about compound variation and do problems on it.
- To solve time and work problems.

4.1 Introduction

The following conversation happens in a Math class of VIII Std.

Teacher : Dear students, money collection is being made for the Flag Day and so far 32 out of 40 students in VII Std and 42 out of 50 students from our class have contributed. Can anyone of you say, whose contribution is better?

Sankar : Teacher, 32 out of 40 is $\frac{32}{40}$ and 42 out of 50 is $\frac{42}{50}$. The corresponding like fractions of them are $\frac{160}{200}$ and $\frac{168}{200}$ respectively. So, our class students' contribution is better.

Teacher : Very Good, Sankar. Students, is there any other way to compare?

Bumrah : Yes Teacher, percentages will help. Here, $\frac{32}{40} = \frac{32}{40} \times 100 = 80\ \%$ and $\frac{42}{50} = \frac{42}{50} \times 100 = 84\%$ Clearly, our class contribution is 4 % more than VII Std.

Teacher : Well done, Bumrah. You are exactly correct. Can anyone of you say where the use of percentages is seen more?

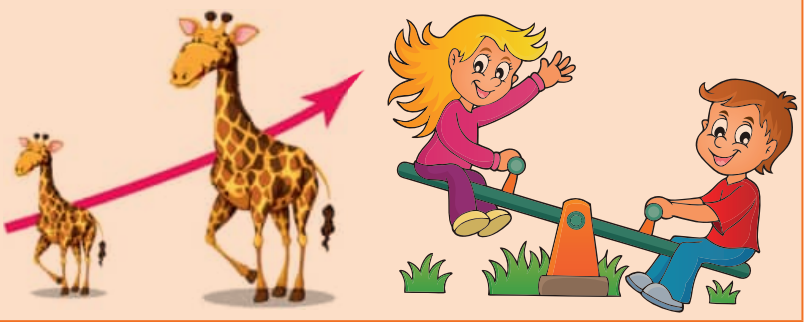
Bhuvi : Yes Teacher. My father told me that percentages are used in the calculation of profit, loss, discount, calculation of tax (eg: GST), interest on investment, growth in population and depreciation of machines and almost everywhere where comparison is made. He also had told me that it is an easy tool which is helpful to compare values.

Teacher : Well said, Bhuvi. These are the topics what we are going to see in this chapter using percentages.

The above conversation leads the way to know the application of percentages in various situations and in the commonly seen problems in our day-to-day life. We will also see direct and inverse proportions, compound variation and time and work problems topics later.

MATHEMATICS ALIVE – LIFE MATHEMATICS IN REALITY

Money grows faster by compound interest.



The growth of a giraffe over time is an example for direct proportion and see-saw is an example for inverse proportion

Try these

Find the indicated percentage value of the given numbers.

Number / %	60	240	660	852	1200
10 %					
20 %					
25 %					
$33\frac{1}{3}$ %					

4.2 Applications of Percentage in Word Problems

We know that Per Cent means per hundred or out of a hundred. It is denoted by the symbol %. x% denotes the fraction $\frac{x}{100}$. It is very useful in comparing quantities easily. Let us see its uses in the following word problems.

Example 4.1

If x % of 600 is 450, then find the value of x.

Solution:

Given, $x\%$ of $600 = 450$

$$\frac{x}{100} \times 600 = 450$$

$$x = \frac{450}{6}$$

$$x = 75$$

Example 4.2

When a number is decreased by 25%, it becomes 120. Find the number.

Solution:

Let the number be x.

Given, $x - \frac{25}{100}x = 120$

$$\frac{100x - 25x}{100} = 120$$

$$\frac{75x}{100} = 120$$

$$\Rightarrow x = \frac{120 \times 100}{75}$$

$$\Rightarrow x = 160$$

If we start with a quantity A and then decrease that quantity by x%, we will get the decreased quantity as,

$$D = \left(1 - \frac{x}{100}\right) A$$

Aliter: Using the above formula,

$$D = \left(1 - \frac{25}{100}\right)A \Rightarrow 120 = \frac{75}{100} \times A$$

$$A = 120 \times \frac{100}{75} = 160$$

Example 4.3

Akila scored 80% of marks in an examination. If her score was 576 marks, then find the maximum marks of the examination.

Solution:

Let the maximum marks be x.

Now, 80% of $x = 576$

$$\frac{80}{100} \times x = 576$$

$$\Rightarrow x = 576 \times \frac{100}{80} = 720 \text{ marks}$$

Therefore, the maximum marks of the examination = 720.

Example 4.4

If the price of Orid dhall after 20% increase is ₹96 per kg, then find the original price of Orid dhall per kg.

Solution:

Let the original price of Orid dhall be ₹x.

New price after 20% increase

$$= x + \frac{20}{100}x = \frac{120x}{100}$$

Given that, $\frac{120x}{100} = 96$

$$\therefore x = \frac{96 \times 100}{120}$$

$\therefore$ Original price of Orid dhall per kg, x = ₹80

If we start with a quantity A and then increase that quantity by x%, we will get the increased quantity as,

$$I = \left(1 + \frac{x}{100}\right) A$$

Aliter: Using the above formula,

$$I = \left(1 + \frac{20}{100}\right)A$$

$$\Rightarrow 96 = \frac{120}{100} \times A$$

$$A = 96 \times \frac{100}{120}$$

$$A = ₹80$$

Try these

1. What percentage of a day is 10 hours?
2. Divide ₹350 among P, Q and R such that P gets 50% of what Q gets and Q gets 50% of what R gets.

Think

With a lot of pride, the traffic police commissioner of a city reported that the accidents had decreased by 200% in one year. He came up with this number by stating that the increase in accidents from 200 to 600 is clearly a 200% rise and now that it had gone down from 600 last year to 200 this year should be a 200% fall. Is this decrease from 600 to 200, the same 200% as reported by him? Justify.

Example 4.5

The income of a person is increased by 10% and then decreased by 10%. Find the change in his income.

Solution:

Let his initial income be ₹x.

Income after 10% increase is

$$₹x + \left(\frac{10}{100} \times x\right) = ₹\frac{110}{100}\ x \text{ (or) } ₹\frac{11}{10}\ x$$

Now, income after 10% decrease is,

$$₹\frac{11x}{10} - \frac{10}{100}\left(\frac{11x}{10}\right)$$

$$\frac{11x}{10} - \frac{11x}{100} = \frac{110x - 11x}{100} = ₹\frac{99x}{100}$$

$$\therefore \text{ Net change in his income } = x - \frac{99x}{100} = \frac{x}{100}$$

$$\therefore \text{ Percentage change } = \frac{\frac{x}{100}}{x} \times 100\% = 1\%$$

That is, income of the person is reduced by 1%.

(or)

Aliter:

Let his income be ₹100

Income after 10% increase is

$$100 + 100 \times \frac{10}{100} = ₹110.$$

Now, income after 10% decrease is,

$$110 - 110 \times \frac{10}{100} = 110 - 11 = ₹99$$

$\therefore$ Net change in his income = 100 – 99 = 1

$$\therefore \text{ Percentage change } = \frac{1}{100} \times 100\% = 1\%.$$

That is, income of the person is reduced by 1%.

Note

For any given number, when it is increased first by x % and then decreased by y %, then the value of the number is increased or decreased by $\left(x + y + \frac{xy}{100}\right)\%$. Use 'negative' sign for decrease and also assume 'decrease' if the sign is negative. Use this note to check the answer for the Example 4.5.

Exercise 4.1

1. **Fill in the blanks:**
   (i) If 30 % of x is 150, then x is _____.
   (ii) 2 minutes is _____ % to an hour.
   (iii) If x % of x = 25, then x = _____.
   (iv) In a school of 1400 students, there are 420 girls. The percentage of boys in the school is _____.
   (v) 0.5252 is _____ % .

2. **Rewrite each underlined part using percentage language.**
   (i) One half of the cake is distributed to the children.
   (ii) Aparna scored 7.5 points out of 10 in a competition.
   (iii) The statue was made of pure silver.
   (iv) 48 out of 50 students participated in sports.
   (v) Only 2 persons out of 3 will be selected in the interview.

3. 48 is 32% of which number?
4. What is 25% of 30% of 400?
5. If a car is sold for ₹200000 from its original price of ₹300000, then find the percentage of decrease in the value of the car.

6. If the difference between 75% of a number and 60% of the same number is 82.5, then find 20% of that number.
7. A number when increased by 18% gives 236. Find the number.
8. A number when decreased by 20% gives 80. Find the number.
9. A number is increased by 25% and then decreased by 20%. Find the percentage change in that number.
10. The ratio of boys and girls in a class is 5:3. If 16% of boys and 8% of girls failed in an examination, then find the percentage of passed students.

Objective Type Questions

11. 12% of 250 litre is the same as _____of 150 litre.
    (A) 10% (B) 15% (C) 20% (D) 30%
12. If three candidates A, B and C in a school election got 153,245 and 102 votes respectively, then the percentage of votes got by the winner is_____.
    (A) 48% (B) 49% (C) 50% (D) 45%
13. 15% of 25% of 10000 =_____.
    (A) 375 B) 400 C) 425 (D) 475
14. When 60 is subtracted from 60% of a number to give 60, the number is
    (A) 60 (B) 100 (C) 150 (D) 200
15. If 48% of 48 = 64% of x, then x =
    (A) 64 (B) 56 (C) 42 (D) 36

4.3 Profit, Loss, Discount, Overhead Expenses and GST

4.3.1 Profit and Loss

Try these

1. If the selling price of an article is less than the cost price of the article, then there is a _____.
2. An article costing ₹5000 is sold for ₹4850. Is there a profit or loss? What percentage is it?
3. If the ratio of cost price and the selling price of an article is 5:7, then the profit/ gain is_____%.

Cost Price (C.P)

The amount for which an article is bought is called its ***Cost Price*** (C.P).

Selling Price (S.P)

The amount for which an article is sold is called its ***Selling Price*** (S.P).

Profit or Gain

When the *S.P* is more than the *C.P,* then there is a ***profit or gain***.

Therefore, Profit/Gain = S.P – C.P

Loss

When the *S.P* is less than the *C.P,* then there is a ***loss***.

Therefore, Loss = C.P – S.P

It is to be noted that the profit or loss is always calculated on the cost price.

Formulae:

(i) Profit or Gain % $= \left(\frac{Profit}{C.P} \times 100\right)\%$

(ii) Loss % $= \left(\frac{Loss}{C.P} \times 100\right)\%$

(iii) Selling Price, S.P $= \frac{(100 + Profit\%)}{100} \times$ C.P (or) Cost Price, C.P $= \frac{100}{(100 + Profit\%)} \times$ S.P

(iv) Selling Price, S.P $= \frac{(100 - Loss\%)}{100} \times$ C.P (or) Cost Price, C.P $= \frac{100}{(100 - Loss\%)} \times$ S.P

4.3.2 Discount

During the month of Aadi and festival seasons, shopkeepers offer a certain percentage of rebate on the marked price of the articles in order to increase the sales and also to clear the old stock. This rebate is known as ***discount***.

Marked price

In big shops and departmental stores, we see that every product is tagged with a card and a price marked on it. This price marked on the card is called the ***marked price***.

Based on this marked price, the shopkeeper offers a certain percentage of discount. The price payable by the customer after deduction of discount is called the ***selling price***.

That is, Selling Price = Marked Price – Discount

4.3.3 Overhead Expenses

Traders, retailers and shopkeepers are involved in the buying and selling of goods. Sometimes, when articles like machinery, furniture, electronic items etc., are bought, a few expenses may happen on their repairs, transportation and labour charges etc., These expenses are included in the cost price and are called as ***overhead expenses.***

Total Cost Price = Cost Price + Overhead Expenses

$$\text{Discount \%} = \frac{\text{Discount}}{\text{marked price}} \times 100\%$$

4.3.4 Goods and Services Tax (GST)

The goods and services tax (GST) is the only common tax in India levied on almost all the goods and the services meant for domestic consumption. The GST is remitted by the traders and the consumers alike and is also one of the main sources of income to both the Central and State Governments.There are 3 types of GST namely Central GST (CGST), State GST (SGST) and Integrated GST (IGST). For union territories, there is UTGST.

The GST is shared by the Central and the State Governments equally. There are also many products like Egg, Honey, Milk, Salt etc., which are exempted from GST. Products like Petrol, Diesel etc., do not come under GST and they are taxed separately. The GST council has fitted over 1300 goods and 500 services under four tax slabs namely 5%, 12%, 18% and 28%.

Example 4.6

Ranjith bought a washing machine for ₹16150 and paid ₹1350 for its transportation. Then, he sold it for ₹19250. Find his gain or loss percentage.

Solution:

Total C.P of the washing machine

= C.P + Overhead Expenses

= 16150 + 1350 = ₹17500

S.P = ₹19250

Here, S.P > C.P. Hence, there is a gain.

$$\text{Gain \%} = \left(\frac{Gain}{C.P} \times 100\right)\% = \left(\frac{19250 - 17500}{17500} \times 100\right)\%$$

$$= \left(\frac{1750}{17500} \times 100\right)\% = 10\%$$

Example 4.7

If the selling price of an LED TV is equal to $\frac{5}{4}$ of its cost price, then find the gain / profit percentage.

Solution:

Let the C.P of the LED TV be ₹x.

$$\therefore \text{ S.P} = \frac{5}{4}x$$

$$\text{Profit} = \text{S.P} - \text{C.P} = \frac{5}{4}x - x = \frac{x}{4}$$

$$\text{Profit \%} = \left(\frac{Profit}{C.P} \times 100\right)\%$$

$$\therefore \text{ Profit \%} = \left(\frac{x/4}{x} \times 100\right)\% = \left(\frac{1}{4} \times 100\right)\% = 25\%$$

Example 4.8

The cost price of 16 boxes of strawberries is equal to the selling price of 20 boxes of strawberries. Find the gain or loss percentage.

Solution:

Let the C.P of one strawberry box be ₹ x .

C.P of 20 strawberry boxes = $20x$ and

S.P of 20 strawberry boxes = C.P of 16 strawberry boxes = $16x$ (given)

Here, S.P < C.P, hence there is a loss.

$$\text{Loss} = \text{C.P} - \text{S.P} = 20x - 16x = 4x$$

$$\text{Loss \%} = \left(\frac{Loss}{C.P} \times 100\right)\%$$

$$= \left(\frac{4x}{20x} \times 100\right)\%$$

$$\therefore \text{ Loss \%} = 20\ \%$$

If the cost price of x articles = selling price of y articles, then profit % $= \left(\frac{x-y}{y} \times 100\right)\%$ .If the answer is negative, then it is treated as loss. Check and verify this for Example 4.8.

Example 4.9

By selling a bicycle for ₹4275, a shopkeeper loses 5%. For how much should he sell it to have a profit of 5%?

Solution:

S.P of the bicycle = ₹4275

Loss = 5%

$$\therefore \text{ C.P} = \frac{100}{(100 - Loss\%)} \times \text{ S.P}$$

$$= \frac{100}{95} \times 4275 = ₹4500$$

Now, C.P = ₹4500 and the desired profit = 5%

$$\therefore \text{ Desired S.P } = \frac{(100 + Profit\%)}{100} \times \text{C.P}$$

$$= \frac{100+5}{100} \times 4500 = 105 \times 45 = ₹4725$$

∴ To have a profit of 5%, he should sell the bicycle for ₹4725.

Example 4.10

The price of a rain coat was slashed from ₹1060 to ₹901 by a shopkeeper in the rainy season to boost the sales. Find the rate of discount given by him.

Solution:

Discount = Marked Price –Selling Price

= 1060 – 901

= ₹159

$$\therefore \text{ Discount \%} = \left(\frac{159}{1060} \times 100\right)\%$$

= 15%

Think

A shopkeeper marks the price of a marker board 15% above the cost price and then allows a discount of 15% on the marked price. Does he gain or lose in the transaction?

Example 4.11

Find the single discount in percentage which is equivalent to two successive discounts of 25% and 20% given on an article.

Solution:

Let the marked price of an article be ₹100.

First discount of 25% $= 100 \times \frac{25}{100} = ₹25$

Price after first discount $= 100 - 25 = ₹75$

Second discount of 20% $= 75 \times \frac{20}{100} = ₹15$

Price after second discount = 75 – 15 = ₹60

$\therefore$ Net selling price = ₹60

$\therefore$ Single discount in percentage equivalent to two given successive discounts = (100–60)% = 40%.

Note

- If there are 2 successive discounts of a % and b % respectively, then
  $$\text{S.P} = \left(1-\frac{a}{100}\right)\left(1-\frac{b}{100}\right)\times \text{M.P.}$$
- Single discount in % equivalent to 3 successive discounts of a %, b % and c % respectively
  $$= \left\{1-\left(1-\frac{a}{100}\right)\left(1-\frac{b}{100}\right)\left(1-\frac{c}{100}\right)\right\}\times 100\%.$$

(Use this formula for the Example 4.11 and check the answer).

Example 4.12

A water heater is sold by a trader for ₹10502 including GST at 18%. Find the marked price of the water heater and GST.

Solution:

Let the marked price be ₹ x.

$$\text{Now, } x+\frac{18x}{100} = 10502$$

$$\frac{118x}{100} = 10502$$

$\therefore$ Marked price, x = ₹8900

GST at 18% = ₹10502 – ₹8900 = ₹1602

(or)

$$= 8900 \times \frac{18}{100} = ₹1602$$

Example 4.13

A family went to a hotel and spent ₹350 for food and paid extra 5% as GST. Calculate the CGST and SGST.

Solution:

Cost of food = ₹350

Extra 5% paid as GST is equally shared by the Central and the State Governments at 2.5% each.

$$\therefore \text{CGST} = \text{SGST} = 350 \times \frac{2.5}{100} = ₹8.75$$

Exercise 4.2

1. Fill in the blanks:

(i) Loss or gain percentage is always calculated on the_____.

(ii) A mobile phone is sold for ₹8400 at a gain of 20%. The cost price of the mobile phone is_____.

(iii) An article is sold for ₹555 at a loss of $7\frac{1}{2}$ %. The cost price of the article is _____.

(iv) A mixer grinder marked at ₹4500 is sold for ₹4140 after discount. The rate of discount is _____.

(v) The total bill amount of a shirt costing ₹575 and a T-shirt costing ₹325 with GST of 5% is_____.

2. If selling an article for ₹820 causes 10% loss on the selling price, then find its cost price.

3. If the profit earned on selling an article for ₹810 is the same as loss on selling it for ₹530, then find the cost price of the article.

4. If the selling price of 10 rulers is the same as the cost price of 15 rulers, then find the profit percentage.

5. Some articles are bought at 2 for ₹15 and sold at 3 for ₹25. Find the gain percentage.

6. By selling a speaker for ₹768, a man loses 20%. In order to gain 20%, how much should he sell the speaker?

7. Find the unknowns x, y and z.

S.No	Name of the item	Marked Price	Selling Price	Discount
(i)	Book	₹225	x	8%
(ii)	LED TV	y	₹11970	5%
(iii)	Digital clock	₹750	₹615	z

8. Find the total bill amount for the data given below:

S.No	Name of the item	Marked Price	Discount	GST
(i)	School bag	₹500	5%	12%
(ii)	Hair dryer	₹2000	10%	28%

9. A branded Air-Conditioner (AC) has a marked price of ₹38000. There are 2 options given for the customer.

(i) Selling Price is the same ₹38000 but with attractive gifts worth ₹3000

(or)

(ii) Discount of 8% on the marked price but no free gifts.

Which offer is better?

10. If a mattress is marked for ₹7500 and is available at two successive discounts of 10% and 20%, find the amount to be paid by the customer.

Objective Type Questions

11. A fruit vendor sells fruits for ₹200 gaining ₹40. His gain percentage is

(A) 20% (B) 22% (C) 25% (D) $16\frac{2}{3}\%$

12. By selling a flower pot for ₹528, a woman gains 20%. At what price should she sell it to gain 25%?

(A) ₹500 (B) ₹550 (C) ₹553 (D) ₹573

13. A man buys an article for ₹150 and makes overhead expenses which are 12% of the cost price. At what price must he sell it to gain 5%?

(A) ₹180 (B) ₹168 (C) ₹176.40 (D) ₹88.20

14. What is the marked price of a hat which is bought for ₹210 at 16% discount?

(A) ₹243 (B) ₹176 (C) ₹230 (D) ₹250

15. The single discount in % which is equivalent to two successive discounts of 20% and 25% is

(A) 40% (B) 45% (C) 5% (D) 22.5%

4.4 Compound Interest

The most powerful force in this universe is_____. How would you complete this quote? The world renowned physicist Albert Einstein completed this quote with the word **Compound Interest.**

When money is borrowed or deposited on simple interest ($I = \frac{PNR}{100}$), then the interest is calculated evenly on the principal throughout the loan or deposit period.

Try these

1. The formula to find the simple interest for a given principal is _____.
2. Find the simple interest on ₹900 for 73 days at 8% p.a.
3. In how many years will ₹2000 become ₹3600 at 10% p.a simple interest?

In post offices, banks, insurance companies and other financial institutions, there is another type of interest calculation on offer. Here, the interest accrued during the first time period (say 6 months) is added to the original principal and the amount so obtained is taken as the principal for the second time period (that is, the next 6 months) and this keeps going on, up to the fixed time agreement between the banker and the depositer.

After a certain period, the difference between the amount and the money deposited is called the compound interest which is abbreviated as C.I. Clearly, the compound interest will be more than the simple interest just because the principal keeps on changing for every time period.

We call the time period after which the interest is added to the principal, as the conversion period. For example, if the interest is compounded say quarterly, there will be four conversion periods in a year, every 3 months. In such cases, the interest rate will be one fourth of the annual rate and the number of times that interest will be compounded is four times the number of years.

In case of simple interest, the principal remains the same for the whole duration where as in case of compound interest, the principal keeps on changing as per the conversion period. The simple interest and the compound interest remains the same for the first conversion period.

Illustration 1

To find the compound interest on ₹20000 for 4 years at 10% p.a compounded annually and compare it with the simple interest obtained for the same.

Calculating Compound Interest

Principal for the I year = ₹20000

Interest for the I year $\left(\frac{20000\times10\times1}{100}\right)$ = ₹ 2000

Amount at the end of I Year (P+I) = ₹22000

That is, Principal for the II year = ₹22000

Interest for the II year $\left(\frac{22000\times10\times1}{100}\right)$ = ₹ 2200

Amount at the end of II year = ₹24200

That is, Principal for the III year = ₹24200

Interest for the III year $\left(\frac{24200\times10\times1}{100}\right)$ = ₹ 2420

Amount at the end of III year = ₹26620

That is, Principal for the IV year = ₹26620

Interest for the IV year $\left(\frac{26620\times10\times1}{100}\right)$ = ₹ 2662

∴ Amount at the end of IV year = ₹29282

∴ Compound Interest for 4 years = A – P = 29282 – 20000 = ₹9282

Calculating Simple Interest

Recall that,

Simple Interest, $I = \frac{PNR}{100}$

Here, P = ₹20000

N = 4 years

R = 10%

$\therefore I = \frac{20000\times4\times10}{100}$

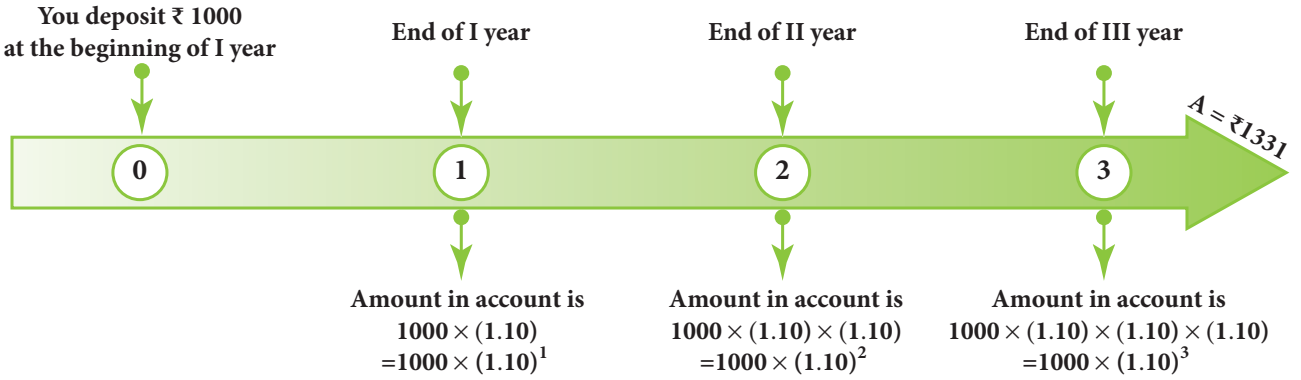
That is, I = ₹ 8000

What we observe from the calculation of C.I is a repeated multiplication by the factor 1.1 as 20000 (×1.1), 22000 (×1.1), 24200 (×1.1), 26620 (×1.1) for 4 years. We also note that the compound interest (₹9282) grows faster and is clearly more than the simple interest (₹8000) obtained. When the time period is longer, the above method is time

consuming. So, to save time and to find the amount and the compound interest easily we have a formula as explained in the following illustration.

Illustration 2

To calculate the amount and compound interest for ₹1000 for 3 years at 10% p.a compound annually.


You deposit ₹ 1000 at the beginning of I year
End of I year
End of II year
End of III year
A = ₹1331
0
1
2
3
Amount in account is 1000 × (1.10) =1000 × (1.10)^1
Amount in account is 1000 × (1.10) × (1.10) =1000 × (1.10)^2
Amount in account is 1000 × (1.10) × (1.10) × (1.10) =1000 × (1.10)^3

Flow chart showing 10% interest being compounded annually

This leads to the pattern for Amount as $P\left(1+\frac{r}{100}\right)$ for the I year, $P\left(1+\frac{r}{100}\right)\left(1+\frac{r}{100}\right)$ for the II year, $P\left(1+\frac{r}{100}\right)\left(1+\frac{r}{100}\right)\left(1+\frac{r}{100}\right)$ for the III year and so on. In general, $A = P\left(1+\frac{r}{100}\right)^n$ for the nth year. Here, the amount after 3 years, $A = 1000\left(1+\frac{10}{100}\right)^3 = ₹1331$ and so, C.I = A – P = ₹331.

In 1626, Peter Minuit convinced the Wappinger Indians to sell him the Manhattan Island for 24 dollars. If the native Americans had put the 24 dollars into a bank account at 5% interest rate compounded monthly, by the year 2020 there would be well over 5.5 billion (550 crores) dollars in the account! This is the might of compound interest!

The following formulae will be helpful in calculating the compound interest easily for the following time periods.

(i) When the interest is compounded annually, we have

$$A = P\left(1+\frac{r}{100}\right)^n$$

where A is the amount, P is the principal, r is the rate of interest per annum and n is the number of years and we shall get the compound interest as C.I = Amount – Principal.

(ii) When the interest is compounded half yearly, we have

$$A = P\left(1+\frac{r}{200}\right)^{2n}$$

(iii) When the interest is compounded quarterly, we have

$$A = P\left(1+\frac{r}{400}\right)^{4n}$$

(iv) When the interest is compounded annually but rate of interest differs year by year, we have

$$A = P\left(1+\frac{a}{100}\right)\left(1+\frac{b}{100}\right)\left(1+\frac{c}{100}\right)\ldots$$

where a, b and c are interest rates for I, II and III years respectively.

(v) When interest is compounded annually but time period is in fraction say $a\frac{b}{c}$ years, we have

$$A = P\left(1+\frac{r}{100}\right)^{a}\left(1+\frac{\frac{b}{c}\times r}{100}\right)$$

(Use of calculators are permitted for lengthy calculations and also to verify answers).

Example 4.14

Find the C.I for the data given below:

(i) Principal = ₹4000, r = 5% p.a, n = 2 years, interest compounded annually.

(ii) Principal = ₹5000, r = 4% p.a, n = $1\frac{1}{2}$ years, interest compounded half-yearly.

(iii) Principal = ₹30000, r = 7% for I year, r = 8% for II year, compounded annually.

(iv) Principal = ₹10000, r = 8% p.a, n = $2\frac{3}{4}$ years, interest compounded yearly.

Solution:

(i) Amount, $A = P\left(1+\frac{r}{100}\right)^{n}$

$$= 4000\left(1+\frac{5}{100}\right)^{2}$$

$$= 4000\times\frac{21}{20}\times\frac{21}{20}$$

$$A = ₹4410$$

$\therefore$ C.I = A − P = 4410 − 4000 = ₹410

(ii) Amount, $A = P\left(1+\frac{r}{200}\right)^{2n} = 5000\left(1+\frac{4}{200}\right)^{2\times\frac{3}{2}} = 5000\times\frac{51}{50}\times\frac{51}{50}\times\frac{51}{50}$

$$= 51 \times 10.2 \times 10.2$$

$$= ₹5306.04$$

$\therefore$ C.I = A − P = 5306.04 − 5000

$$= ₹306.04$$

(iii) Amount, $A = P\left(1+\frac{a}{100}\right)\left(1+\frac{b}{100}\right)$

$= 30000\left(1+\frac{7}{100}\right)\left(1+\frac{8}{100}\right)$

$= 30000\times\frac{107}{100}\times\frac{108}{100}$

$A = ₹34668$

$\therefore$ C.I = A − P = 34668 − 30000 = ₹4668

(iv) Amount, $A = P\left(1+\frac{r}{100}\right)^{a}\left(1+\frac{\frac{b}{c}\times r}{100}\right) = 10000\left(1+\frac{8}{100}\right)^{2}\left(1+\frac{\frac{3}{4}\times 8}{100}\right)$

$= 10000\left(\frac{27}{25}\right)^{2}\left(\frac{53}{50}\right)$

$= 10000\times\frac{27}{25}\times\frac{27}{25}\times\frac{53}{50}$

$A = ₹12363.84$

$\therefore$ C.I = A − P = 12363.84 − 10000 = ₹2363.84

4.4.1 Applications of Compound Interest Formula

The compound interest formula is used in the following situations.

(i) To find the increase $\left[P\left(1+\frac{r}{100}\right)^{n}\right]$ or decrease $\left[P\left(1-\frac{r}{100}\right)^{n}\right]$ in population.

(ii) To find the growth of cells when the rate of growth is given.

(iii) To find the depreciation in the values of machines, vehicles, utility appliances etc.,

Example 4.15

The value of a motor cycle 2 years ago was ₹70000. It depreciates at the rate of 4% p.a. Find its present value.

Solution:

Depreciated value $= P\left(1-\frac{r}{100}\right)^{n}$

$= 70000\left(1-\frac{4}{100}\right)^{2}$

$= 70000\times\frac{96}{100}\times\frac{96}{100}$

$= ₹64512$

Example 4.16

The bacteria in a culture grows by 5% in the first hour, decreases by 8% in the second hour and again increases by 10% in the third hour. Find the count of the bacteria at the end of 3 hours, if its initial count was 10000.

Solution:

Bacteria at the end of 3 hours

$$A = P\left(1+\frac{a}{100}\right)\left(1+\frac{b}{100}\right)\left(1+\frac{c}{100}\right)$$

$$= 10000\left(1+\frac{5}{100}\right)\left(1-\frac{8}{100}\right)\left(1+\frac{10}{100}\right) \text{ ('–' because 'decrease')}$$

$$= 10000 \times \frac{105}{100} \times \frac{92}{100} \times \frac{110}{100}$$

$$A = ₹10626$$

Example 4.17

The population of a town is increasing at the rate of 6% p.a. It was 238765 in the year 2018. Find the population in the year 2016 and 2020.

Solution:

Let the population in 2016 be P.

Then, $A = P\left(1+\frac{r}{100}\right)^n$

$$\Rightarrow 238765 = P\left(1+\frac{6}{100}\right)^2 = P\left(\frac{53}{50}\right)^2$$

$$\Rightarrow P = 238765 \times \frac{50}{53} \times \frac{50}{53}$$

$$\therefore P = 212500$$

Let the population in 2020 be A

Then, $A = P\left(1+\frac{r}{100}\right)^n$

$$\therefore A = 238765\left(1+\frac{6}{100}\right)^2$$

$$= 238765 \times \frac{53}{50} \times \frac{53}{50}$$

$$= 95.506 \times 53 \times 53$$

$$A = 268276$$

$\therefore$ The population in the year 2016 was 212500 and that in the year 2020 will be 268276.

4.4.2 Difference between C.I and S.I

- There is no difference in C.I and S.I for the first year or the first conversion period.
- For 2 years, the difference in C.I and S.I is

$$\text{C.I} - \text{S.I} = \text{P}\left(\frac{r}{100}\right)^2$$

- For 3 years, the difference in C.I and S.I is

$$\text{C.I} - \text{S.I} = \text{P}\left(\frac{r}{100}\right)^2\left(3+\frac{r}{100}\right)$$

Example 4.18

Find the difference in C.I and S.I for

(i) P = ₹5000, r = 4% p.a, n = 2 years.

(ii) P = ₹8000, r = 5% p.a, n = 3 years.

Solution:

(i) For 2 years, $\text{C.I} - \text{S.I} = \text{P}\left(\frac{r}{100}\right)^2 = 5000 \times \frac{4}{100} \times \frac{4}{100} = ₹8$

(ii) For 3 years, $\text{C.I} - \text{S.I} = \text{P}\left(\frac{r}{100}\right)^2\left(3+\frac{r}{100}\right)$

$$= 8000 \times \frac{5}{100} \times \frac{5}{100} \times \left(3+\frac{5}{100}\right)$$

$$= 20 \times \frac{61}{20} = ₹61$$

Exercise 4.3

1. **Fill in the blanks:**

(i) The compound interest on ₹5000 at 12% p.a for 2 years, compounded annually is _____.

(ii) The compound interest on ₹8000 at 10% p.a for 1 year, compounded half yearly is _____.

(iii) The annual rate of growth in population of a town is 10%. If its present population is 26620, then the population 3 years ago was_____.

(iv) If the compound interest is calculated quarterly, the amount is found using the formula _____.

(v) The difference between the C.I and S.I for 2 years for a principal of ₹5000 at the rate of interest 8% p.a is _____.

2. **Say True or False.**

   (i) Depreciation value is calculated by the formula, $P\left(1-\frac{r}{100}\right)^n$.

   (ii) If the present population of a city is P and it increases at the rate of r% p.a, then the population n years ago would be $P\left(1+\frac{r}{100}\right)^n$.

   (iii) The present value of a machine is ₹16800. It depreciates at 25% p.a. Its worth after 2 years is ₹9450.

   (iv) The time taken for ₹1000 to become ₹1331 at 20% p.a, compounded annually is 3 years.

   (v) The compound interest on ₹16000 for 9 months at 20% p.a, compounded quarterly is ₹2522.

3. Find the compound interest on ₹3200 at 2.5% p.a for 2 years, compounded annually.

4. Find the compound interest for $2\frac{1}{2}$ years on ₹4000 at 10% p.a, if the interest is compounded yearly.

5. A principal becomes ₹2028 in 2 years at 4% p.a compound interest. Find the principal.

6. In how many years will ₹3375 become ₹4096 at $13\frac{1}{3}$ % p.a if the interest is compounded half-yearly?

7. Find the C.I on ₹15000 for 3 years if the rates of interest are 15%, 20% and 25% for the I, II and III years respectively.

8. Find the difference between C.I and S.I on ₹5000 for 1 year at 2% p.a, if the interest is compounded half yearly.

9. Find the rate of interest if the difference between C.I and S.I on ₹8000 compounded annually for 2 years is ₹20.

10. Find the principal if the difference between C.I and S.I on it at 15% p.a for 3 years is ₹1134.

Objective Type Questions

11. The number of conversion periods in a year, if the interest on a principal is compounded every two months is_____.

    (A) 2 (B) 4 (C) 6 (D) 12

12. The time taken for ₹4400 to become ₹4851 at 10%, compounded half yearly is _____.

    (A) 6 months (B) 1 year (C) 1 years (D) 2 years

13. The cost of a machine is ₹18000 and it depreciates at $16\frac{2}{3}$% annually. Its value after 2 years will be_____.

    (A) ₹12000 (B) ₹12500 (C) ₹15000 (D) ₹16500

14. The sum which amounts to ₹2662 at 10% p.a in 3 years, compounded yearly is_____.
(A) ₹2000 (B) ₹1800 (C) ₹1500 (D) ₹2500

15. The difference between compound and simple interest on a certain sum of money for 2 years at 2% p.a is ₹1. The sum of money is _____ .
(A) ₹2000 (B) ₹1500 (C) ₹3000 (D) ₹2500

4.5 Compound Variation

Before we could learn about compound variation, let us recall the concepts on direct and inverse proportions.

If two quantities are such that an increase or decrease in one quantity makes a corresponding increase or decrease (same effect) in the other quantity, then they are said to be in direct proportion or said to vary directly. In other words, x and y are said to vary directly if $\boldsymbol{y=kx}$ always, where k is called the proportionality constant and $k > 0$ assuming that y depends on x and so $\boldsymbol{k = \frac{y}{x}}$.

For example, let us assume that one of you plan to give 2 pens to each of your friends in the birthday party. Then the number of pens to be bought will be in direct proportion with the number of friends who will attend the party. Isn't it? The following table will help you understand this clearly.

Number of friends (x)	1	2	5	12	15
Number of pens (y)	2	4	10	24	30

In this case, we find that the proportionality constant, $k = \frac{y}{x} = \frac{2}{1} = \frac{4}{2} = \frac{10}{5} = \frac{24}{12} = \frac{30}{15} = 2$.

Few more examples of Direct Proportion:

1. Distance –Time (under constant speed): If the distance increases, then the time taken to reach that distance will also increase and vice- versa.
2. Purchase – Spending: If the purchase on utilities for a family during the festival time increases, then the spending limit also increases and vice versa.
3. WorkTime – Earnings: If the number of hours worked is less, then the pay earned will also be less and vice-versa.

Similarly, if two quantities are such that an increase or decrease in one quantity makes a corresponding decrease or increase (opposite effect) in the other quantity, then they are said to be in inverse (indirect) proportion or said to vary inversely. In other words, x and y are said to vary inversely, if $\boldsymbol{xy = k}$ always, where k is called proportionality constant and $k > 0$.

For example, let us assume that a class of 30 students in a school walks on streets in a village for health awareness campaign in an orderly manner, then we can see an inverse proportion in the number of rows and columns they walk. The following table will help you understand this clearly.

Number of students in columns (x)	1	2	3	5	6
Number of students in rows (y)	30	15	10	6	5

We can map a few of these arrangements as (5 rows/6 columns) and (6 rows/5 columns) and see the opposite variations in rows and columns.

In this case, we find that the proportionality constant, k is 30.

Few more examples of Inverse Proportion:

1. Price – Consumption: If the price of consumable products increases, then naturally its consumption will decrease and vice-versa.
2. Workers – Time: If more workers are employed to complete a work, then the time taken to complete the work will be less and vice-versa.
3. Speed – Time (Fixed Distance): If we travel with less speed, then the time taken to cover a given distance will be more and vice-versa.

Now, use the concept of direct and inverse proportions and try to answer the following questions:

Try these

1. Classify the given examples as direct or inverse proportion:
   (i) Weight of pulses to their cost.
   (ii) Distance travelled by bus to the price of ticket.
   (iii) Speed of the athelete to cover a certain distance.
   (iv) Number of workers employed to complete a construction in a specified time.
   (v) Area of a circle to its radius.
2. A student can type 21 pages in 15 minutes. At the same rate, how long will it take the student to type 84 pages?
3. If 35 women can do a piece of work in 16 days, in how many days will 28 women do the same work?

Let us now see what a Compound Variation is? There will be problems which may involve a chain of two or more variations in them. This is called as **Compound Variation**. The different possibilities of two variations are:

Direct-Direct, Direct-Inverse, Inverse-Direct, Inverse-Inverse.

Note

There are situations where neither direct proportion nor indirect proportion can be applied. For example, if one can see a parrot at a distance through one eye, it does not mean that he can see two parrots at the same distance through both the eyes. Also, if it takes 5 minutes to fry a vadai, it does not mean that it will take 100 minutes to fry 20 vadais!

Let us now solve a few problems on compound variation. Here, we compare the known quantity with the unknown (x). There are a few methods in practice by which problems on compound variation are solved. They are given as follows:

4.5.1 Proportion Method

In this method, we shall compare the given data and find whether they are in direct or inverse proportion. By finding the proportion, we can use the fact that

the product of the extremes = the product of the means

and get the value of the unknown (x).

4.5.2 Multiplicative Factor Method

Illustration:

Men	Hours	Days
D $\begin{matrix} a \\ x \end{matrix}$ I	D $\begin{matrix} c \\ d \end{matrix}$	$\begin{matrix} e \\ f \end{matrix}$ I

Here, the unknown (x) is in men and so it is compared to the known, namely hours and days. If men and hours are in direct proportion (D), then take the multiplying factor as $\frac{d}{c}$ (take the reciprocal). Also, if men and days are in inverse proportion (I), then take the multiplying factor as $\frac{e}{f}$ (no change). Thus, we can find the unknown (x) in men as $x = a \times \frac{d}{c} \times \frac{e}{f}$.

4.5.3 Formula Method

Identify the data from the given statement as Persons (P), Days (D), Hours (H) and Work (W) and use the formula,

$$\frac{P_1 \times D_1 \times H_1}{W_1} = \frac{P_2 \times D_2 \times H_2}{W_2}$$

where the suffix 1 contains the complete data from the first statement of the given problem and the suffix 2 contains the unknown data to be found out in the second statement of the problem. That is, this formula says, P_1 men doing W_1 units of work in D_1 days working H_1 hours per day is the same as P_2 men doing W_2 units of work in D_2 days working H_2 hours per day. Identifying the work W_1 and W_2 correctly is more important in these problems. This method will be easy for finding the unknown (x) quickly.

Example 4.19 **(Direct – Direct Variation)**

If a company pays ₹6 lakh for 15 workers for 20 days, how much would it need to pay for 5 workers for 12 days?

Solution:

Proportion Method

Workers	Pay (Work)	Days
D $\begin{matrix} 15 \\ 5 \end{matrix}$	D $\begin{matrix} 6 \\ x \end{matrix}$ D	$\begin{matrix} 20 \\ 12 \end{matrix}$ D

Here, the unknown is the pay (x). It is to be compared with the workers and the days.

Step 1:

Here, less days means less pay. So, it is in direct proportion.

$\therefore$ The proportion is $20 : 12 :: 6 : x$ $\longrightarrow$ (1)

Step 2:

Also, less workers means less pay. So, it is in direct proportion again.

$\therefore$ The proportion is $15 : 5 :: 6 : x$ $\longrightarrow$ (2)

Step 3:

Combining (1) and (2),

$$\left.\begin{matrix} 20 : 12 \\ 15 : 5 \end{matrix}\right\} :: 6 : x$$

We know that, **the product of the extremes = the product of the means**

Extremes	:	Means	:	Extremes
20	:	12 : 6	:	x
15	:	5		

So, $20 \times 15 \times x = 12 \times 6 \times 5 \Rightarrow x = \dfrac{12 \times 6 \times 5}{20 \times 15}$ = ₹ 1.2 lakh.

Multiplicative Factor Method

Workers	Pay (Work)	Days
D 15 / 5	6 / D x D	20 / 12 D

Here, the unknown is the pay (x). It is to be compared with the workers and the days.

Step 1:

Here, less days means less pay. So, it is in direct proportion.

$\therefore$ The multiplying factor is $\dfrac{12}{20}$ (take the reciprocal).

Step 2:

Also, less workers means less pay. So, it is in direct proportion again.

$\therefore$ The multiplying factor is $\dfrac{5}{15}$ (take the reciprocal).

Step 3:

$\therefore x = 6 \times \dfrac{12}{20} \times \dfrac{5}{15}$ = ₹ 1.2 lakh.

Formula Method

Here, $P_1 = 15, D_1 = 20$ and $W_1 = 6$. Also, $P_2 = 5, D_2 = 12$ and $W_2 = x$

Using the formula, $\dfrac{P_1 \times D_1}{W_1} = \dfrac{P_2 \times D_2}{W_2}$

$$\Rightarrow \frac{15\times 20}{6} = \frac{5\times 12}{x}$$

$$\Rightarrow x = \frac{5\times 12\times 6}{15\times 20} = ₹\ 1.2 \text{ lakh.}$$

Example 4.20 **(Direct – Inverse Variation)**

A mat of length 180 m is made by 15 women in 12 days. How long will it take for 32 women to make a mat of length 512 m?

Solution:

Proportion Method

Length (Work)	Women	Days
D 180 / 512	15 / 32 I	12 / x D I

Here, the unknown is the days (x). It is to be compared with the length and the women.

Step 1:

Here, more length means more days. So, it is in direct proportion.

$\therefore$ The proportion is $180 : 512 :: 12 : x$ ⟶ (1)

Step 2:

Also, more women means less days. So, it is in inverse proportion.

$\therefore$ The proportion is $32 : 15 :: 12 : x$ ⟶ (2)

Step 3:

Combining (1) and (2),

$$\left.\begin{matrix} 180 : 512 \\ 32 : 15 \end{matrix}\right\} :: 12 : x$$

We know that, **the product of the extremes = the product of the means**

Extremes	:	Means	:	Extremes
180 / 32	:	512 : 12 / 15	:	x

So, $180\times 32\times x = 512\times 12\times 15 \Rightarrow x = \frac{512\times 12\times 15}{180\times 32} = 16$ days.

Multiplicative Factor Method

Length (Work)	Women	Days
D 180 / 512	15 / 32 I	12 / x D I

Here, the unknown is the days (x). It is to be compared with the length and the women.

Step 1:

Here, more length means more days. So, it is in direct proportion.

$\therefore$ The multiplying factor is $\frac{512}{180}$ (take the reciprocal).

Step 2:

Also, more women means less days. So, it is in inverse proportion.

$\therefore$ The multiplying factor is $\frac{15}{32}$ (no change).

Step 3:

$\therefore x = 12 \times \frac{512}{180} \times \frac{15}{32} = 16$ days.

Formula Method:

Here, $P_1 = 15$, $D_1 = 12$ and $W_1 = 180$ and $P_2 = 32$, $D_2 = x$ and $W_2 = 512$

Using the formula, $\frac{P_1 \times D_1}{W_1} = \frac{P_2 \times D_2}{W_2}$

$$\Rightarrow \frac{15 \times 12}{180} = \frac{32 \times x}{512}$$

$$\Rightarrow 1 = \frac{32 \times x}{512} \Rightarrow x = \frac{512}{32} = 16 \text{ days.}$$

Remark: Students may answer in any of the three given methods dealt here.

Try these

1. If x and y vary directly, find k when $x = y = 5$.
2. If x and y vary inversely, find the constant of proportionality when $x = 64$ and $y = 0.75$

Activity

Draw a circle of a given radius. Then, draw its radii in such a way that the angles between any two consecutive pair of radii are equal. Start drawing 3 radii and end with drawing 12 radii in the circle. List and prepare a table for the number of radii to the angle between a pair of consecutive radii and check whether they are in inverse proportion. What is the proportionality constant?

Example 4.21 **(Inverse – Direct Variation)**

If 81 students can do a painting on a wall of length 448 m in 56 days, then how many students can do the painting on a similar type of wall of length 160 m in 27 days?

Solution:

Multiplicative Factor Method

Students	Days	Length of the wall (Work)
I 81 / x D	I 56 / 27	448 / 160 D

Here, the unknown is the students (x). It is to be compared with the days and the length of the wall.

Step 1:

Here, less days means more students. So, it is in inverse proportion.

$\therefore$ The multiplying factor is $\frac{56}{27}$.

Step 2:

Also, less length means less students. So, it is in direct proportion.

$\therefore$ The multiplying factor is $\frac{160}{448}$.

Step 3:

$$\therefore x = 81 \times \frac{56}{27} \times \frac{160}{448}$$

$x = 60$ students.

Formula Method

Here, $P_1 = 81$, $D_1 = 56$ and $W_1 = 448$ and $P_2 = x$, $D_2 = 27$ and $W_2 = 160$

Using the formula, $\frac{P_1 \times D_1}{W_1} = \frac{P_2 \times D_2}{W_2}$

$$\Rightarrow \frac{81 \times 56}{448} = \frac{x \times 27}{160}$$

$$\Rightarrow x = \frac{81 \times 56}{448} \times \frac{160}{27} \Rightarrow x = 60 \text{ students.}$$

Example 4.22 **(Inverse – Inverse Variation)**

If 48 men working 7 hours a day can do a work in 24 days, then in how many days will 28 men working 8 hours a day can complete the same work?

Solution:

Multiplicative Factor Method

Men	Hours	Days
48	7	24
I	I	I $\quad$ I
28	8	x

Here, the unknown is the days (x). It is to be compared with the men and the hours.

Step 1:

Here, less men means more days. So, it is in inverse proportion.

$\therefore$ The multiplying factor is $\frac{48}{28}$.

Step 2:

Also, more hours means less days. So, it is in inverse proportion again.

$\therefore$ The multiplying factor is $\frac{7}{8}$.

Step 3:

$\therefore x = 24 \times \frac{48}{28} \times \frac{7}{8} = 36$ days.

Formula Method

Here, $P_1 = 48$, $D_1 = 24$, $H_1 = 7$ and $W_1 = 1$ (Why?)

$P_2 = 28$, $D_2 = x$, $H_2 = 8$ and $W_2 = 1$ (Why?)

Using the formula, $\frac{P_1 \times D_1 \times H_1}{W_1} = \frac{P_2 \times D_2 \times H_2}{W_2}$

$$\Rightarrow \frac{48 \times 24 \times 7}{1} = \frac{28 \times x \times 8}{1}$$

$$\Rightarrow x = \frac{48 \times 24 \times 7}{28 \times 8} = 36 \text{ days.}$$

Try these

Identify the different variations present in the following questions:

1. 24 men can make 48 articles in 12 days. Then, 6 men can make _____ articles in 6 days.
2. 15 workers can lay a road of length 4 km in 4 hours. Then, _____ workers can lay a road of length 8 km in 8 hours.
3. 25 women working 12 hours a day can complete a work in 36 days. Then, 20 women must work _____ hours a day to complete the same work in 30 days.
4. In a camp there are 420 kg of rice sufficient for 98 persons for 45 days. The number of days that 60 kg of rice will last for 42 persons is _____.

4.6 Time and Work

How would you find the answer for the following question?

If Kani can finish a given work in 2 hours and Viji in 3 hours, then in what time can they finish it working together? The answer for this question will be known here in this section.

Work to be done is usually considered as one unit. Work can be in any form like building a wall, making a road, filling or emptying a tank or even eating a certain amount of food.

Time is measured in hours, days etc., Certain assumptions are made that the work so done is uniform and each person shares the same work time in case of group work in completing the work.

Unitary Method

If two persons X and Y can complete some work individually in a and b days, then their one day's work will be $\frac{1}{a}$ and $\frac{1}{b}$ respectively.

Working together, their one day's work $= \frac{1}{a}+\frac{1}{b} = \frac{a+b}{ab}$ and so,

X and Y together can complete the work in $\frac{ab}{a+b}$ days.

Example 4.23

A and B together can do a piece of work in 16 days and A alone can do it in 48 days. How long will B take to complete the work?

Solution:

$(A+B)$'s 1 day's work $= \frac{1}{16}$

A's 1 day's work $= \frac{1}{48}$

$\therefore$ B's 1 day's work $= \frac{1}{16}-\frac{1}{48}$

$= \frac{3-1}{48} = \frac{2}{48} = \frac{1}{24}$

$\therefore$ B alone can complete the work in 24 days.

Note

The time taken to complete a work or task depends on various factors such as **number of persons**, their **capacity** to do the work, the **amount of work** and the **time spent per day** for the completion of work.

Example 4.24

P and Q can do a piece of work in 20 days and 30 days respectively. They started the work together and Q left after some days of work and P finished the remaining work in 5 days. After how many days from the start did Q leave?

Solution:

P's 1 day's work $= \frac{1}{20}$ and Q's 1 day's work $= \frac{1}{30}$

P's work for 5 days $= \frac{1}{20}\times 5 = \frac{5}{20} = \frac{1}{4}$

Therefore, the remaining work $= 1-\frac{1}{4} = \frac{3}{4}$ (as the total work is always 1). This remaining work was done by both P and Q.

Work done by P and Q in a day $= \frac{1}{20}+\frac{1}{30} = \frac{5}{60} = \frac{1}{12}$

Therefore, the number of days they worked together $= \frac{3/4}{1/12} = \frac{3}{4}\times\frac{12}{1} = 9$ days.

So, Q left after 9 days from the day the work started.

Example 4.25

A works 3 times as fast as B and is able to complete a work in 24 days less than the days taken by B. Find the time in which they can complete the work together.

Solution:

If B does the work in 3 days, then A will do it in 1 day. That is, the difference is 2 days. Here, given that the difference between A and B in completing the work is 24 days. Therefore, A will take $\frac{24}{2} = 12$ days and B will take $3 \times 12 = 36$ days to complete the work separately.

Hence, the time taken by A and B together to complete the work = $\frac{ab}{a+b}$ days.

$$= \frac{12 \times 36}{12 + 36} = \frac{12 \times 36}{48} = 9 \text{ days.}$$

Note

If A is $\frac{a}{b}$ times as good a worker as B, then A will only $\frac{b}{a}$ of the time taken by B to complete the work. Try to solve the above Example 4.25 using this.

4.6.1 Sharing of the money for work

When a group of people do some work together, based on their individual work they get a share of money themselves. In general, **money earned** is shared by people, who worked together, in the ratio of the **total work** done by each of them.

- If the ratio of the time taken by A and B in doing a work is $x : y$, then the ratio of work done by A and B is $\frac{1}{x} : \frac{1}{y} = y : x$. This is the ratio for their separate wages too.
- If three persons A, B and C can do a work in x, y and z days respectively, then the ratio in which their wages will be distributed to them is $\frac{1}{x} : \frac{1}{y} : \frac{1}{z}$.

Example 4.26

X, Y and Z can do a piece of job in 4, 6 and 10 days respectively. If X,Y and Z work together to complete, then find their separate shares if they will be paid ₹ 31000 for completing the job.

Solution:

Since they all work together for the same number of days $\left(\frac{60}{31}\right)$, the ratio in which they share the money is equal to the ratio of their work done per day.

That is, it is equal to $\frac{1}{4} : \frac{1}{6} : \frac{1}{10} = \frac{15}{60} : \frac{10}{60} : \frac{6}{60} = 15 : 10 : 6$

Here, the total parts = 15 + 10 + 6 = 31

Hence, A's share = $\frac{15}{31} \times 31000 = ₹15000$, B's share = $\frac{10}{31} \times 31000 = ₹10000$ and

C's share is ₹31000 − (₹15000 + ₹10000)= ₹6000.

Try these

1. Vikram can do one-third of a work in p days. He can do $\frac{3}{4}$ th of the same work in _____ days.
2. If m persons can complete a work in n days, then $4m$ persons can complete the work in _____days and $\frac{m}{4}$ persons can complete the same work in _____ days.

Exercise 4.4

SEYHZ

1. **Fill in the blanks:**
   (i) A can finish a job in 3 days whereas B finishes it in 6 days. The time taken to complete the job working together is _____days.
   (ii) If 5 persons can do 5 jobs in 5 days, then 50 persons can do 50 jobs in _____ days.
   (iii) A can do a work in 24 days. If A and B together can finish the work in 6 days, then B alone can finish the work in _____ days.
   (iv) A alone can do a piece of work in 35 days. If B is 40% more efficient than A, then B will finish the work in _____days.
   (v) A alone can do a work in 10 days and B alone in 15 days. They undertook the work for ₹200000. The amount that A will get is _____.
2. 210 men working 12 hours a day can finish a job in 18 days. How many men are required to finish the job in 20 days working 14 hours a day?
3. A cement factory makes 7000 cement bags in 12 days with the help of 36 machines. How many bags can be made in 18 days using 24 machines?
4. A soap factory produces 9600 soaps in 6 days working 15 hours a day. In how many days will it produce 14400 soaps working 3 more hours a day?
5. If 6 container lorries can transport 135 tonnes of goods in 5 days, how many more lorries are required to transport 180 tonnes of goods in 4 days?
6. A can do a piece of work in 12 hours, B and C can do it 3 hours whereas A and C can do it in 6 hours. How long will B alone take to do the same work?
7. A and B can do a piece of work in 12 days, while B and C can do it in 15 days whereas A and C can do it in 20 days. How long would each take to do the same work?
8. Carpenter A takes 15 minutes to fit the parts of a chair while Carpenter B takes 3 minutes more than A to do the same work. Working together, how long will it take for them to fit the parts for 22 chairs?

9. A can do a work in 45 days. He works at it for 15 days and then, B alone finishes the remaining work in 24 days. Find the time taken to complete 80% of the work, if they work together.

10. A is thrice as fast as B. If B can do a piece of work in 24 days, then find the number of days they will take to complete the work together.

Exercise 4.5

Miscellaneous Practice Problems

1. A fruit vendor bought some mangoes of which 10% were rotten. He sold $33\frac{1}{3}$ % of the rest. Find the total number of mangoes bought by him initially, if he still has 240 mangoes with him.

2. A student gets 31% marks in an examination but fails by 12 marks. If the pass percentage is 35%, find the maximum marks of the examination.

3. Sultana bought the following things from a general store. Calculate the total bill amount paid by her.

   (i) Medicines costing ₹800 with GST at 5%

   (ii) Cosmetics costing ₹650 with GST at 12%

   (iii) Cereals costing ₹900 with GST at 0%

   (iv) Sunglass costing ₹1750 with GST at18%

   (v) Air Conditioner costing ₹28500 with GST at 28%

4. P's income is 25% more than that of Q. By what percentage is Q's income less than P's?

5. Vaidegi sold two sarees for ₹2200 each. On one she gains 10% and on the other she loses 12%. Find her total gain or loss percentage in the sale of the sarees.

6. If 32 men working 12 hours a day can do a work in 15 days, then how many men working 10 hours a day can do double that work in 24 days?

7. Amutha can weave a saree in 18 days. Anjali is twice as good a weaver as Amutha.If both of them weave together, then in how many days can they complete weaving the saree?

8. P and Q can do a piece of work in 12 days and 15 days respectively. P started the work alone and then after 3 days, Q joined him till the work was completed. How long did the work last?

Challenging Problems

9. If the numerator of a fraction is increased by 50% and the denominator is decreased by 20%, then it becomes $\frac{3}{5}$. Find the original fraction.
10. Gopi sold a laptop at 12% gain. If it had been sold for ₹1200 more, the gain would have been 20%. Find the cost price of the laptop.

11. A shopkeeper gives two successive discounts on an article whose marked price is ₹180 and selling price is ₹108. Find the first discount percentage if the second discount is 25%.
12. Find the rate of compound interest at which a principal becomes 1.69 times itself in 2 years.
13. A small–scale company undertakes an agreement to make 540 motor pumps in 150 days and employs 40 men for the work. After 75 days, the company could make only 180 motor pumps. How many more men should the company employ so that the work is completed on time as per the agreement?
14. P alone can do $\frac{1}{2}$ of a work in 6 days and Q alone can do $\frac{2}{3}$ of the same work in 4 days. In how many days will they finish $\frac{3}{4}$ of the work, working together?
15. X alone can do a piece of work in 6 days and Y alone in 8 days. X and Y undertook the work for ₹48000. With the help of Z, they completed the work in 3 days. How much is Z's share?

SUMMARY

- When the S.P is more than the C.P, then there is a gain or profit. *Profit/Gain* = *S.P – C.P.*
- When the S.P is less than the C.P, then there is a loss. *Loss* = *C.P – S.P.*
- The profit or loss is always calculated on the cost price.
- Selling price = Marked price – Discount.
- **Formulae**

(i) Profit or Gain % $= \left(\frac{Profit}{C.P} \times 100\right)\%$

(ii) Loss % $= \left(\frac{Loss}{C.P} \times 100\right)\%$

(iii) Selling Price, S.P $= \frac{(100 + Profit\%)}{100} \times$ C.P (or) Cost Price, C.P $= \frac{100}{(100 + Profit\%)} \times$ S.P

(iv) Selling Price, S.P $= \frac{(100 - Loss\%)}{100} \times$ C.P (or) Cost Price, C.P $= \frac{100}{(100 - Loss\%)} \times$ S.P

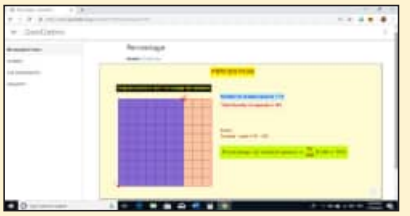
- When the interest is compounded annually, $A = P\left(1+\frac{r}{100}\right)^{n}$.
- When the interest is compounded half yearly, $A = P\left(1+\frac{r}{200}\right)^{2n}$.
- When the interest is compounded quarterly, $A = P\left(1+\frac{r}{400}\right)^{4n}$.
- When the interest is compounded annually but rate of interest differs year by year, $A = P\left(1+\frac{a}{100}\right)\left(1+\frac{b}{100}\right)\left(1+\frac{c}{100}\right)\ldots$ where a, b and c are interest rates for the I, II and III years respectively.
- When interest is compounded annually but time period is in fraction say $a\frac{b}{c}$ years, $A = P\left(1+\frac{r}{100}\right)^{a}\left(1+\frac{\frac{b}{c}\times r}{100}\right)$.
- C.I = A–P (Amount – Principal).
- The simple interest and the compound interest remains the same for the first year or the first conversion period.
- For 2 years, the difference in C.I and S.I is $C.I - S.I = P\left(\frac{r}{100}\right)^{2}$.
- For 3 years, the difference in C.I and S.I is $C.I - S.I = P\left(\frac{r}{100}\right)^{2}\left(3+\frac{r}{100}\right)$.
- x and y are said to vary directly if $\boldsymbol{y=kx}$ always, where k is called the proportionality constant and $k > 0$ assuming that y depends on x and so $\boldsymbol{k = \frac{y}{x}}$.
- x and y are said to vary inversely, if $\boldsymbol{xy = k}$ always, where k is called the proportionality constant and $k > 0$.
- There will be problems which involve a chain of two or more variations in them. This is called as compound variation.
- By finding the proportion, we can use the fact that the product of the extremes is equal to the product of the means to find the unknown (x) in the problem.
- By using the formula $\frac{P_1 \times D_1 \times H_1}{W_1} = \frac{P_2 \times D_2 \times H_2}{W_2}$, we can find the unknown (x).
- We can also find the unknown (x) by Multiplicative Factor Method.
- If two persons X and Y can complete some work individually in $\boldsymbol{a}$ and $\boldsymbol{b}$ days, their one day's work will be $\boldsymbol{\frac{1}{a}}$ and $\boldsymbol{\frac{1}{b}}$ respectively and X and Y together can complete the work in $\boldsymbol{\frac{ab}{a+b}}$ days.

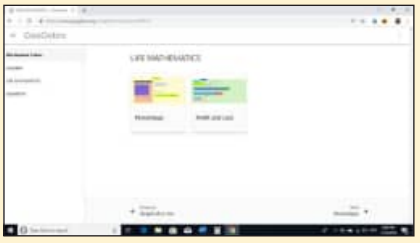
ICT CORNER

Step-1 Open the Browser type the URL Link given below (or) Scan the QR Code. GeoGebra work book named "LIFE MATHEMATICS" will open. Click on the worksheet named "Percentage".

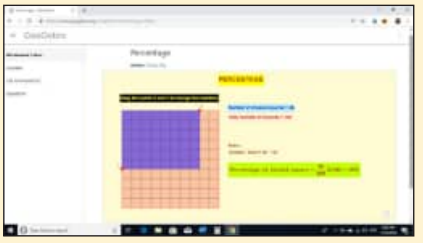
Step-2 In the given worksheet you can drag the red points E and F to change the Blue rectangle. Find the ratio of Blue with the whole by counting the squares and check the ratio and percentage.

Expected Outcome





Step 1



Step 2

SKT0B

Go through the remaining worksheets given for this chapter

Browse in the link

Life Mathematics:

https://www.geogebra.org/m/fqxbd7rz#chapter/409575 or Scan the QR Code.

*Pictures are indicatives only.

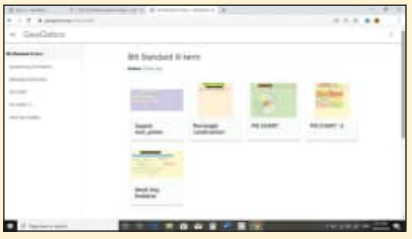
*If browser requires, allow Flash Player or Java Script to load the page.

ICT CORNER

Step-1 Open the Browser type the URL Link given below (or) Scan the QR Code. GeoGebra work sheet named "8th Standard III term" will open. Select the work sheet named "Work Day Problem"

Step-2 Click on " NEW PROBLEM". Check the calculation and work out yourself.

Expected Outcome



Step 1

Step 2

CA6TY

Browse in the link

Life Mathematics:

https://www.geogebra.org/m/xmm5kj9r or Scan the QR Code.

5 GEOMETRY

Learning Objectives

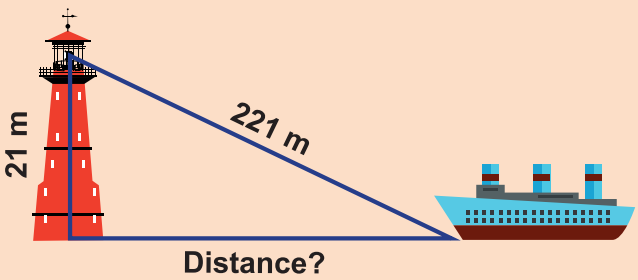
- To recall the similar and congruence properties and also the basic properties of triangles.
- To understand the theorems based on these properties and apply them appropriately to problems.
- To understand the Pythagorean theorem and solve problems using it.
- To know and understand the concurrency of medians, altitudes, angle bisectors and perpendicular bisectors in a triangle.
- To construct quadrilaterals of various types.

S7T8P

5.1 Introduction

Geometry, as we all know studies shapes by looking at the properties and relations of points, circles, triangles of two dimensions and solids. In the earlier classes, we have seen a few properties of triangles. In this class, we are going to recall them and also learn the congruence and the similarity properties in triangles. Also, we shall learn about the Pythagorean theorem and the concurrency of medians, altitudes, angle bisectors and perpendicular bisectors in a triangle. Also, we will also see how to construct quadrilaterals of various types.

MATHEMATICS ALIVE – GEOMETRY IN REAL LIFE



The Pythagoras theorem is useful in finding the distance and the heights of objects.

For better strength and stability, congruent triangles in the construction of buildings.

Answer the following questions by recalling the properties of triangles:

1. The sum of the three angles of a triangle is _____ .
2. The exterior angle of a triangle is equal to the sum of the _____ angles to it.
3. In a triangle, the sum of any two sides is _____ than the third side.
4. Angles opposite to equal sides are _____ and vice – versa.
5. What is $\angle A$ in the triangle ABC?

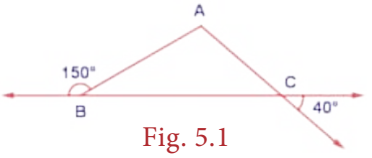
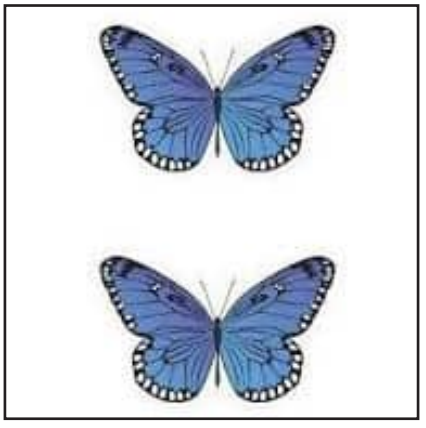


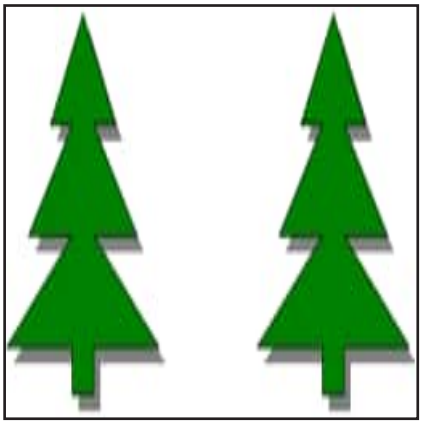
Fig. 5.1

5.2 Congruent and Similar Shapes

Congruent figures are exactly the same in shape and size. In other words, shapes are congruent if one fits exactly over the other.

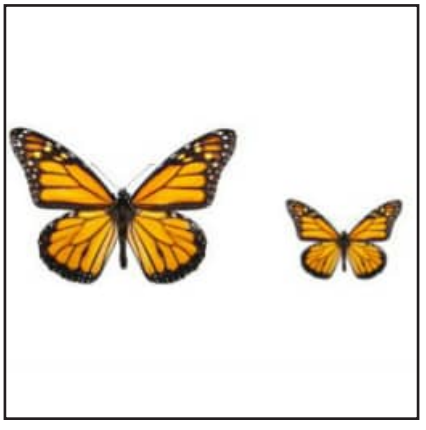
Examples

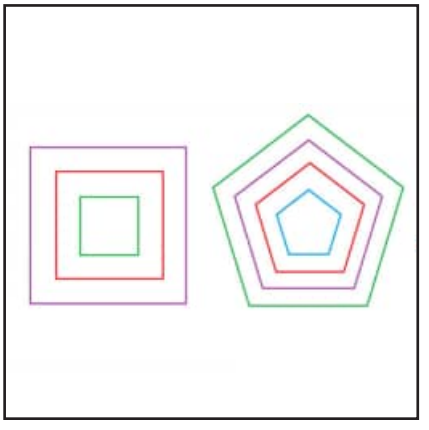




Similar figures mathematically have the same shape but different sizes. Two geometrical figures are said to be similar ($\sim$) if the measures of one to the corresponding measures of the other are in a constant ratio. In other words, every part of a photographic enlargement is similar to the corresponding part of the original.

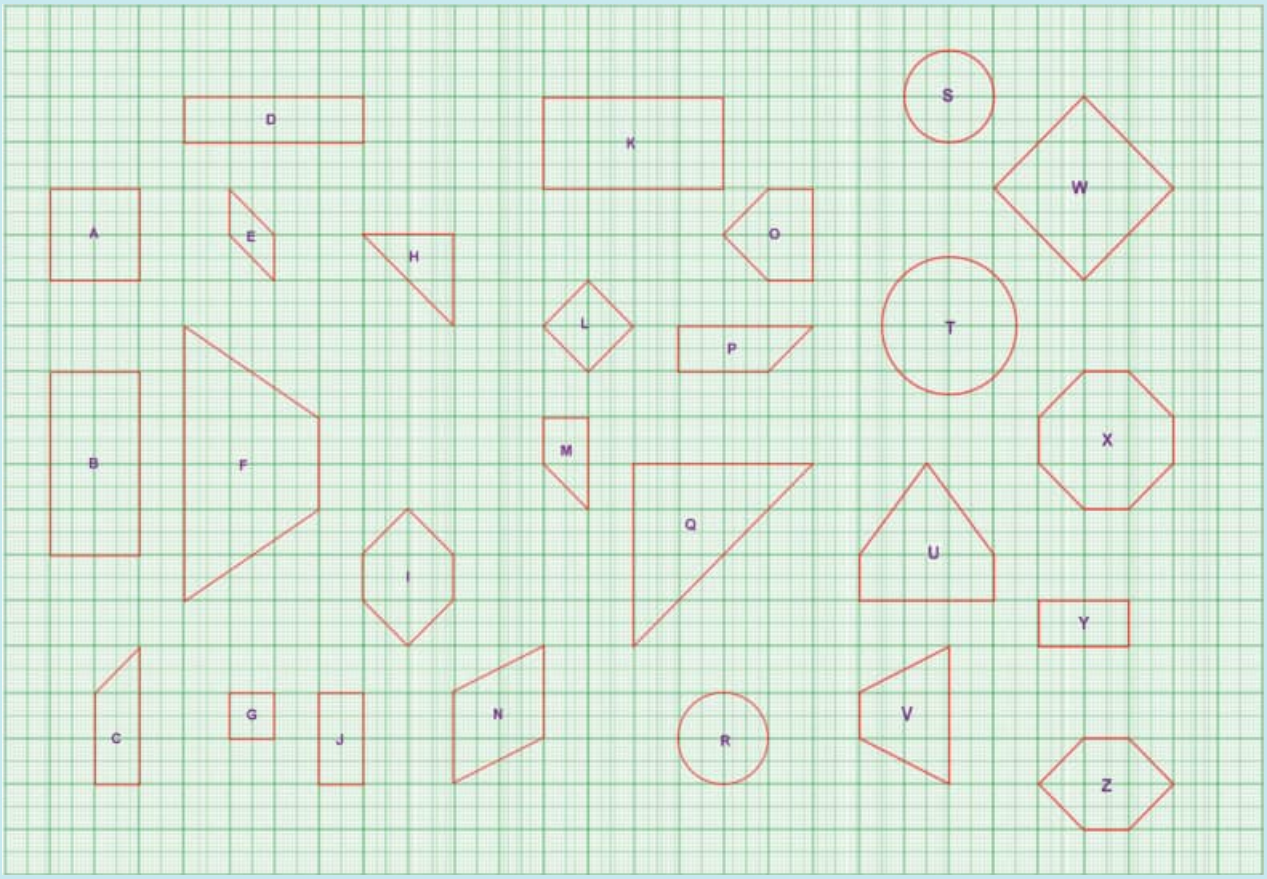
Examples





Try these

Identify the pairs of figures which are similar and congruent and write the letter pairs.



5.2.1 Congruent Triangles

Consider two given triangles PQR and ABC. They are said to be congruent (≡) if their corresponding parts are congruent. That is PQ=AB, QR=BC and PR=AC and also $\angle P = \angle A$, $\angle Q = \angle B$ and $\angle R = \angle C$. This is denoted as $\Delta PQR \equiv \Delta ABC$.

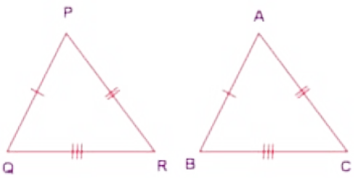


Fig. 5.2

There are 4 ways by which one can prove that two triangles are congruent.

(i) SSS (Side – Side – Side) Congruence

If the three sides of a triangle are congruent to the three sides of another triangle, then the triangles are congruent. That is $AB = PQ$, $BC = QR$ and $AC = PR$

$\Rightarrow \Delta ABC \equiv \Delta PQR$.

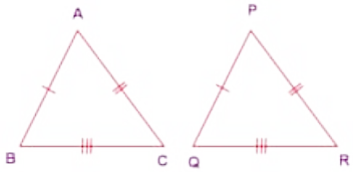


Fig. 5.3

(ii) SAS (Side – Angle – Side) Congruence

If two sides and the included angle (the angle between them) of a triangle are congruent to two sides and the included angle of another triangle, then the triangles are congruent. Here, $AC = PQ$, $\angle A = \angle P$ and $AB = PR$ and hence $\Delta ACB \equiv \Delta PQR$.

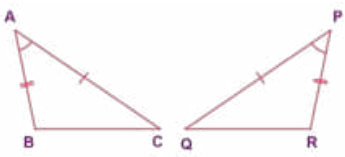


Fig. 5.4

(iii) ASA (Angle-Side-Angle) Congruence

If two angles and the included side of a triangle are congruent to two angles and the included side of another triangle, then the triangles are congruent. Here, $\angle A = \angle R$, CA=PR and $\angle C = \angle P$ and hence $\Delta ABC \equiv \Delta RQP$

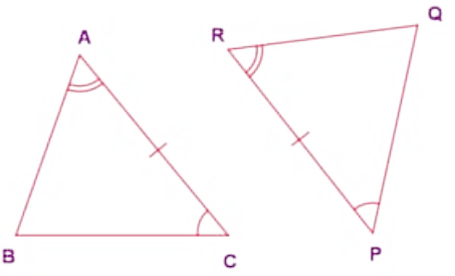


Fig. 5.5

(iv) RHS (Right Angle – Hypotenuse – Side) Congruence

If the hypotenuse and a leg of one right triangle are congruent to the hypotenuse and a leg of another right triangle, then the triangles are congruent. Here, $\angle B = \angle Q = 90°$, $BC = QR$ and $AC = PR$

(right angle) (leg) (hypotenuse)

and hence $\Delta ABC \equiv \Delta PQR$.

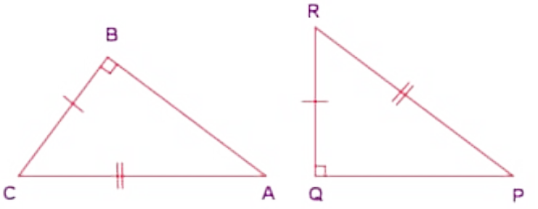


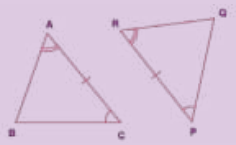
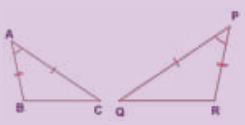
Fig. 5.6

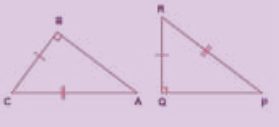
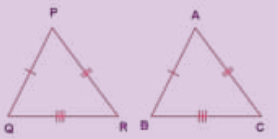
Note

- Any segment or angle is congruent to itself! This is called Reflexive property
- If two triangles are congruent, then their corresponding parts are congruent. This is called **CPCTC (Corresponding parts of Congruent Triangles are Congruent).**
- *If angles then sides* means if two angles are equal in a triangle, then the sides opposite to them are equal.
- *If sides then angles* means if two sides are equal in a triangle, then the angles opposite to them are equal.

Try these

Match the following by their congruence property

S.No.	A	B
1		RHS
2		SSS

S.No.	A	B
3		SAS
4		ASA

Example 5.1

Find the unknowns in the following figures

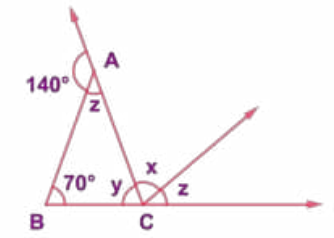
(i) 

Fig. 5.7(i)

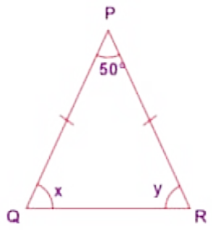
(ii) 

Fig. 5.7(ii)

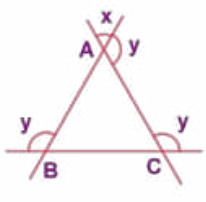
(iii) 

Fig. 5.7(iii)

Solution:

(i) Now, from Fig. 5.7(i), $140° + z = 180°$ (linear pair)

$$\Rightarrow z = 180° - 140° = 40°$$

Also $x + z = 70° + z$ (exterior angle property)

$$\Rightarrow x = 70°$$

Also $z + y + 70° = 180°$ (angle sum property in ΔABC)

$$\Rightarrow 40° + y + 70° = 180°$$

$$\Rightarrow y = 180° - 110° = 70°$$

(ii) Now, from Fig. 5.7(ii), $PQ = PR$

$$\Rightarrow \angle Q = \angle R \quad \text{(angles opposite to equal sides are equal)}$$

$$\Rightarrow x = y$$

$$\Rightarrow x + y + 50° = 180° \quad \text{(angle sum property in } \Delta PQR\text{)}$$

$$\Rightarrow 2x = 130°$$

$$\Rightarrow x = 65°$$

$$\Rightarrow y = 65°$$

(iii) Now, from Fig. 5.7(iii), in ΔABC $\angle A = x$ (vertically opposite angles)

Similarly $\angle B = \angle C = x$ (Why?)

$$\Rightarrow \angle A + \angle B + \angle C = 180° \quad \text{(angle sum property in } \Delta ABC\text{)}$$

$$\Rightarrow 3x = 180°$$

$$\Rightarrow x = 60°$$

$$\Rightarrow y = 180° - 60° = 120°$$

Example 5.2 (Illustrating SSS and SAS Congruence)

If $\angle E = \angle S$ and G is the midpoint of ES,

prove that $\Delta GET \equiv \Delta GST$.

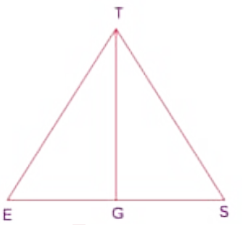
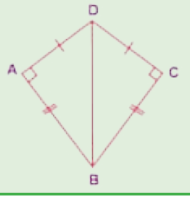


Fig. 5.8

Proof:

	Statements	Reasons
1	$\angle E \equiv \angle S$	given
2	$ET \equiv ST$	if angles, then sides
3	G is the midpoint of ES	given
4	$EG \equiv SG$	follows from 3
5	$TG \equiv TG$	common side (reflexive property)
6	$\Delta GET \equiv \Delta GST$	by SSS (2,4,5) & also by SAS (2,1,4)

Think



In the figure, $DA = DC$ and $BA = BC$.

Are the triangles DBA and DBC congruent?

Why?

Example 5.3 (Illustrating ASA Congruence)

If $\angle YTB \equiv \angle YBT$ and $\angle BOY \equiv \angle TRY$,

prove that $\Delta BOY \equiv \Delta TRY$

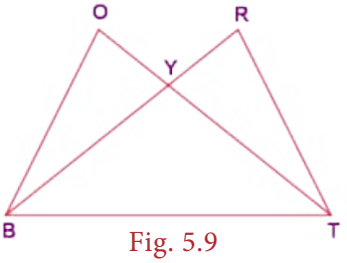


Fig. 5.9

Proof:

	Statements	Reasons
1	$\angle BYO \equiv \angle TYR$	vertical angles are congruent
2	$\angle YTB \equiv \angle YBT$	given
3	$BY \equiv TY$	if angles, then sides
4	$\angle BOY \equiv \angle TRY$	given
5	$\angle OBY \equiv \angle RTY$	follows from 1 and 4
6	$\Delta BOY \equiv \Delta TRY$	by ASA (1,3,5)

Example 5.4 (Illustrating RHS Congruence)

If TAP is an isosceles triangle with $TA = TP$ and $\angle TSA = 90°$.

(i) Is $\Delta TAS \equiv \Delta TPS$? Why?

(ii) Is $\angle P = \angle A$? Why?

(iii) Is $AS = PS$? Why?

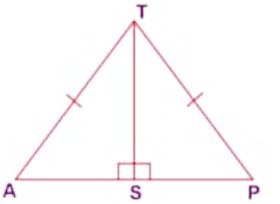


Fig. 5.10

Proof:

(i) TA = TP (hypotenuse) and $\angle TSA = 90$

TS is common (leg)

Hence, by RHS congruence, $\triangle TAS \equiv \triangle TPS$

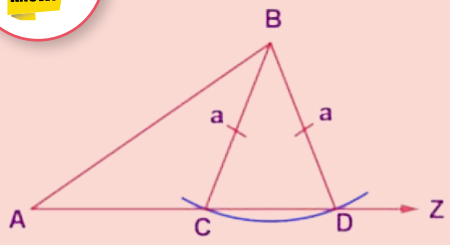
(ii) Given TA = TP

$\therefore \quad \angle P = \angle A$ (if angles then sides)

(iii) From (i) $\triangle TAS \equiv \triangle TPS$,

By CPCTC

$AS = PS$



SSA and ASS properties are not sufficient to prove that two triangles are congruent. This is explained in the given figure. By construction, in triangles ABD and ABC, BC = BD = a. Also, AB and ∠BAZ are common. But AC ≠ AD. So, ΔABD is not congruent to ΔABC and so SSA fails.

5.2.2 Similar Triangles

Consider two given triangles PQR and ABC. They are said to be similar ($\sim$) if their corresponding angles are equal and corresponding sides are proportional. That is $\angle P = \angle A$, $\angle Q = \angle B$ and $\angle R = \angle C$ and also $\frac{PQ}{AB} = \frac{PR}{AC} = \frac{QR}{BC}$. This is denoted as $\Delta PQR \sim \Delta ABC$.

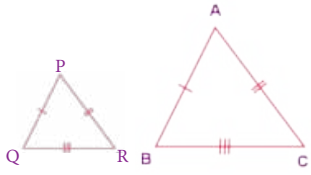


Fig. 5.11

There are 4 ways by which one can prove that two triangles are similar.

(i) AAA (Angle – Angle – Angle) or AA (Angle – Angle) Similarity

Two triangles are similar if two angles of one triangle are equal respectively to two angles of the other triangle. In the given figure, $\angle A = \angle P$, $\angle B = \angle Q$. Therefore, ΔABC $\sim$ ΔPQR.

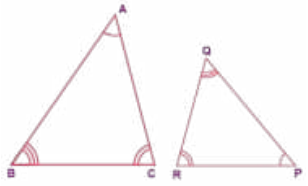


Fig. 5.12

(ii) SAS (Side – Angle – Side) Similarity

Two triangles are similar if two sides of one triangle are proportional to two sides of the other triangle and the included angles are equal. In the given figure, $\frac{AC}{PQ} = \frac{AB}{PR}$ and $\angle A = \angle P$ and hence ΔACB $\sim$ ΔPQR.

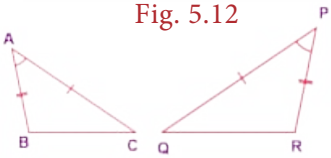


Fig. 5.13

(iii) SSS (Side-Side-Side) Similarity

Two triangles are similar if their corresponding sides are in the same ratio. That is, if $\frac{AB}{PQ}=\frac{AC}{PR}=\frac{BC}{QR}$, then $\Delta ABC \sim \Delta PQR$.

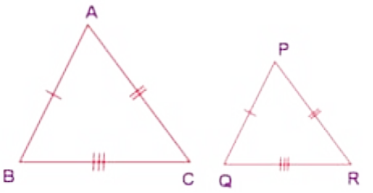


Fig. 5.14

(iv) RHS (Right Angle – Hypotenuse – Side) Similarity

Two right triangles are similar if the hypotenuse and a leg of one triangle are respectively proportional to the hypotenuse and a leg of the other triangle. That is, if $\angle B=\angle Q=90^\circ$ and $\frac{AC}{PR}=\frac{BC}{QR}$ then, $\Delta ABC \sim \Delta PQR$.

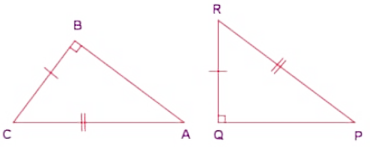
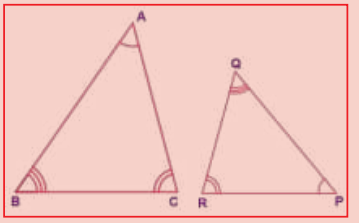


Fig. 5.15

DO YOU KNOW?

If $\Delta ABC \sim \Delta PQR$, then the corresponding sides to AB, BC and AC of ΔABC are PQ, QR and PR respectively and the corresponding angles to A, B and C are P, Q and R respectively. Naming a triangle has a significance. For example, if $\Delta ABC \sim \Delta PQR$ then, ΔBAC is not similar to ΔPQR.



Example 5.5

In the Fig. 5.16, if $\Delta PQR \sim \Delta XYZ$, find a and b.

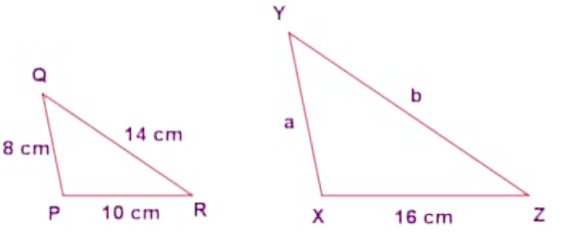


Fig. 5.16

Solution:

Given that $\Delta PQR \sim \Delta XYZ$

$\therefore$ Their corresponding sides are proportional.

$$\Rightarrow \frac{PQ}{XY}=\frac{QR}{YZ}=\frac{PR}{XZ}$$

$$\Rightarrow \frac{8}{a}=\frac{14}{b}=\frac{10}{16}$$

$$\Rightarrow \frac{8}{a}=\frac{10}{16}$$

$$\Rightarrow a=\frac{8\times 16}{10}=\frac{128}{10}$$

$$a = 12.8\ cm$$

Also, $\frac{14}{b}=\frac{10}{16}$

$$\Rightarrow b=\frac{14\times 16}{10}=\frac{224}{10}$$

$$\therefore \quad b = 22.4\ cm$$

Note

- All circles and squares are similar to each other.
- Not all rectangles need to be similar always.
- If two angles are both congruent and supplementary then, they are right angles.
- All congruent triangles are similar.
- The symbol ~ is used to denote similarity.

Example 5.6 (Illustrating AA Similarity)

In the Fig. 5.17 , $\angle ABC \equiv \angle EDC$ and the perimeter of ΔCDE is 27 *units*, prove that $AB \equiv EC$.

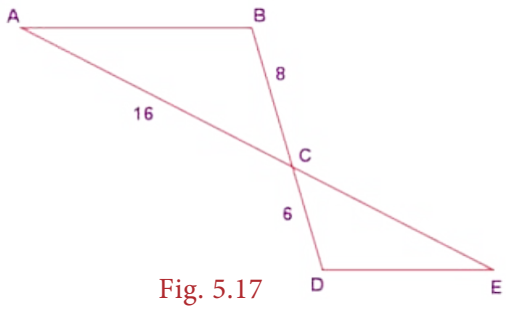


Fig. 5.17

Proof:

	Statements	Reasons
1	$\angle ABC \equiv \angle EDC$	given
2	$\angle BCA \equiv \angle DCE$	vertically opposite angles are equal
3	$\Delta ABC \sim \Delta EDC$	by AA property (1, 2)
4	$\frac{AB}{ED} = \frac{BC}{DC} = \frac{AC}{EC}$	corresponding sides are proportional by 3

$\Rightarrow \frac{8}{6} = \frac{16}{EC} \Rightarrow EC = 12$ *units*

Given, the perimeter of $\Delta CDE = 27$ *units*,

$\therefore$ $ED + DC + EC = 27 \Rightarrow ED + 6 + 12 = 27 \Rightarrow ED = 27 - 18 = 9$ *units*

$\therefore$ $\frac{AB}{9} = \frac{8}{6} \Rightarrow AB = 12$ units and hence $AB = EC$.

Example 5.7 (Illustrating AA Similarity)

In the given Fig. 5.18,

if $\angle 1 \equiv \angle 3$ and $\angle 2 \equiv \angle 4$ then,

prove that $\Delta BIG \sim \Delta FAT$. Also find FA.

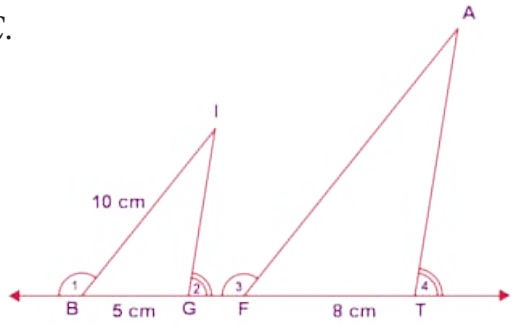


Fig. 5.18

Proof:

	Statements	Reasons
1	$\angle 1 \equiv \angle 3$	given
2	$\angle IBG \equiv \angle AFT$	supplements of congruent angles are congruent.
3	$\angle 2 \equiv \angle 4$	given
4	$\angle IGB \equiv \angle ATF$	supplement of congruent angles are congruent
5	$\Delta BIG \sim \Delta FAT$	by AA property (2, 4)

Also, their corresponding sides are proportional

$\Rightarrow \frac{BI}{FA} = \frac{BG}{FT} \Rightarrow \frac{10}{FA} = \frac{5}{8} \Rightarrow FA = \frac{10 \times 8}{5} = \frac{80}{5} = 16\ cm$

Example 5.8 (Illustrating SAS Similarity)

If A is the midpoint of RU and T is the midpoint of RN, prove that $\Delta RAT \sim \Delta RUN$.

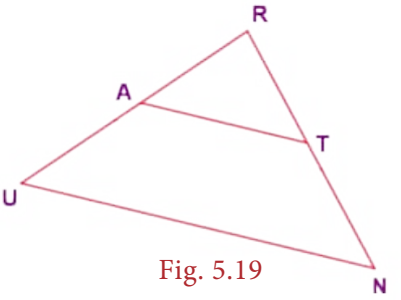


Fig. 5.19

Proof:

	Statements	Reasons
1	$\angle ART = \angle URN$	$\angle R$ is common in ΔRAT and ΔRUN
2	$RA = AU = \frac{1}{2}RU$	A is the midpoint of RU
3	$RT = TN = \frac{1}{2}RN$	T is the midpoint of RN
4	$\frac{RA}{RU} = \frac{RT}{RN} = \frac{1}{2}$	the sides are proportional from 2 and 3
5	$\Delta RAT \sim \Delta RUN$	by SAS (1 and 4)

Example 5.9 (Illustrating SSS similarity)

Prove that $\Delta PQR \sim \Delta PRS$ in the given Fig. 5.20.

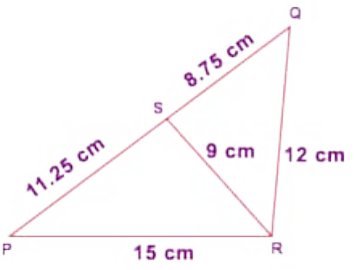


Fig. 5.20

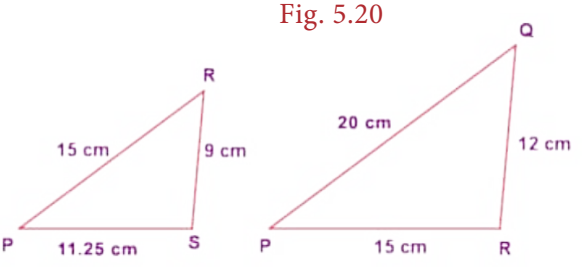
Solution:

Now, $\frac{PQ}{PR} = \frac{20}{15} = \frac{4}{3}$

$\frac{PR}{PS} = \frac{15}{11.25} = \frac{4}{3}$

Also, $\frac{QR}{RS} = \frac{12}{9} = \frac{4}{3}$

We find $\frac{PQ}{PR} = \frac{PR}{PS} = \frac{QR}{RS}$



That is, their corresponding sides are proportional.

$\therefore$ By SSS Similarity, $\Delta PQR \sim \Delta PRS$

Example 5.10 (Illustrating RHS similarity)

The height of a man and his shadow form a triangle similar to that formed by a nearby tree and its shadow. What is the height of the tree?

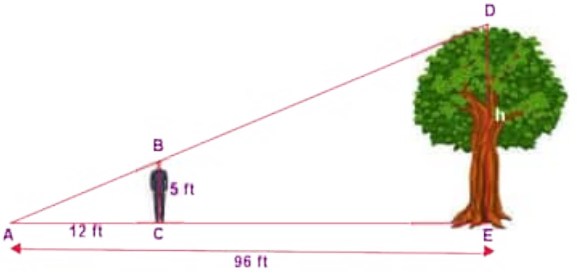


Fig. 5.21

Solution:

Here, $\Delta ABC \sim \Delta ADE$ (given)

$\therefore$ Their corresponding sides are proportional (by RHS similarity).

$$\therefore \quad \frac{AC}{AE} = \frac{BC}{DE}$$

$$\Rightarrow \frac{12}{96} = \frac{5}{h}$$

$$\Rightarrow h = \frac{5 \times 96}{12} = 40 \text{ } feet$$

$\therefore$ The height of the tree is 40 *feet*.

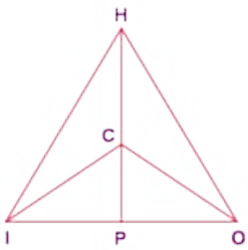
Activity

The teacher cuts many triangles that are similar or congruent from a card board (or) chart sheet. The students are asked to find which pair of triangles are similar or congruent based on the measures indicated in the triangles.

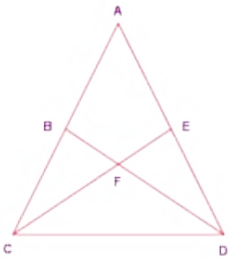
Exercise 5.1

1. Fill in the blanks with the correct term from the given list.
(in proportion, similar, corresponding, congruent, shape, area, equal)
   (i) Corresponding sides of similar triangles are _____.
   (ii) Similar triangles have the same _____ but not necessarily the same size.
   (iii) In any triangle _____ sides are opposite to equal angles.
   (iv) The symbol $\equiv$ is used to represent _____ triangles.
   (v) The symbol $\sim$ is used to represent _____ triangles.

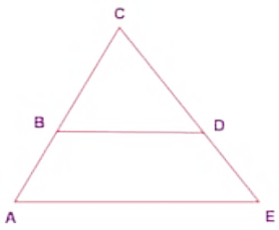
2. In the given figure, $\angle CIP \equiv \angle COP$ and $\angle HIP \equiv \angle HOP$. Prove that $IP \equiv OP$.



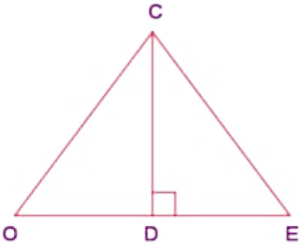
3. In the given figure, $AC \equiv AD$ and $\angle CBD \equiv \angle DEC$. Prove that $\Delta BCF \equiv \Delta EDF$.



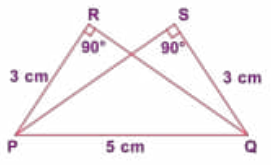
4. In the given figure, ΔBCD is isosceles with base BD and $\angle BAE \equiv \angle DEA$. Prove that $AB \equiv ED$.



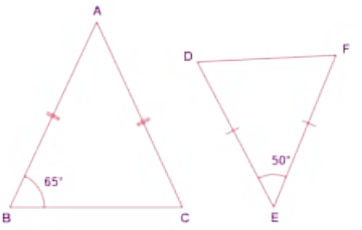
5. In the given figure, D is the midpoint of OE and $\angle CDE = 90°$. Prove that $\Delta ODC \equiv \Delta EDC$



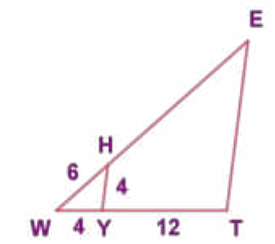
6. Is $\triangle PRQ \equiv \triangle QSP$? Why?



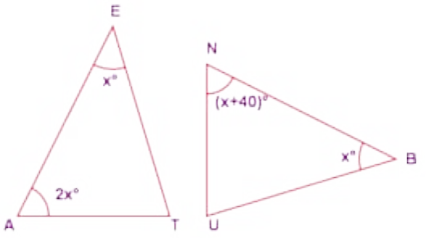
7. From the given figure, prove that $\Delta ABC \sim \Delta EDF$



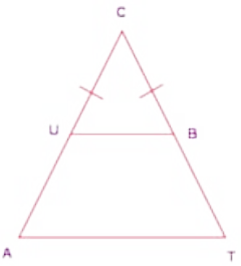
8. In the given figure $YH \parallel TE$. Prove that $\Delta WHY \sim \Delta WET$ and also find HE and TE.



9. In the given figure, if $\Delta EAT \sim \Delta BUN$, find the measure of all angles.



10. In the given figure, $UB \parallel AT$ and $CU \equiv CB$ Prove that $\Delta CUB \sim \Delta CAT$ and hence ΔCAT is isosceles.



1SRAU

Objective Type Questions

11. Two similar triangles will always have _____angles

(A) acute (B) obtuse (C) right (D) matching

12. If in triangles PQR and XYZ, $\frac{PQ}{XY} = \frac{QR}{YZ}$ then they will be similar if

(A) $\angle Q = \angle Y$ (B) $\angle P = \angle Y$ (C) $\angle Q = \angle X$ (D) $\angle P = \angle Z$

13. A flag pole 15 *m* high casts a shadow of 3 *m* at 10 a.m. The shadow cast by a building at the same time is 18.6 *m*. The height of the building is

(A) 90 *m* (B) 91 *m* (C) 92 *m* (D) 93 *m*

14. If $\Delta ABC \sim \Delta PQR$ in which $\angle A = 53^\circ$ and $\angle Q = 77^\circ$, then $\angle R$ is

(A) 50° (B) 60° (C) 70° (D) 80°

15. In the figure, which of the following statements is true?

(A) $AB = BD$ (B) $BD < CD$ (C) $AC = CD$ (D) $BC = CD$

5.3 The Pythagorean Theorem

The Pythagorean theorem or simply Pythagoras theorem, named after the ancient Greek Mathematician Pythagoras BC 570-495 (BCE) is one of the most famous and celebrated theorems in Mathematics. People have proved this theorem in numerous ways possibly the most for any mathematical theorem. The proofs are very diverse which include both geometric and algebraic methods dating back to thousands of years.

Statement of the theorem

In a right angled triangle, the square on the hypotenuse is equal to the sum of the squares on the other two sides.

In $\varnothing ABC$, $BC^2 = AB^2 + AC^2$

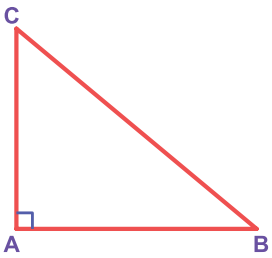


Fig. 5.22

Visual Illustraton:

The given figure contains a triangle of sides of measures 3 units, 4 units and 5 units. From this well known $3-4-5$ triangle, one can easily visualise and understand the meaning of the Pythagorean theorem.

In the figure, the sides of measure 3 units and 4 units are called the legs or sides of the right angled triangle. The side of measure 5 units is called the hypotenuse. Recall that the hypotenuse is the greatest side in a right angled triangle

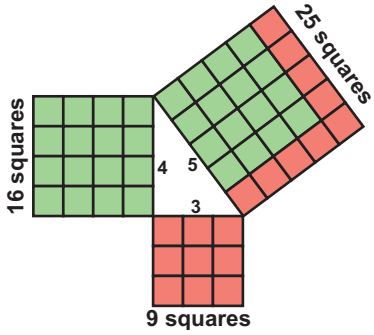


Fig. 5.23

Now, it is easily seen that a square formed with side 5 units (hypotenuse) has 5 × 5 = 25 unit squares (small squares) and the squares formed with side 3 units and 4 units have $3\times3=9$ unit squares and 4 × 4 = 16 unit squares respectively. As per the statement of the theorem, the number of unit squares on the hypotenuse is exactly the sum of the unit squares on the other two legs (sides) of the right angled triangle. Isn't this amazing?

Yes, we find that

5×5 = 3×3+4×4

i.e. 25 = 9+16 (True)

JD9Q5

5.4 Converse of Pythagoras Theorem

If in a triangle, the square on the greatest side is equal to the sum of squares on the other two sides, then the triangle is right angled triangle.

Example:

In the triangle ABC,

$AB^2 + AC^2 = 11^2 + 60^2 = 3721 = 61^2 = BC^2$

Hence, ΔABC is a right angled triangle.

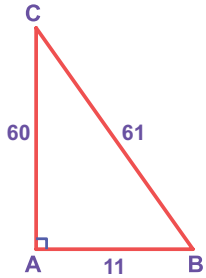


Fig. 5.24

DO YOU KNOW?

(i) There are special sets of numbers a, b and c that makes the Pythagorean relationship true and these sets of special numbers are called Pythagorean triplets. Example: (3, 4, 5) is a Pythagorean triplet.

(ii) Let **k** be any positive integer greater than 1 and (a, b, c) be a Pythagorean triplet, then (ka, kb, kc) is also a Pythagorean triplet.

Examples:

k	**(3,4,5)**	**(5,12,13)**
2k	(6,8,10)	(10,24,26)
3k	(9,12,15)	(15,36,39)
4k	(12,16,20)	(20,48,52)

So, it is clear that we can have infinitely many Pythagorean triplets just by multiplying any Pythagorean triplet by **k.**

We shall now see a few examples on the use of Pythagoras theorem in problems.

Example 5.11

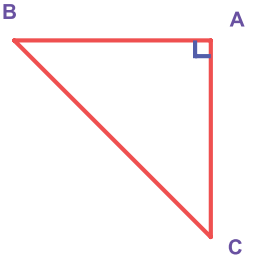


Fig. 5.25

In the figure, $AB \perp AC$

a) What type of $\varnothing$ is ABC?

b) What are AB and AC of the ΔABC ?

c) What is CB called as ?

d) If $AC = AB$ then, what is the measure of $\angle B$ and $\angle C$?

Solution:

a) ΔABC is right angled as $AB \perp AC$ at A.

b) AB and AC are legs of ΔABC.

c) CB is called as the hypotenuse.

d) $\angle B + \angle C = 90^\circ$ and equal angles are opposite to equal sides. Hence, $\angle B = \angle C = \frac{90^\circ}{2} = 45^\circ$

Example 5.12

Can a right triangle have sides that measure 5*cm*, 12*cm* and 13*cm*?

Solution:

Take $a = 5$, $b = 12$ and $c = 13$

Now, $a^2 + b^2 = 5^2 + 12^2 = 25 + 144 = 169 = 13^2 = c^2$

By the converse of Pythagoras theorem, the triangle with given measures is a right angled triangle.

Example 5.13

A 20-feet ladder leans against a wall at height of 16 feet from the ground. How far is the base of the ladder from the wall?

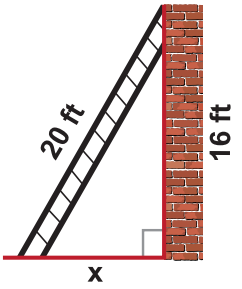


Fig. 5.26

Solution:

The ladder, wall and the ground form a right triangle with the ladder as the hypotenuse. From the figure, by Pythagoras theorem,

$$20^2 = 16^2 + x^2$$

$$\Rightarrow 400 = 256 + x^2$$

$$\Rightarrow \quad x^2 = 400 - 256 = 144 = 12^2 \quad \Rightarrow \quad x = 12 \text{ feet}$$

Therefore, the base (foot) of the ladder is 12 feet away from the wall.

Activity

1. We can construct sets of Pythagorean triplets as follows.
   Let **m** and **n** be any two positive integers (m > n):
   (a, b, c) is a Pythagorean triplet if $a = m^2 - n^2, b = 2mn$ and $c = m^2 + n^2$ (Think, why?)
   Complete the table.

m	**n**	$a = m^2 - n^2$	$b = 2mn$	$c = m^2 + n^2$	Pythagorean triplet
2	1	___	___	___	_____
3	2	___	___	___	_____
4	1	15	8	17	(15, 8, 17)
7	2	45	28	53	(45, 28, 53)

2. Find all integer-sided right angled triangles with hypotenuse 85.

Example 5.14

Find LM, MN, LN and also the area of ΔLON.

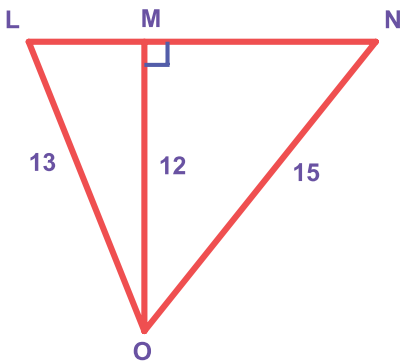


Fig. 5.27

Solution:

From ΔLMO, by Pythagoras theorem,

$$LM^2 = OL^2 - OM^2$$

$$\Rightarrow LM^2 = 13^2 - 12^2 = 169 - 144 = 25 = 5^2$$

$$\therefore \quad LM = 5 \text{ units}$$

From ΔNMO, by Pythagoras theorem,

$$MN^2 = ON^2 - OM^2$$

$$= 15^2 - 12^2 = 225 - 144 = 81 = 9^2$$

$$\therefore \quad MN = 9 \text{ units}$$

Hence, $LN = LM + MN = 5 + 9 = 14$ *units*

$$\text{Area of } \Delta LON = \frac{1}{2} \times \text{base} \times \text{height}$$

$$= \frac{1}{2} \times LN \times OM$$

$$= \frac{1}{2} \times 14 \times 12 = 84 \text{ square units.}$$

Example 5.15

ΔABC is equilateral and CD of the right triangle BCD is 8 *cm*. Find the side of the equilateral ΔABC and also BD.

Solution:

As ΔABC is equilateral from the figure, AB=BC=AC= $(x-2)$ *cm*.

$\therefore$ From ΔBCD, by Pythagoras theorem

$$BD^2 = BC^2 + CD^2$$

$$\Rightarrow (x+2)^2 = (x-2)^2 + 8^2$$

$$x^2 + 4x + 4 = x^2 - 4x + 4 + 8^2$$

$$\Rightarrow 8x = 8^2$$

$$\Rightarrow x = 8\,cm$$

$\therefore$ The side of the equilateral $\Delta ABC = 6$ *cm* and BD =10 *cm*.

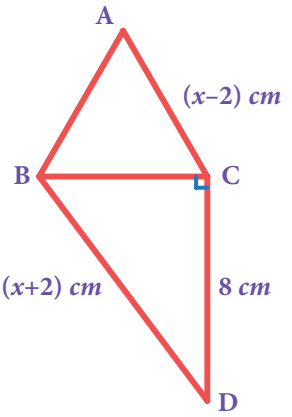
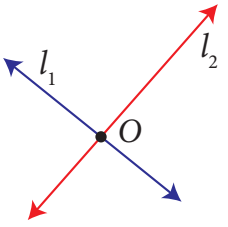
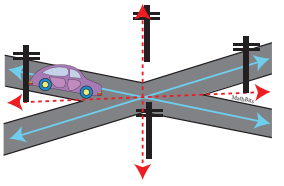


Fig. 5.28

5.5 Point of Concurrency

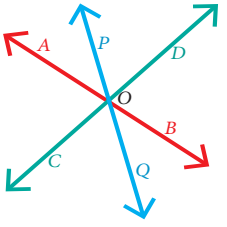
When two lines in a plane cross each other, they are called *intersecting lines*. Here, lines l_1 and l_2 intersect at point O and hence it is called the *point of intersection* of l_1 and l_2 .




Intersecting Roads

Three or more lines in a plane are said to be concurrent, if all of them pass through the same point.

In this figure, $\overleftrightarrow{AB}$, $\overleftrightarrow{CD}$ and $\overleftrightarrow{PQ}$ are concurrent lines and *O* is the *point of concurrency*.



Concurring wooden pieces.

5.6 Medians of a Triangle

A median of a triangle is a line segment from a vertex to the midpoint of the side opposite that vertex.

In the figure $\overline{AM}$ is a median of ΔABC.

Are there any more medians for ΔABC ? Yes, since there are three vertices in a triangle, one can identify three medians in a triangle.

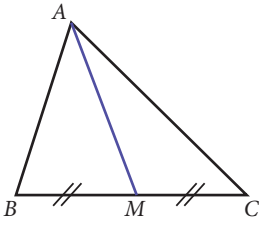


Fig. 5.29

Example 5.16

In the figure, ABC is a triangle and AM is one of its medians. If BM = 3.5 cm, find the length of the side BC.

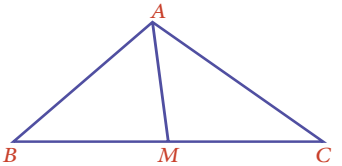


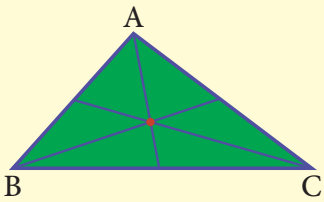
Fig. 5.30

Solution:

AM is median $\Rightarrow$ *M* is the midpoint of *BC*.

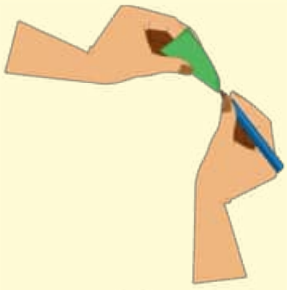
Given that, *BM* = 3.5 cm, hence *BC* = twice the length *BM* = 2 × 3.5 cm = 7 cm.

Activity

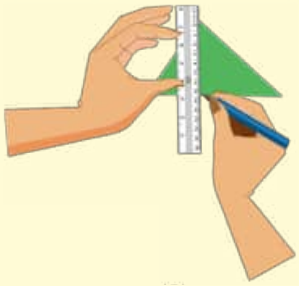


1. Consider a paper cut-out of a triangle. (Let us have an acute-angled triangle, to start with). Name it, say ABC.

2. Fold the paper along the line that passes through the point A and meets the line BC such that point B falls on C. Make a crease and unfold the sheet.

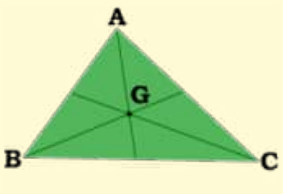


3. Mark the mid point M of BC.



4. You can now draw the median AM, if you want to see it clearly. (Or you can leave it as a fold).

5. In the same way, fold and draw the other two medians.



6. Do the medians pass through the same point?

Now you can repeat this activity for an obtuse-angled triangle and a right triangle. What is the conclusion? We see that,

The three medians of *any* triangle are concurrent.

5.6.1 Centroid

The point of concurrence of the three medians in a triangle is called its **Centroid**, denoted by the letter **G**. Interestingly, it happens to be the centre of mass of the triangle. One can easily verify this fact. Take a stiff cut out of triangle of paper. It can be balanced horizontally at this point on a finger tip or a pencil tip.

Should you fold all the three medians to find the centroid? Now you can explore among yourself the following questions:

(i) How can you find the centroid of a triangle?

(ii) Is the centroid equidistant from the vertices?

(iii) Is the centroid of a triangle always in its interior?

(iv) Is there anything special about the medians of an

(a) Isosceles triangle? (b) Equilateral triangle?

Properties of the centroid of a triangle

The location of the centroid of a triangle exhibits some nice properties.

- It is always *located inside the triangle.*
- We have already seen that it *serves as the Centre of gravity* for any triangular lamina.
- Observe the figure given. The lines drawn from each vertex to G form the three triangles ΔABG, ΔBCG, and ΔCAG. Surprisingly, the areas of these triangles are equal.

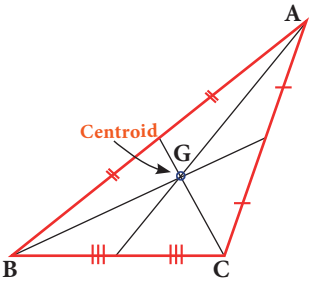


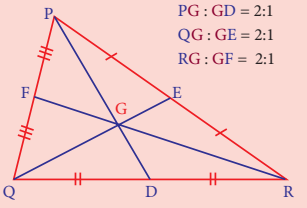
Fig. 5.31

That is, the medians of a triangle divide it into three smaller triangles of equal area!

DO YOU KNOW?

The centroid of a triangle splits each of the medians in two segments, the one closer to the vertex being twice as long as the other one.

This means the centroid divides each median in a ratio of 2:1. (For example, GD is ⅓ of PD). *(Try to verify this by paper folding).*



Example 5.17

In the figure *G* is the centroid of the triangle XYZ.

(i) If GL = 2.5 cm, find the length XL.

(ii) If YM = 9.3 cm, find the length GM.

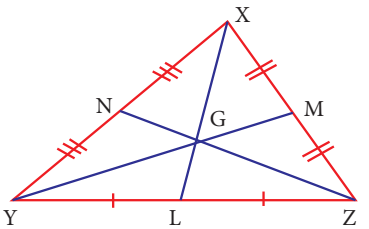


Fig. 5.32

Solution:

(i) Since G is the centroid, XG : GL = 2 : 1 which gives XG : 2.5 = 2 : 1.
Therefore, we get 1 × (XG) = 2 × (2.5) ⇒ XG = 5 cm.
Hence, length XL = XG + GL = 5 + 2.5 = 7.5 cm.

(ii) If YG is of 2 parts then GM will be 1 part. (Why?)
This means YM has 3 parts.
3 parts is 9.3 cm long. So GM (made of 1 part) must be 9.3 ÷ 3 = 3.1 cm.

Example 5.18

ABC is a triangle and G is its centroid. If AD=12 cm, BC=8 cm and BE=9 cm,find the perimeter of ΔBDG.

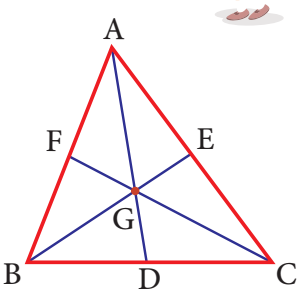


Fig. 5.33

Solution:

ABC is a triangle and G is its centroid. If,

$AD = 12\,cm \Rightarrow GD = \frac{1}{3}$ of $AD = \frac{1}{3}(12) = 4\,cm$ and $BE = 9\,cm \Rightarrow BG = \frac{2}{3}$ of $BE = \frac{2}{3}(9) = 6\,cm$.

Also D is a midpoint of BC $\Rightarrow BD = \frac{1}{2}$ of $BC = \frac{1}{2}(8) = 4\,cm$.

$\therefore$ The perimeter of $\Delta BDG = BD+GD+BG = 4+4+6 = 14$ cm

5.7 Altitude of a Triangle

Altitude of a triangle also known as the height of the triangle, is the perpendicular drawn from the vertex of the triangle to the opposite side. The altitude makes a right angle with the base of a triangle. Here, in ΔABC, AD is one of the altitudes as $AD \perp BC$.

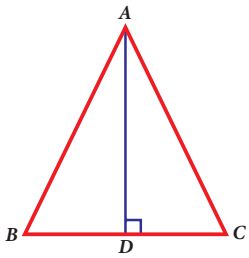
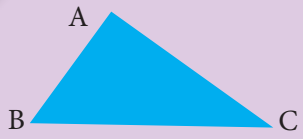
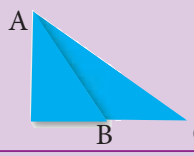
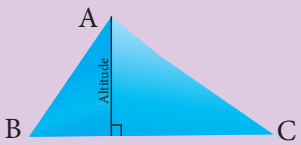


Fig. 5.34

Activity

		
1. Consider a paper cut-out of an acute angled triangle. Name it, say ABC.	2. Fold the triangle so that a side overlaps itself and the fold contains the vertex opposite to that side.	3. You can now draw the altitude AM, if you want to see it clearly.

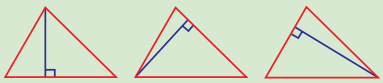
In the same way, you find altitudes of other two sides. Also, with the help of your teacher, you find altitudes of right angled triangle and obtuse angled triangle. Do the altitudes of triangle pass through the same point? What is your conclusion? We see that,

The three altitudes of *any* triangle are concurrent.

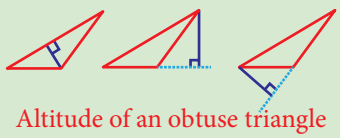
The point of concurrence is known as its Orthocentre, denoted by the letter H.

Think

1. **In any acute angled triangle,** all three altitudes are inside the triangle. Where will be the orthocentre? In the interior of the triangle or in its exterior?
2. **In any right angled triangle,** the altitude perpendicular to the hypotenuse is inside the triangle; the other two altitudes are the legs of the triangle. Can you identify the orthocentre in this case?
3. **In any obtuse angled triangle,** the altitude connected to the obtuse vertex is inside the triangle, and the two altitudes connected to the acute vertices are outside the triangle. Can you identify the orthocentre in this case?

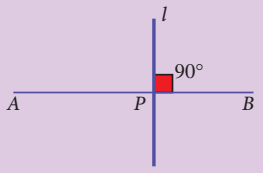
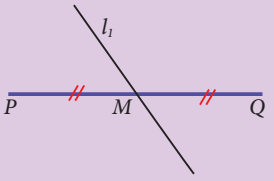
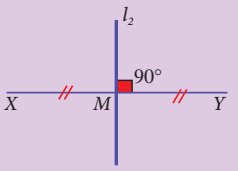

Altitude of an acute triangle

Altitude of a right triangle

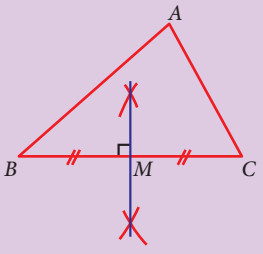
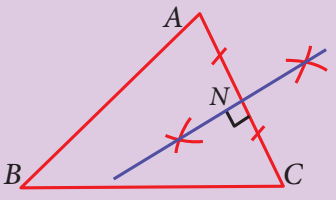
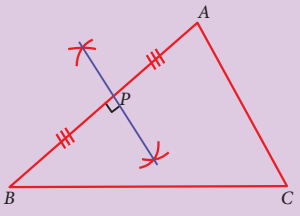

Altitude of an obtuse triangle

5.8 Perpendicular Bisectors of a Triangle

Let us first recall the following ideas.

Perpendicular	Bisector	Perpendicular bisector
		
$\overline{AB}$ is a line segment. l is perpendicular to $\overline{AB}$. P is the foot of the $\perp^r$. Note that $\overline{AP} \uparrow \overline{PB}$ here.	$\overline{PQ}$ is a line segment. l_1 is a bisector to $\overline{PQ}$. M is the midpoint of $\overline{PQ}$. l_1 need not be $\perp^r$ to $\overline{PQ}$	$\overline{XY}$ is a line segment. l_2 is a bisector to $\overline{XY}$. l_2 is also $\perp^r$ to $\overline{XY}$. M is the midpoint of $\overline{XY}$.

Consider a triangle ABC. It has three sides. For each side you can have a perpendicular bisector as follows:

		
Perpendicular bisector of side BC	Perpendicular bisector of side AC	Perpendicular bisector of side AB
M is the midpoint of $\overline{BC}$ We have a perpendicular at M.	N is the midpoint of $\overline{AC}$ We have a perpendicular at N.	P is the midpoint of $\overline{AB}$ We have a perpendicular at P.

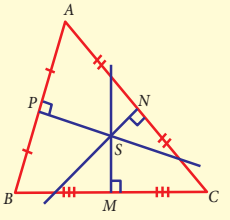
Surprisingly, all the three perpendicular bisectors of the sides of a triangle are concurrent at a point!

Activity

One can visualize the point of concurrence of the perpendicular bisectors, through simple paper folding. Try and see that,

The perpendicular bisectors of the sides of *any* triangle are concurrent.

As done in the earlier activity on Centroid, you can repeat the experiment for various types of triangle, acute, obtuse, right, isosceles and equilateral. Do you find anything special with the equilateral triangle in this case?



5.8.1 Circumcentre

The point of concurrence of the three perpendicular bisectors of a triangle is called as its **Circumcentre,** denoted by the letter **S**.

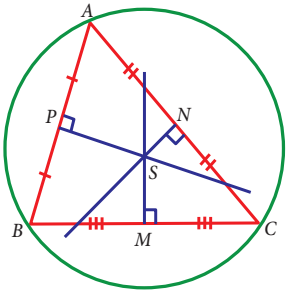


Fig. 5.35

Why should it be called so? Because one can draw a circle exactly passing through the three vertices of the triangle, with centre at the point of concurrence of the perpendicular bisectors of sides. Thus, the circumcentre is equidistant from the vertices of the triangle.

Activity

Check if the following are true by paper-folding:

1. The circumcentre of an acute angled triangle lies in the interior of the triangle.
2. The circumcentre of an obtuse angled triangle lies in the exterior of the triangle.
3. The circumcentre of a right triangle lies at the midpoint of its hypotenuse.

Example 5.19

In ΔABC, S is the circumcentre, BC = 72 cm and DS = 15 cm. Find the radius of its circumcircle.

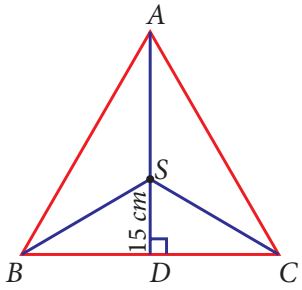


Fig. 5.36

Solution:

As S is the circumcentre of ΔABC, it is equidistant from A,B and C. So AS=BS=CS=radius of its circumcircle. As AD is the perpendicular bisector of BC, $BD = \frac{1}{2} \times BC = \frac{1}{2} \times 72 = 36$ cm

In right angled triangle BDS, by Pythagoras theorem,

$BS^2 = BD^2 + SD^2 = 36^2 + 15^2 = 1521 = 39^2 \Rightarrow BS = 39$ cm.

∴ The radius of the circumcircle of ΔABC is 39 cm.

5.9 Angle Bisectors of a Triangle

We have learnt about angle bisectors in the previous class. An **angle bisector** is a line or ray that divides an angle into two congruent angles. In the figure, ∠ABC is bisected by the line BD such that ∠ABD = ∠CBD.

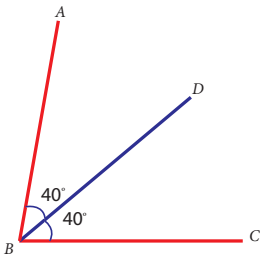
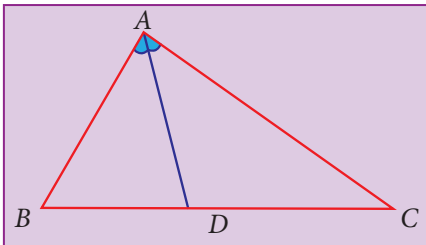
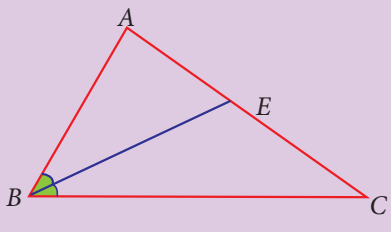
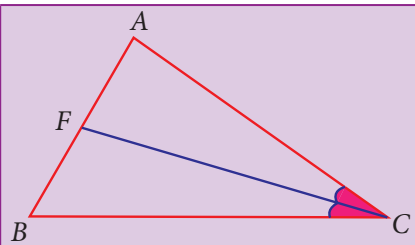
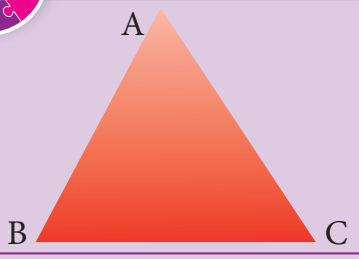
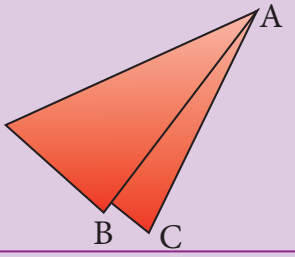
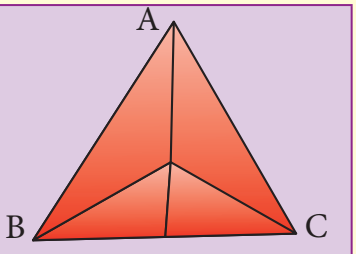


Fig. 5.37

Consider a triangle ABC. How many angles does a triangle have ? 3 angles. For each angle you can have an angle bisector as follows:

		
AD bisects ∠A into two congruent angles. Hence it is an angle bisector of ∠A.	BE bisects ∠B into two congruent angles. Hence it is an angle bisector of ∠B.	CF bisects ∠C into two congruent angles. Hence it is an angle bisector of ∠C.

Activity

		
1. Consider a paper cut-out of a triangle. Name it, say ABC.	2. Fold the triangle so that the opposite sides meet and contain the vertex. Repeat the same to find angle bisectors of other two angles also.	3. Trace all of the folds. Do the angle bisectors pass through the same point?

Now you can repeat this activity for an obtuse-angled triangle and a right angled triangle. What is the conclusion? Do the angle bisectors pass through the same point in all the cases? Yes, we see that,

The three angle bisectors of any triangle are concurrent.

5.9.1 Incentre

The point of concurrence of the three angle bisectors of a triangle is called as its **incentre,** denoted by the letter **I**.

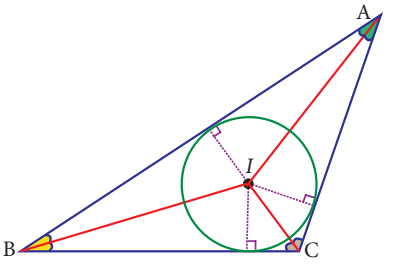


Fig. 5.38

Why should it be called so? Because one can draw a circle inside of the triangle so that it touches all three sides internally, with centre at the point of concurrence of the angle bisectors. The lengths of a perpendicular line drawn from incentre to each side is found to be same. Thus, the incentre is equidistant from the sides of the triangle.

Example 5.20

Identify the incentre of the triangle PQR.

Solution:

Incentre is the point of intersection of angle bisectors.

Here, PM and QN are angle bisectors of ∠P and ∠Q respectively, intersecting at B.

So, the incentre of the triangle PQR is B.

(Can A and C be the incentre of ΔPQR? Why?)

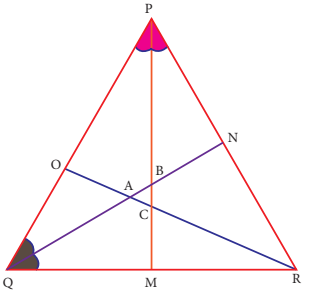


Fig. 5.39

The position of the Centroid, Orthocentre, Circumcentre and Incentre differs depending on the type of triangles given. The following points will help us in locating and remembering these.

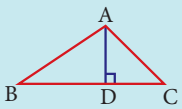
(i) For all types of triangles, Centroid (G) and Incentre (I) will be inside the triangle.

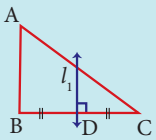
(ii) The Orthocentre (H) will be inside in an acute angled triangle, outside in an obtuse angled triangle and on the vertex containing 90° in a right angled triangle.

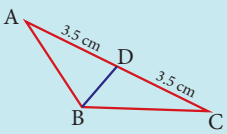
(iii) The Circumcentre (S) will be inside in an acute angled triangle, outside in an obtuse angled triangle and on the hypotenuse in a right-angled triangle.

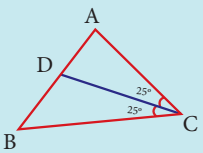
Try these

Identify the type of segment required in each triangle:
(median, altitude, perpendicular bisector, angle bisector)









(i) AD =_____ (ii) l_1 =_____

(iii) BD =_____ (iv) CD =_____

Activity

1. By paper folding, find the centroid, orthocentre, circumcentre and incentre of an equilateral triangle. Do they coinside?
2. By paper folding, find the centroid (G), orthocentre(H), circumcentre (S) and incentre (I) of a triangle. Join G,H,S and I. Are they collinear?

Exercise 5.2

1. **Fill in the blanks:**

(i) If in a ΔPQR, $PR^2 = PQ^2 + QR^2$, then the right angle of ΔPQR is at the vertex _____.

(ii) If 'l' and 'm' are the legs and 'n' is the hypotenuse of a right angled triangle then, l^2 = _____.

(iii) If the sides of a triangle are in the ratio 5:12:13 then, it is _____.

(iv) The medians of a triangle cross each other at _____.

(v) The centroid of a triangle divides each medians in the ratio _____.

2. **Say True or False.**

(i) 8, 15, 17 is a Pythagorean triplet.

(ii) In a right angled triangle, the hypotenuse is the greatest side.

(iii) In any triangle the centroid and the incentre are located inside the triangle.

(iv) The centroid, orthocentre, and incentre of a triangle are collinear.

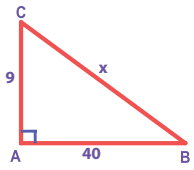
(v) The incentre is equidistant from all the vertices of a triangle.

3. Check whether given sides are the sides of right-angled triangles, using Pythagoras theorem.

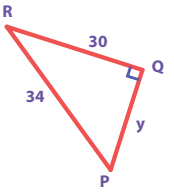
(i) 8,15,17 (ii) 12,13,15 (iii) 30,40,50 (iv) 9,40,41 (v) 24,45,51

4. Find the unknown side in the following triangles.

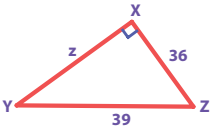
(i)



(ii)

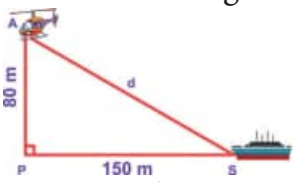


(iii)

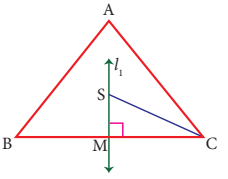


5. An isosceles triangle has equal sides each 13 *cm* and a base 24 *cm* in length. Find its height.

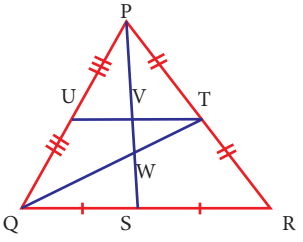
6. Find the distance between the helicopter and the ship.



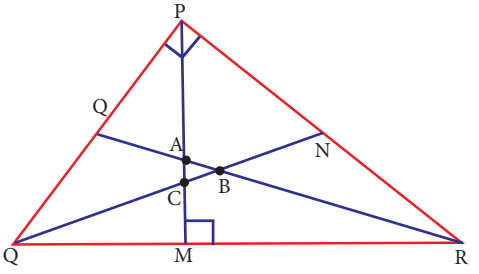
7. In triangle ABC, line l_1 is a perpendicular bisector of BC. If BC=12 cm, SM=8 cm, find CS.



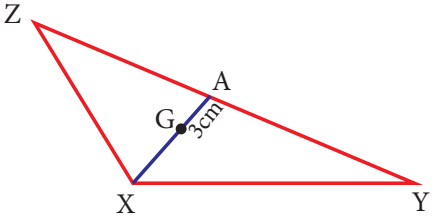
8. Identify the centroid of ΔPQR.



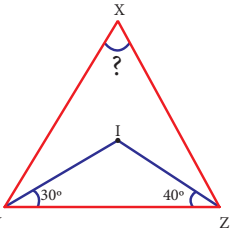
9. Name the orthocentre of ΔPQR.



10. In the given figure, A is the midpoint of YZ and G is the centroid of the triangle XYZ. If the length of GA is 3 cm, find XA.

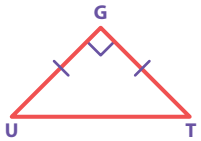


11. If I is the incentre of ΔXYZ, $\angle IYZ = 30°$ and $\angle IZY = 40°$, find $\angle YXZ$.



Objective Type Questions

12. If ΔGUT is isosceles and right angled, then $\angle TUG$ is _____.



(A) 30° (B) 40° (C) 45° (D) 55°

13. The hypotenuse of a right angled triangle of sides $12cm$ and $16cm$ is _____.

(A) 28 cm (B) 20 cm (C) 24 cm (D) 21 cm

14. The area of a rectangle of length $21cm$ and diagonal $29cm$ is _____.

(A) 609 cm^2 (B) 580 cm^2 (C) 420 cm^2 (D) 210 cm^2

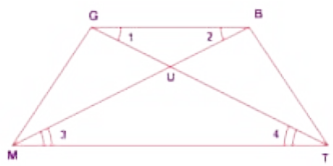
15. The sides of a right angled triangle are in the ratio 5:12:13 and its perimeter is 120 units then, the sides are _____.

(A) 25, 36, 59 (B) 10,24,26 (C) 36, 39, 45 (D) 20,48,52

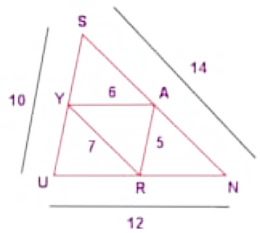
Exercise 5.3

Miscellaneous Practice Problems

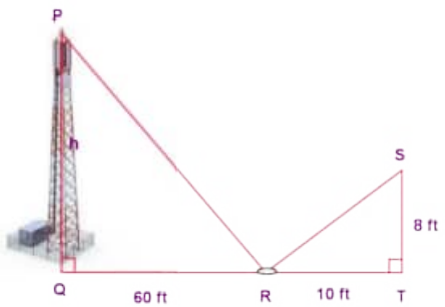
1. In the figure, given that $\angle 1 \equiv \angle 2$ and $\angle 3 \equiv \angle 4$. Prove that $\Delta MUG \equiv \Delta TUB$.



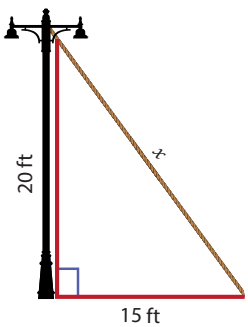
2. From the figure, prove that $\Delta SUN \sim \Delta RAY$.



3. The height of a tower is measured by a mirror on the ground at R by which the top of the tower's reflection is seen. Find the height of the tower. If $\Delta PQR \sim \Delta STR$

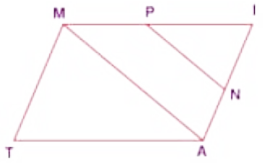


4. Find the length of the support cable required to support the tower with the floor.

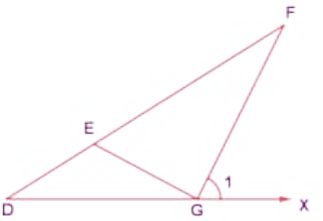


5. Rithika buys an LED TV which has a 25 inches screen. If its height is 7 inches, how wide is the screen? Her TV cabinet is 20 inches wide. Will the TV fit into the cabinet? Give reason.

6. In the figure, $\angle TMA \equiv \angle IAM$ and $\angle TAM \equiv \angle IMA$. P is the midpoint of MI and N is the midpoint of AI. Prove that $\Delta\ PIN \sim \Delta\, ATM$.

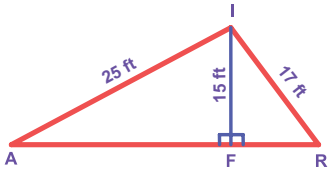


7. In the figure, if $\angle FEG \equiv \angle 1$ then, prove that $DG^2 = DE.DF$.



8. The diagonals of the rhombus is 12 *cm* and 16 *cm*. Find its perimeter. (Hint: the diagonals of rhombus bisect each other at right angles).

9. In the figure, find AR.



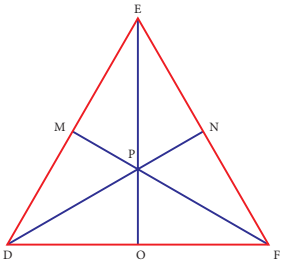
10. In ΔDEF, DN, EO, FM are medians and point P is the centroid. Find the following.

(i) IF DE = 44, then DM = ?

(ii) IF PD = 12, then PN = ?

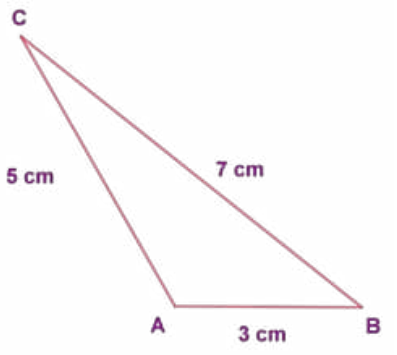
(iii) If DO = 8, then FD = ?

(iv) IF OE = 36 then EP = ?

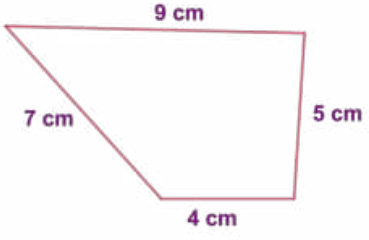


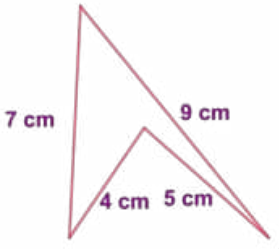
5.10 Construction of Quadrilaterals

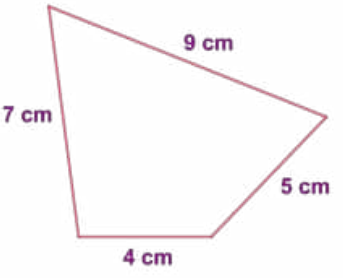
We have already learnt how to draw triangles in the earlier classes. A polygon that has got 3 sides is a triangle. To draw a triangle, we need 3 independent measures. Also, there is only one way to construct a triangle, given its 3 sides. For example, to construct a triangle with sides 3*cm*, 5*cm* and 7*cm*, there is only one way to do it.



Now, let us move on to quadrilaterals. A polygon that is formed by 4 sides is called a quadrilateral. Isn't it? But, a quadrilateral can be of different shapes. They need not look like the same for the given 4 measures. For example, some of the quadrilaterals having their sides as 4 *cm*, 5 *cm*, 7 *cm* and 9 *cm* are given below.







So, to construct a particular quadrilateral, we need a 5th measure. That can be its diagonal or an angle measure. Moreover, even if 2 or 3 sides are given, using the measures of the diagonals and angles, we can construct quadrilaterals.

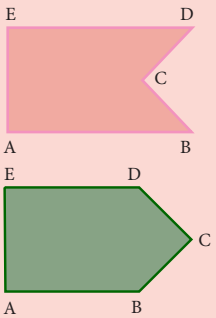
Note

We can split any quadrilateral into two triangles by drawing a diagonal.

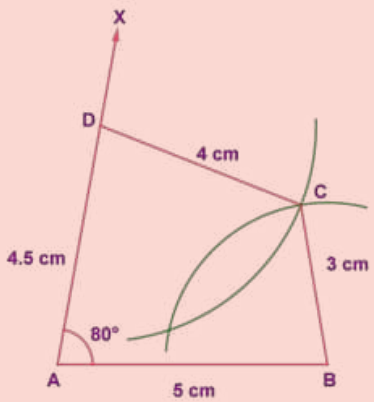
In the above figures, a quadrilateral is split in two ways by its diagonals. So, if a diagonal is given, first draw the lower triangle with two sides and one diagonal. Then, draw the upper triangle with other two measures.

DO YOU KNOW?

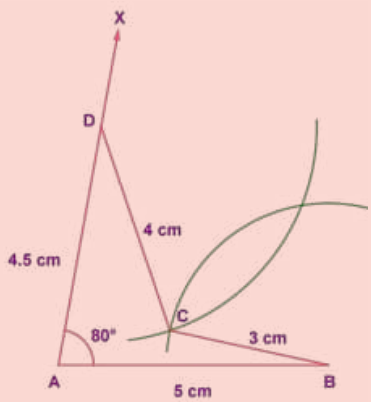
(i)
- A polygon in which atleast one interior angle is more than 180°, is called a concave polygon. In the given polygon, interior angle at C is more than 180°.
- A polygon in which each interior angle is less than 180°, is called a convex polygon. In the given polygon, all interior angles are less than 180°.



(ii) Look at the following quadrilaterals.



Convex quadrilateral



Concave quadrilateral

Though, we can construct a quadrilateral in two ways as shown above, we do not take into account the concave quadrilaterals in this chapter. Hence, the construction of only convex quadrilaterals are treated here.

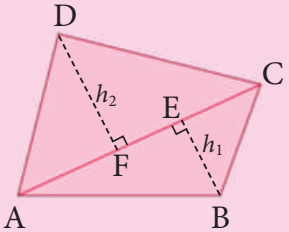
Note

Consider the given Quadrilateral ABCD. In which AC is a diagonal (d), BE (h_1) and DF (h_2) are the perpendiculars drawn from the vertices B and D on diagonal AC.

Area of quadrilateral ABCD = Area of ΔABC + Area of ΔACD

$$= \frac{1}{2} \times AC \times BE + \frac{1}{2} \times AC \times FD$$

$$= \frac{1}{2} \times d \times h_1 + \frac{1}{2} \times d \times h_2 = \frac{1}{2} \times d \times (h_1 + h_2) \text{ sq.units.}$$



We shall now see a few types on constructing these quadrilaterals when its:

(i) 4 sides and a diagonal are given.
(ii) 3 sides and 2 diagonals are given.
(iii) 4 sides and an angle are given.
(iv) 3 sides and 2 angles are given.
(v) 2 sides and 3 angles are given.

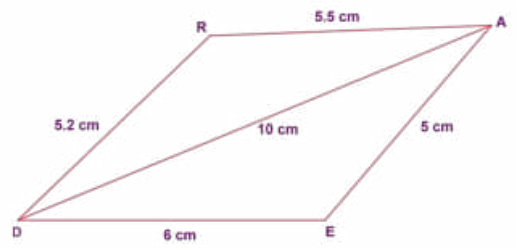
5.10.1 Constructing a quadrilateral when its 4 sides and a diagonal are given

Example 5.21

Construct a quadrilateral *DEAR* with *DE*=6 *cm*, *EA* = 5 *cm*, *AR* = 5.5cm, *RD* = 5.2 *cm* and *DA* = 10 *cm*. Also find its area.

Solution:

Given:
DE = 6 *cm*, *EA* = 5 *cm*, *AR* = 5.5 *cm*,
RD = 5.2 *cm* and a diagonal
DA = 10 *cm*



Rough diagram

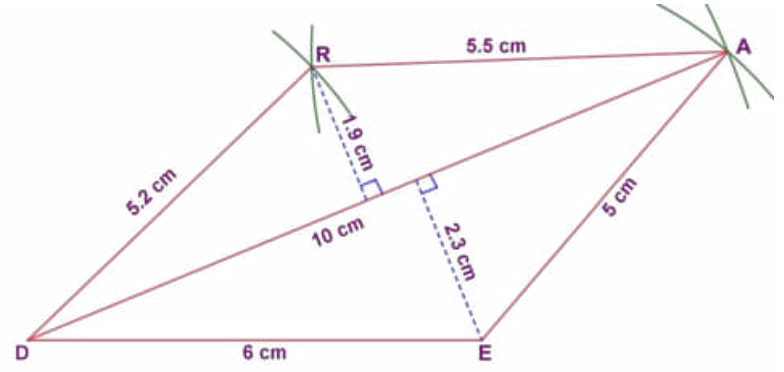


Fig. 5.40

Steps:

1. Draw a line segment *DE* = 6 *cm*.
2. With *D* and *E* as centres, draw arcs of radii 10 *cm* and 5 *cm* respectively and let them cut at *A*.
3. Join *DA* and *EA*.
4. With *D* and *A* as centres, draw arcs of radii 5.2 *cm* and 5.5 *cm* respectively and let them cut at *R*.
5. Join *DR* and *AR* .
6. *DEAR* is the required quadrilateral.

Calculation of Area:

Area of the quadrilateral *DEAR* $= \frac{1}{2} \times d \times (h_1 + h_2)$ *sq. units*

$= \frac{1}{2} \times 10 \times (1.9 + 2.3) = 5 \times 4.2 = 21\ cm^2$

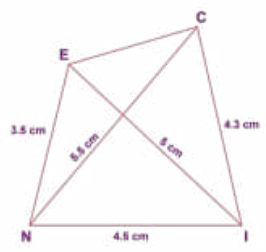
5.10.2 Construct a quadrilateral when its 3 sides and 2 diagonals are given

Example 5.22

Construct a quadrilateral *NICE* with *NI*=4.5 *cm*, *IC*= 4.3 *cm*, *NE*= 3.5 *cm*, *NC*= 5.5 *cm* and *IE* = 5 *cm*. Also find its area.

Solution:

Given:
NI = 4.5 *cm*, *IC*= 4.3 *cm*,
NE = 3.5 *cm* and two diagonals,
NC = 5.5 *cm* and *IE* = 5 *cm*



Rough diagram

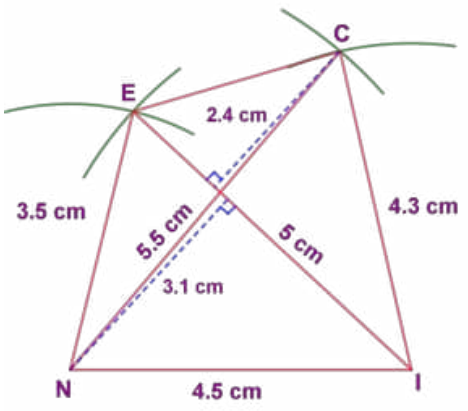


Fig. 5.41

Steps:

1. Draw a line segment *NI* = 4.5 *cm*.
2. With *N* and *I* as centres, draw arcs of radii 5.5 *cm* and 4.3 *cm* respectively and let them cut at *C*.
3. Join *NC* and *IC*.
4. With *N* and *I* as centres, draw arcs of radii 3.5 *cm* and 5 *cm* respectively and let them cut at *E*.
5. Join *NE*, *IE* and *CE* .
6. *NICE* is the required quadrilateral.

Calculation of Area:

Area of the quadrilateral *NICE* $= \frac{1}{2} \times d \times (h_1 + h_2)$ *sq. units*

$= \frac{1}{2} \times 5 \times (2.4 + 3.1)$

$= 2.5 \times 5.5 = 13.75\ cm^2$

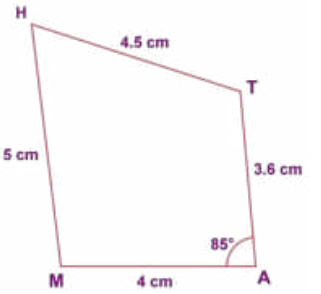
5.10.3 Construct a quadrilateral when its 4 sides and one angle are given

Example 5.23

Construct a quadrilateral $MATH$ with MA=4 cm, AT= 3.6 cm, TH = 4.5 cm, MH= 5 cm and $\angle A = 85°$. Also find its area.

Solution:

Given:
MA=4 cm, AT= 3.6 cm,
TH = 4.5 cm, MH= 5 cm and $\angle A = 85°$



Rough diagram

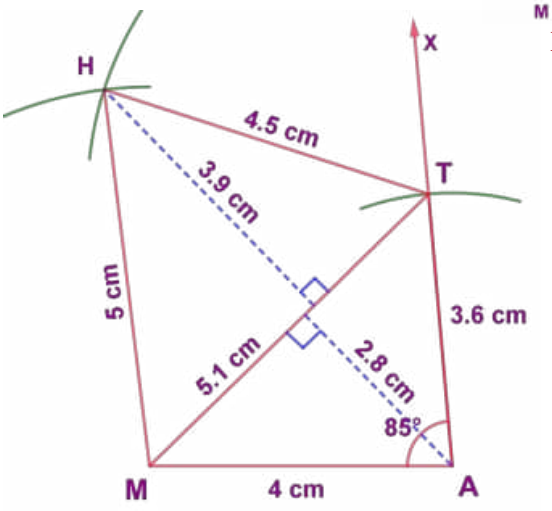


Fig. 5.42

Steps:

1. Draw a line segment MA = 4 cm.
2. Make $\angle A = 85°$.
3. With A as centre, draw an arc of radius 3.6 cm. Let it cut the ray AX at T.
4. With M and T as centres, draw arcs of radii 5 cm and 4.5 cm respectively and let them cut at H.
5. Join MH and TH.
6. $MATH$ is the required quadrilateral.

Calculation of Area:

Area of the quadrilateral $MATH$ $= \frac{1}{2} \times d \times (h_1 + h_2)$ *sq. units*

$= \frac{1}{2} \times 5.1 \times (3.9 + 2.8)$

$= 2.55 \times 6.7 = 17.09\ cm^2$

5.10.4 Construct a quadrilateral when its 3 sides and 2 angles are given

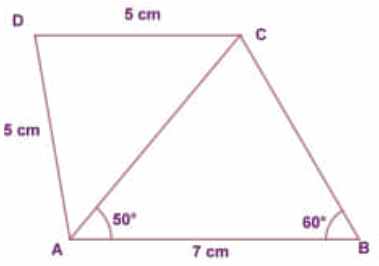
Example 5.24

Construct a quadrilateral $ABCD$ with $AB=7\ cm$, $AD=5\ cm$, $CD=5\ cm$, $\angle BAC=50°$ and $\angle ABC=60°$. Also find its area.

Solution:

Given:

$AB=7\ cm$, $AD=5\ cm$, $CD=5\ cm$ and two angles $\angle BAC=50°$ and $\angle ABC=60°$



Rough diagram

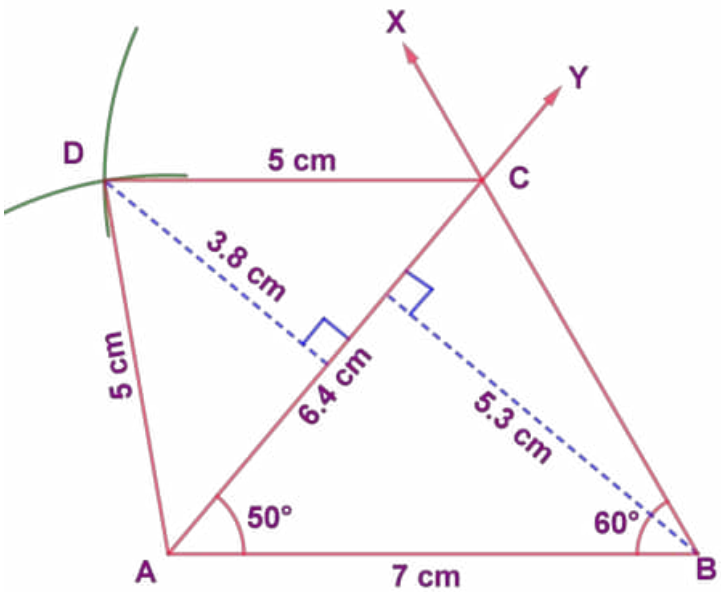


Fig. 5.43

Steps:

1. Draw a line segment $AB = 7\ cm$.
2. At A on AB, make $\angle BAY = 50°$ and at B on AB, make $\angle ABX = 60°$. Let them intersect at C.
3. With A and C as centres, draw arcs of radius 5 cm each. Let them intersect at D.
4. Join AD and CD.
5. $ABCD$ is the required quadrilateral.

Calculation of Area:

$$\text{Area of the quadrilateral } ABCD = \frac{1}{2}\times d\times(h_1+h_2)\ \ sq.\ units$$

$$= \frac{1}{2}\times 6.4\times(3.8+5.3)$$

$$= 3.2 \times 9.1 = 29.12\ cm^2$$

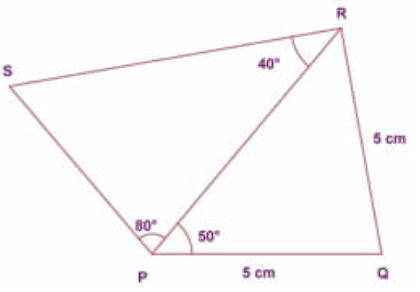
5.10.5 Construct a quadrilateral when its 2 sides and 3 angles are given

Example 5.25

Construct a quadrilateral $PQRS$ with $PQ=QR=5\ cm$, $\angle QPR = 50^\circ$, $\angle PRS = 40^\circ$ and $\angle RPS = 80^\circ$. Also find its area.

Solution:

Given:
$PQ= 5\ cm$, $QR=5\ cm$, $\angle QPR = 50^\circ$, $\angle PRS = 40^\circ$ and $\angle RPS = 80^\circ$



Rough diagram

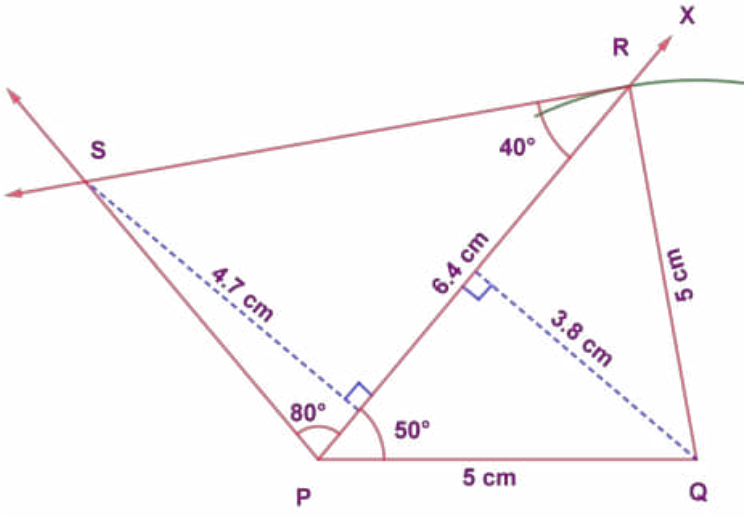


Fig. 5.44

Steps:

1. Draw a line segment $PQ = 5\ cm$.
2. At P on PQ, make $\angle QPX = 50^\circ$.
3. With Q as centre, draw an arc of radius 5 cm. Let it cut PX at R.
4. At R on PR, make $\angle PRS = 40^\circ$ and at P on PR, make $\angle RPS = 80^\circ$. Let them intersect at S.
5. $PQRS$ is the required quadrilateral.

Calculation of Area:

Area of the quadrilateral $PQRS$ $= \frac{1}{2} \times d \times (h_1 + h_2)$ *sq. units*

$= \frac{1}{2} \times 6.4 \times (4.7 + 3.8)$

$= 3.2 \times 8.5 = 27.2\ cm^2$

Think

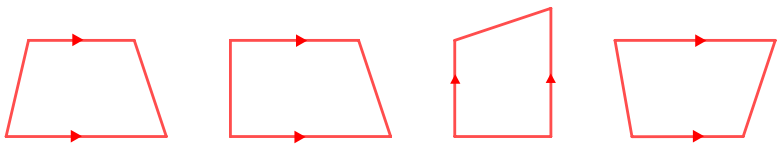
Is it possible to construct a quadrilateral $PQRS$ with $PQ = 5$ cm, $QR = 3$ cm, $RS = 6$ cm, $PS = 7$ cm and $PR = 10$ cm. If not, why?

5.11 Construction of Trapeziums

In the first term, we have learnt how to construct the quadrilaterals. To draw a quadrilateral, how many measurements do you need? 5 measurements. Isn't it? Let us see the special quadrilaterals which need less than 5 measurements. Based on the nature of sides and angles of a quadrilateral, it gets special names like trapezium, parallelogram, rhombus, rectangle, square and kite.

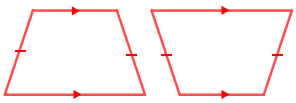
Now, you will learn how to construct trapeziums.

Trapezium is a quadrilateral in which a pair of opposite sides are parallel. To construct a trapezium, draw one of the parallel sides as a base and on that base construct a triangle with the 2 more measurements. Now, through the vertex of that triangle, construct the parallel line opposite to the base so that the triangle lies between the parallel sides. As the fourth vertex lies on this parallel line, mark it with the remaining measure. Hence, we need four independent measures to construct a trapezium. The given shapes are examples of trapeziums.



Note: The arrow marks in the above shapes represent parallel sides.

If the non-parallel sides of a trapezium are equal in length and form equal angles at one of its bases, then it is called an **isosceles trapezium**.



Try these

1. The area of the trapezium is _____.
2. The distance between the parallel sides of a trapezium is called as _____.
3. If the height and parallel sides of a trapezium are 5cm, 7cm and 5cm respectively, then its area is _____.
4. In an isosceles trapezium, the non-parallel sides are _____ in length.
5. To construct a trapezium, _____ measurements are enough.
6. If the area and sum of the parallel sides are 60 cm^2 and 12 cm , its height is _____.

Let us construct a trapezium with the given measurements

1. Three sides and one diagonal.
2. Three sides and one angle.
3. Two sides and two angles.
4. Four sides.

5.11.1 Constructing a trapezium when its three sides and one diagonal are given

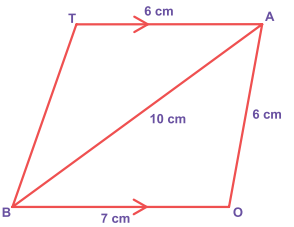
Example 5.26

Construct a trapezium **BOAT** in which $\overline{BO}$ is parallel to $\overline{TA}$, BO=7*cm*, OA=6*cm*, BA=10*cm* and TA=6*cm*. Also find its area.

Solution:

Given:

BO=7*cm*, OA=6*cm*, BA=10*cm* ,
TA=6*cm* and $\overline{BO} \parallel \overline{TA}$



Rough diagram

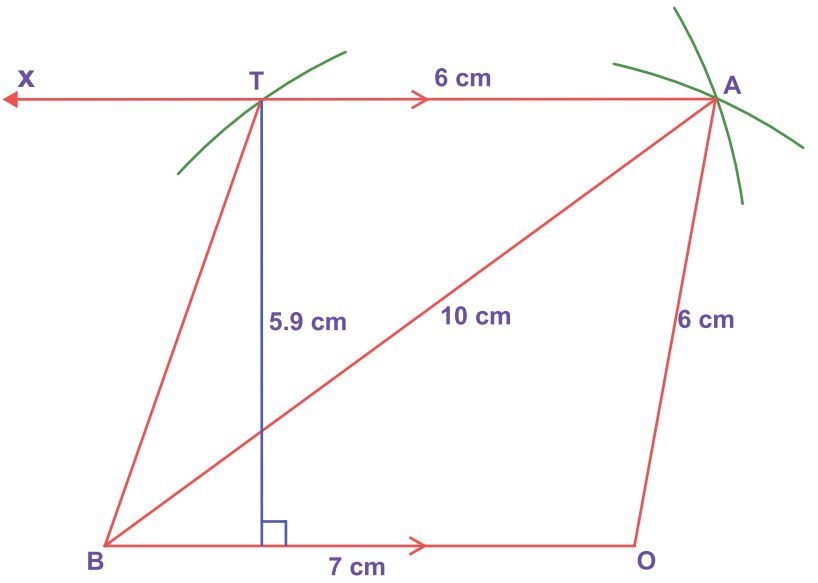


Fig. 5.45

Steps:

1. Draw a line segment BO = 7*cm*.
2. With B and O as centres, draw arcs of radii 10*cm* and 6*cm* respectively and let them cut at A.
3. Join BA and OA.
4. Draw AX parallel to BO
5. With A as centre, draw an arc of radius 6*cm* cutting AX at T.
6. Join BT. BOAT is the required trapezium.

Calculation of Area:

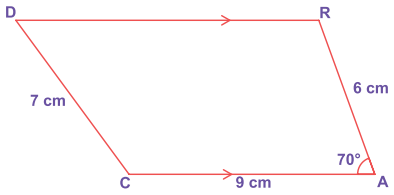
Area of the trapezium BOAT $= \frac{1}{2} \times h \times (a+b)\, sq.units$

$= \frac{1}{2} \times 5.9 \times (7+6) = 38.35\, sq.cm$

5.11.2 Constructing a trapezium when its three sides and one angle are given

Example 5.27

Construct a trapezium **CARD** in which $\overline{CA}$ is parallel to $\overline{DR}$, CA=9*cm*, $\angle CAR = 70°$, AR=6*cm* and CD=7*cm*. Also find its area.



Rough diagram

Solution:

Given:

CA=9*cm*, $\angle CAR = 70°$ AR=6*cm*, and CD=7*cm* and $\overline{CA} \parallel \overline{DR}$

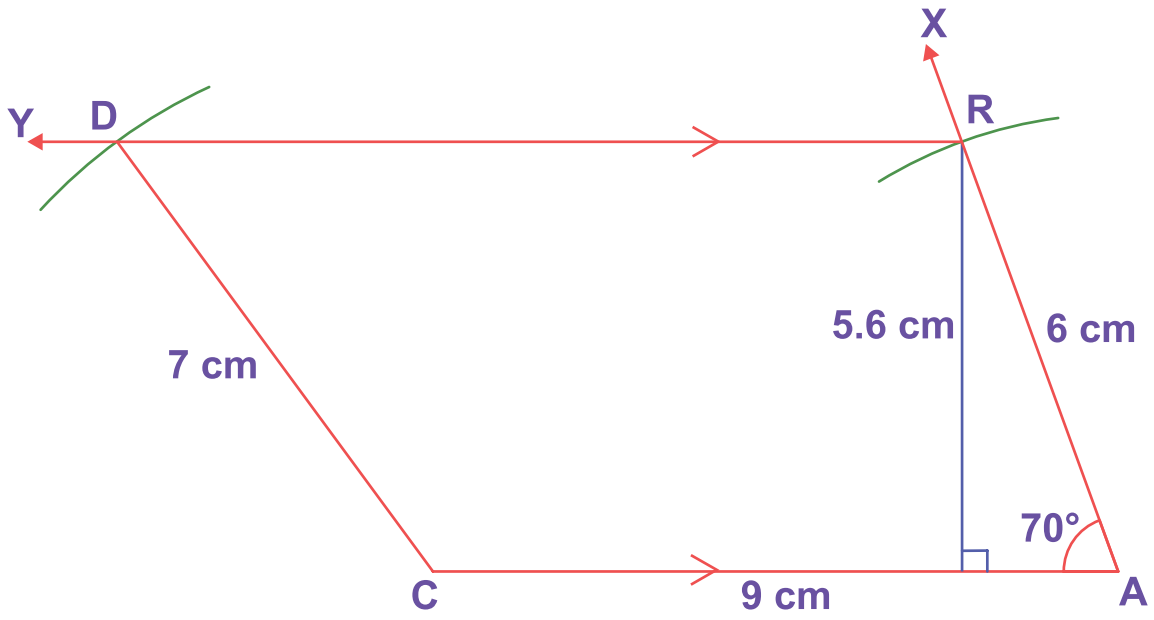


Fig. 5.46

Steps:

1. Draw a line segment CA= 9*cm*.
2. Construct an angle $\angle CAX = 70°$ at A.
3. With A as centre, draw an arc of radius 6*cm* cutting AX at R.
4. Draw RY parallel to CA.
5. With C as centre, draw an arc of radius 7*cm* cutting RY at D.
6. Join CD. CARD is the required trapezium.

Calculation of Area:

Area of the trapezium CARD $= \frac{1}{2} \times h \times (a+b)\, sq.units$

$= \frac{1}{2} \times 5.6 \times (9+11) = 56\, sq.cm$

5.11.3 Constructing a trapezium when its two sides and two angles are given

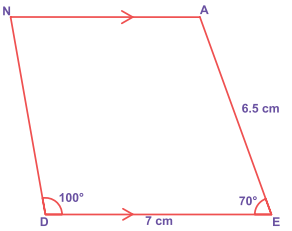
Example 5.28

Construct a trapezium DEAN in which $\overline{DE}$ is parallel to $\overline{NA}$, DE=7*cm*, EA=6.5*cm* $\angle EDN = 100°$ and $\angle DEA = 70°$. Also find its area.

Solution:

Given:

DE=7*cm*, EA=6.5*cm* $\angle EDN = 100°$ and $\angle DEA = 70°$ and $\overline{DE} \parallel \overline{NA}$



Rough diagram

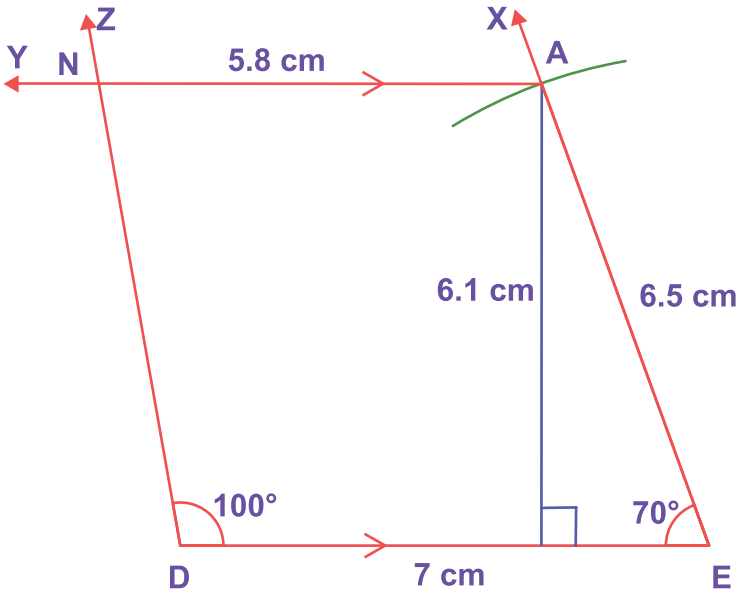


Fig. 5.47

Steps:

1. Draw a line segment DE= 7*cm*.
2. Construct an angle $\angle DEX = 70°$ at E.
3. With E as centre draw an arc of radius 6.5*cm* cutting EX at A.
4. Draw AY parallel to DE.
5. Construct an angle $\angle EDZ = 100°$ at D cutting AY at N.
6. DEAN is the required trapezium.

Calculation of Area:

Area of the trapezium DEAN $= \frac{1}{2} \times h \times (a+b)\, sq.units$

$= \frac{1}{2} \times 6.1 \times (7+5.8) = 39.04\, sq.cm$

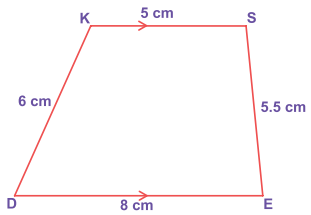
5.11.4 Constructing a trapezium when its four sides are given

Example 5.29

Construct a trapezium DESK in which $\overline{DE}$ is parallel to $\overline{KS}$, DE=8*cm*, ES=5.5*cm*, KS =5*cm* and KD=6*cm*. Find also its area.

Solution:

Given:
DE=8*cm*, ES=5.5*cm*, KS =5*cm*, KD=6*cm* and $\overline{DE} \parallel \overline{KS}$



Rough diagram

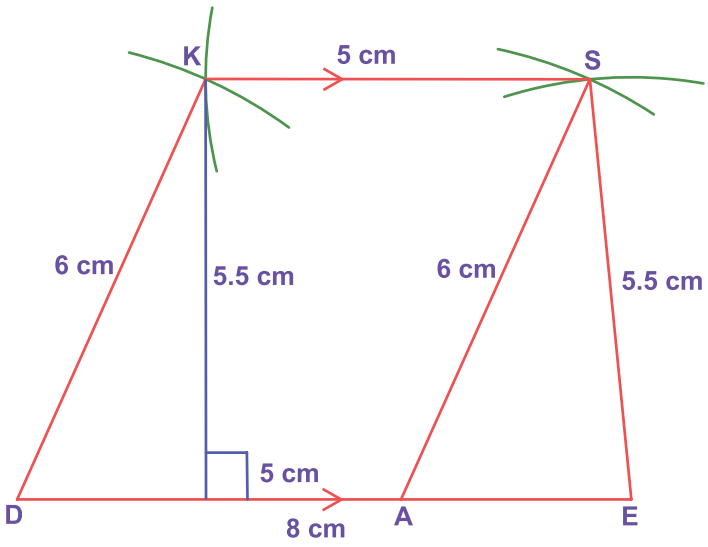


Fig. 5.48

Steps:

1. Draw a line segment DE= 8*cm*.
2. Mark the point A on DE such that DA=5*cm*.
3. With A and E as centres, draw arcs of radii 6*cm* and 5.5*cm* respectively. Let them cut at S. Join AS and ES.
4. With D and S as centres, draw arcs of radii 6*cm* and 5*cm* respectively. Let them cut at K. Join DK and KS.
5. DESK is the required trapezium.

Calculation of Area:

Area of the trapezium DESK $= \frac{1}{2} \times h \times (a+b)\, sq.units$

$= \frac{1}{2} \times 5.5 \times (8+5) = 35.75\, sq.cm$

Exercise 5.4

I. Construct the following quadrilaterals with the given measurements and also find their area.

1. ABCD, AB = 5 cm, BC = 4.5 cm, CD = 3.8 cm, DA = 4.4 cm and AC= 6.2 cm.
2. PLAY, PL= 7 cm, LA = 6 cm, AY= 6 cm, PA = 8 cm and LY = 7 cm.
3. PQRS, PQ=QR= 3.5 cm, RS= 5.2 cm, SP = 5.3 cm and $\angle Q = 120°$.
4. MIND, MI =3.6 cm, ND = 4 cm, MD= 4 cm, $\angle M = 50°$ and $\angle D = 100°$.
5. AGRI, AG= 4.5 cm, GR = 3.8 cm, $\angle A = 60°$, $\angle G = 110°$ and $\angle R = 90°$.

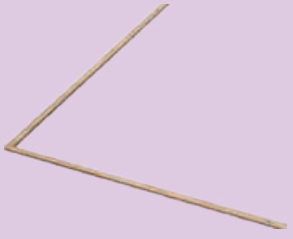
II. Construct the following trapeziums with the given measures and also find their area.

1. AIMS with $\overline{AI} \parallel \overline{SM}$, AI=6*cm*, IM=5*cm*, AM=9*cm* and MS=6.5*cm*.
2. CUTE with $\overline{CU} \parallel \overline{ET}$, CU=7*cm*, $\angle UCE = 80°$ CE=6*cm* and TE=5*cm*.
3. ARMY with $\overline{AR} \parallel \overline{YM}$, AR=7*cm*, RM=6.5*cm* $\angle RAY = 100°$ and $\angle ARM = 60°$
4. CITY with $\overline{CI} \parallel \overline{YT}$, CI=7*cm*, IT=5.5*cm*, TY=4*cm* and YC=6*cm*.

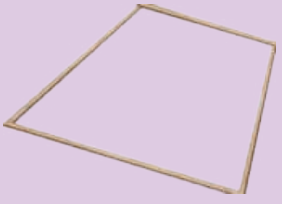
5.12 Construction of Special Quadrilaterals

Before we begin to learn constructing certain quadrilaterals, it is essential to recall their basic properties that would help us during the process. We will try to do this by performing some activities and then sum them up.

Activity



1. Place a pair of *unequal* sticks (say pieces of broomstick) such that they have their end points joined at one end.



2. Now place another such pair meeting the free ends of the first pair.

What is the figure enclosed? It is a quadrilateral. Name it as ABCD. How many sides are there? What are its diagonals? Are the diagonals equal? Are the angles equal?

In the above activity can you get a quadrilateral in which

(i) All the four angles are acute.

(ii) One of the angles is obtuse.

(iii) Two of the angles are obtuse.

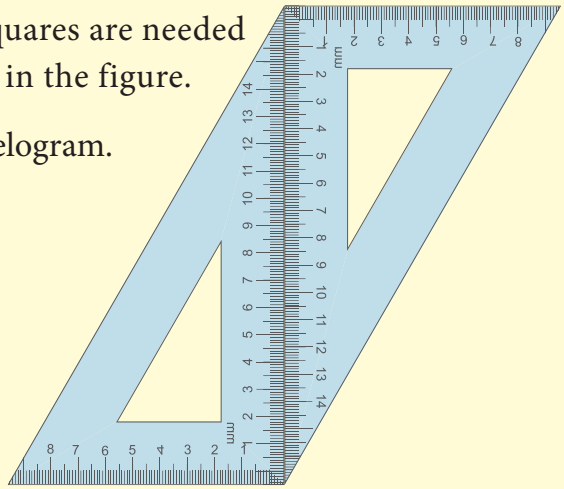
(iv) One of the angles is a right angle.

(v) Two of the angles are right angles.

(vi) The diagonals are mutually $\perp^r$.

Activity

1. A pair of identical 30°–60°–90° set-squares are needed for this activity. Place them as shown in the figure.



(i) What is the shape we get? It is a parallelogram.

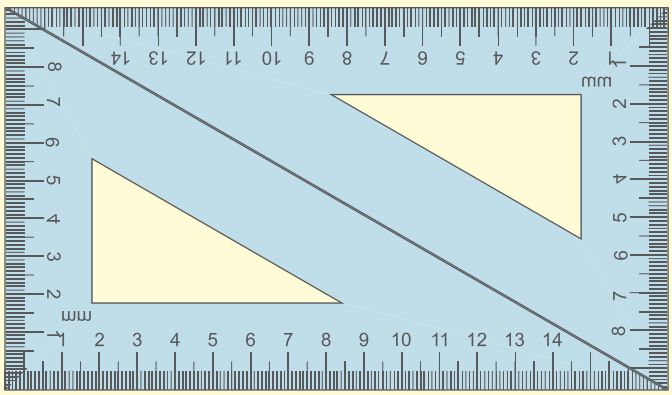
(ii) Are the opposite sides parallel?

(iii) Are the opposite sides equal?

(iv) Are the diagonals equal?

(v) Can you get this shape by using any other pair of identical set-squares?

2. We need a pair of 30°–60°–90° set- squares for this activity. Place them as shown in the figure.



(i) What is the shape we get?

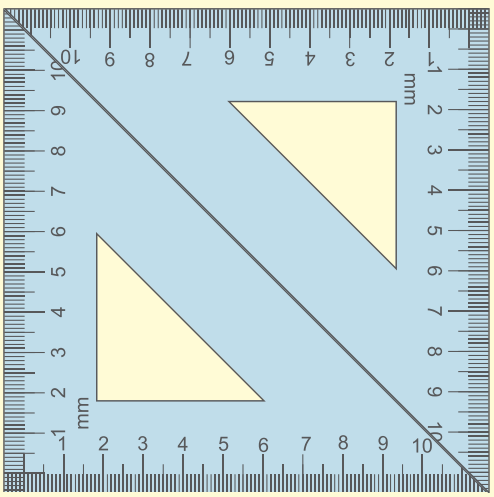
(ii) Is it a parallelogram?

It is a quadrilateral; infact it is a rectangle. (How?)

(iii) What can we say about its lengths of sides, angles and diagonals?

Discuss and list them out.

3. Repeat the above activity, this time with a pair of 45°–45°–90° set-squares.



(i) How does the figure change now? Is it a parallelogram? It becomes a square! (How did it happen?)

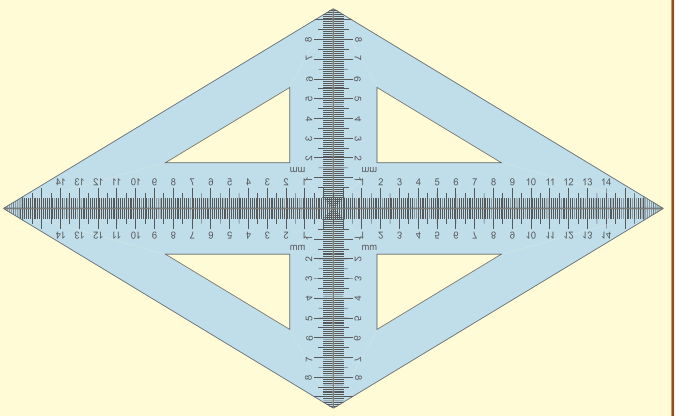
(ii) What can we say about its lengths of sides, angles and diagonals?

Discuss and list them out.

(iii) How does it differ from the list we prepared for the rectangle?

Activity

4. We again use four identical 30°–60°–90° set- squares for this activity. Note carefully how they are placed touching one another.



   (i) Do we get a parallelogram now?

   (ii) What can we say about its lengths of sides, angles and diagonals?

   (iii) What is special about their diagonals?

Based on the outcome of the above activities, we can list out the various properties of the above quadrilaterals, all of which happen to be parallelograms!

Special Quadrilaterals	All sides	All angles	Opposite Sides		All angles	Opposite angles	Diagonals	
	Equal	Equal	Equal	Parallel	90°	Supple-mentary	Bisect each other	Cut at rt.angles
Parallelogram	Sometimes	Sometimes	Always	Always	Sometimes	Sometimes	Always	Sometimes
Rhombus	Always	Sometimes	Always	Always	Sometimes	Sometimes	Always	Always
Rectangle	Sometimes	Always	Always	Always	Always	Always	Always	Sometimes
Square	Always	Always	Always	Always	Always	Always	Always	Always

Try these

1. Say True or False:

(a) A square is a special rectangle.

(b) A square is a parallelogram.

(c) A square is a special rhombus.

(d) A rectangle is a parallelogram

2. Name the quadrilaterals

(a) which have diagonals bisecting each other.

(b) In which the diagonals are perpendicular bisectors of each other.

(c) Which have diagonals of different lengths.

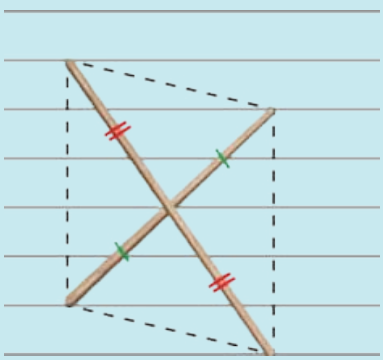
(d) Which have equal diagonals.

(e) Which have parallel opposite sides.

(f) In which opposite angles are equal.

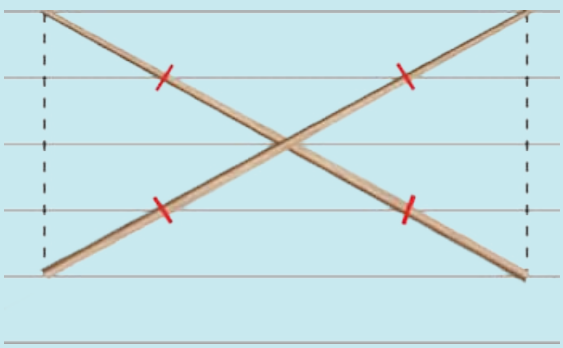
3. Two sticks are placed on a ruled sheet as shown. What figure is formed if the four corners of the sticks are joined?

(a)



Two unequal sticks. Placed such that their midpoints coincide.

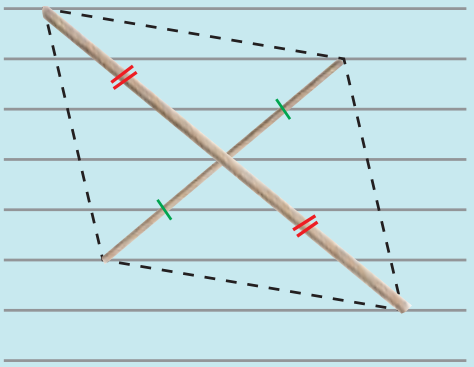
(b)



Two equal sticks. Placed such that their midpoints coincide.

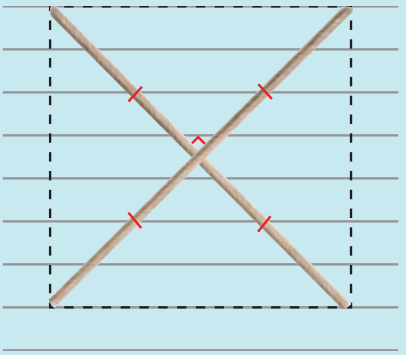
Try these

(c)



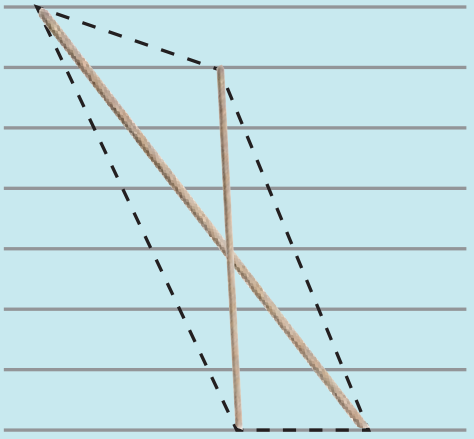
Two unequal sticks. Placed intersecting at mid points perpendicularly.

(d)



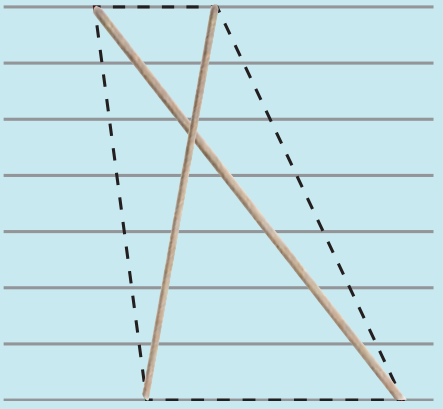
Two equal sticks. Placed intersecting at mid points perpendicularly.

(e)



Two unequal sticks. Tops are not on the same ruling. Bottoms on the same ruling. Not cutting at the mid point of either.

(f)



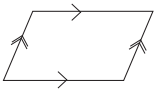
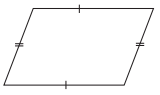
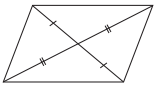
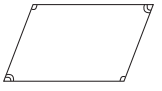
Two unequal sticks. Tops on the same ruling. Bottoms on the same ruling. Not necessarily cutting at the mid point of either.

5.13 Construction of a Parallelogram

Let us construct a parallelogram with the given measurements

1. Two adjacent sides and one angle.
2. Two adjacent sides and one diagonal.
3. Two diagonals and one included angle.
4. One side, one diagonal and one angle.

Note

 Similar arrows indicates the parallel sides.	 Similar lines indicates the congruent sides.	 Figure shows that the diagonals bisect each other.	 Figure shows that the opposite angles are congruent.

5.13.1 Constructing a parallelogram when its two adjacent sides and one angle are given

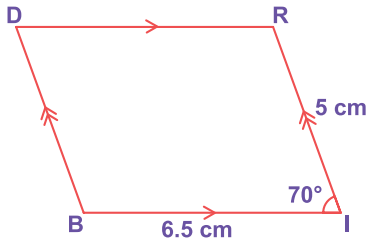
Example 5.30

Construct a parallelogram BIRD with BI=6.5*cm*, IR=5*cm* and $\angle BIR = 70°$. Also find its area.

Solution:

Given:

BI=6.5*cm*, IR=5*cm* and $\angle BIR = 70°$



Rough diagram

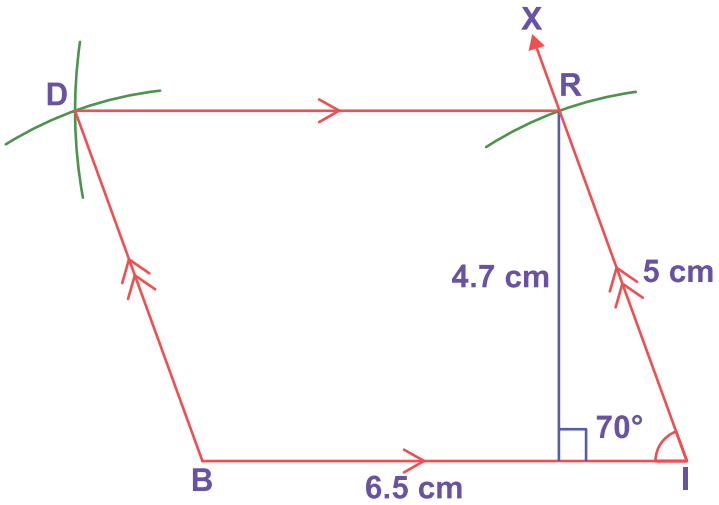


Fig. 5.49

Steps:

1. Draw a line segment BI=6.5*cm*.
2. Make an angle $\angle BIX = 70°$ at I on $\overline{BI}$.
3. With I as centre, draw an arc of radius 5*cm* cutting IX at R.
4. With B and R as centres, draw arcs of radii 5*cm* and 6.5*cm* respectively. Let them cut at D.
5. Join BD and RD.
6. BIRD is the required parallelogram.

Calculation of Area:

Area of the parallelogram BIRD $= bh$ sq.units

$= 6.5 \times 4.7 = 30.55\, sq.cm$

5.13.2 Constructing a parallelogram when its two adjacent sides and one diagonal are given

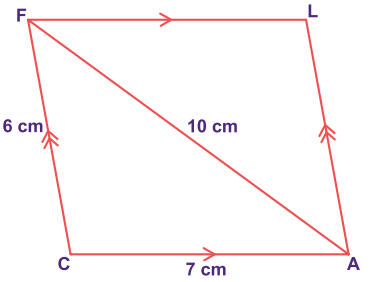
Example 5.31

Construct a parallelogram CALF with CA=7*cm*, CF=6*cm* and AF=10*cm*. Also find its area.

Solution:

Given:

CA=7*cm*, CF=6*cm* and AF=10*cm*



Rough diagram

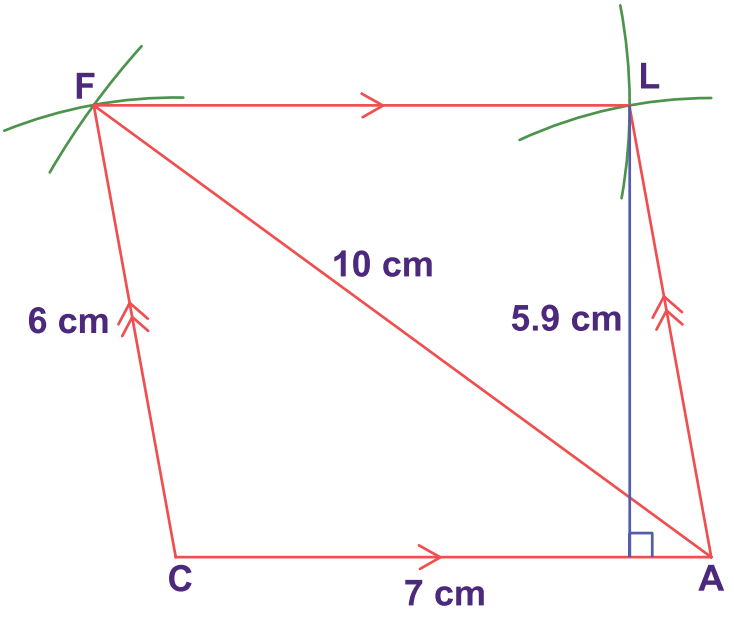


Fig. 5.50

Steps:

1. Draw a line segment CA=7*cm*.
2. With C and A as centres, draw arcs of radii 6*cm* and 10*cm* respectively. Let them cut at F.
3. Join CF and AF.
4. With A and F as centres, draw arcs of radii 6*cm* and 7*cm* respectively. Let them cut at L.
5. Join AL and FL.
6. CALF is the required parallelogram.

Calculation of Area:

Area of the parallelogram CALF = bh sq.units

$= 7 \times 5.9 = 41.3\, sq.cm$

5.13.3 Constructing a parallelogram when its two diagonals and one included angle are given

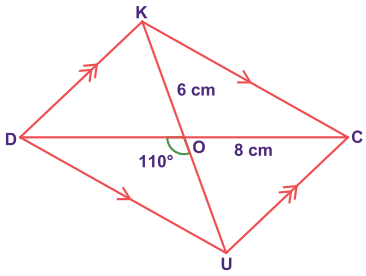
Example 5.32

Construct a parallelogram DUCK with DC=8*cm*, UK=6*cm* and $\angle DOU = 110°$. Also find its area.

Solution:

Given:

DC=8*cm*, UK=6*cm* and $\angle DOU = 110°$



Rough diagram

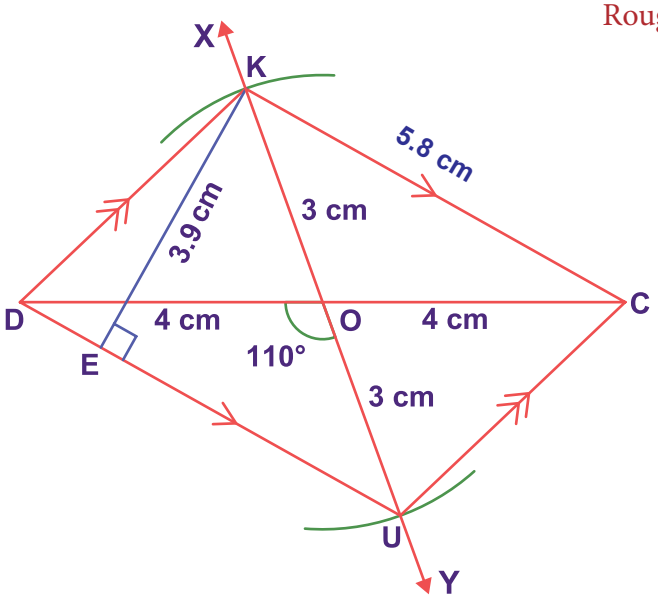


Fig. 5.51

Steps:

1. Draw a line segment DC=8*cm*.
2. Mark O the midpoint of $\overline{DC}$.
3. Draw a line $\overleftrightarrow{XY}$ through O which makes $\angle DOY = 110°$.
4. With O as centre and 3*cm* as radius draw two arcs on $\overleftrightarrow{XY}$ on either sides of $\overline{DC}$. Let the arcs cut $\overrightarrow{OX}$ at K and $\overrightarrow{OY}$ at U
5. Join $\overline{DU}, \overline{UC}, \overline{CK}$ and $\overline{KD}$.
6. DUCK is the required parallelogram.

Calculation of Area:

Area of the parallelogram DUCK = bh sq.units

$= 5.8 \times 3.9 = 22.62\, sq.cm$

5.13.4 Constructing a parallelogram when its one side, one diagonal and one angle are given

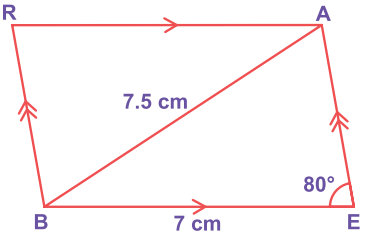
Example 5.33

Construct a parallelogram BEAR with BE=7*cm*, BA=7.5*cm* and $\angle BEA = 80°$. Also find its area.

Solution:

Given:

BE=7*cm*, BA=7.5*cm* and $\angle BEA = 80°$



Rough diagram

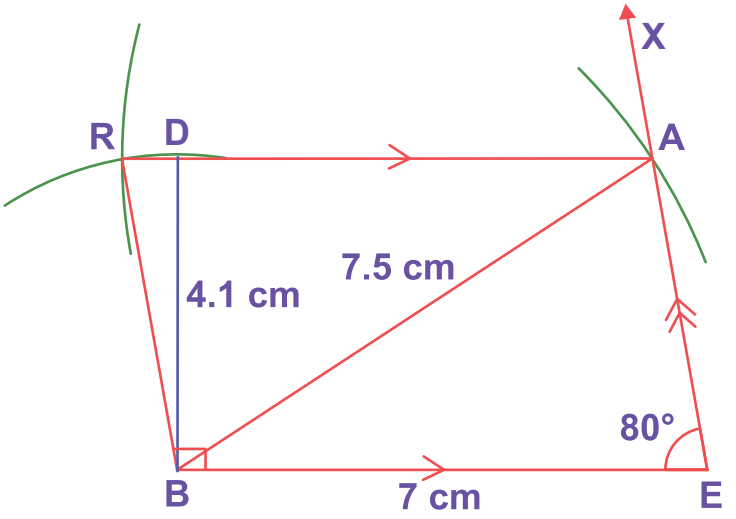


Fig. 5.52

Steps:

1. Draw a line segment BE=7*cm*.
2. Make an angle $\angle BEX = 80°$ at E on $\overline{BE}$.
3. With B as centre, draw an arc of radius 7.5*cm* cutting EX at A and Join BA.
4. With B as centre, draw an arc of radius equal to the length of $\overline{AE}$.
5. With A as centre, draw an arc of radius 7*cm*. Let both arcs cut at R.
6. Join BR and AR.
7. BEAR is the required parallelogram.

Calculation of Area:

Area of the parallelogram BEAR = bh sq.units

$= 7 \times 4.1 = 28.7\ sq.cm$

5.14 Construction of a Rhombus

Let us now construct a rhombus with the given measurements

(i) One side and one diagonal

(ii) One side and one angle

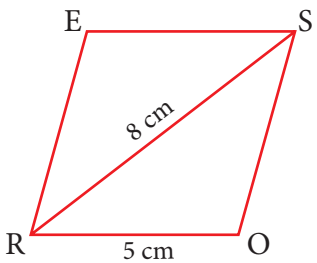
(iii) Two diagonals

(iv) One diagonal and one angle

5.14.1 Construction of a rhombus when one side and one diagonal are given

Example 5.34

Construct a rhombus ROSE with RO = 5 cm and RS = 8 cm. Also find its area.



Rough diagram

Solution:

Given: RO = 5 cm and RS = 8 cm

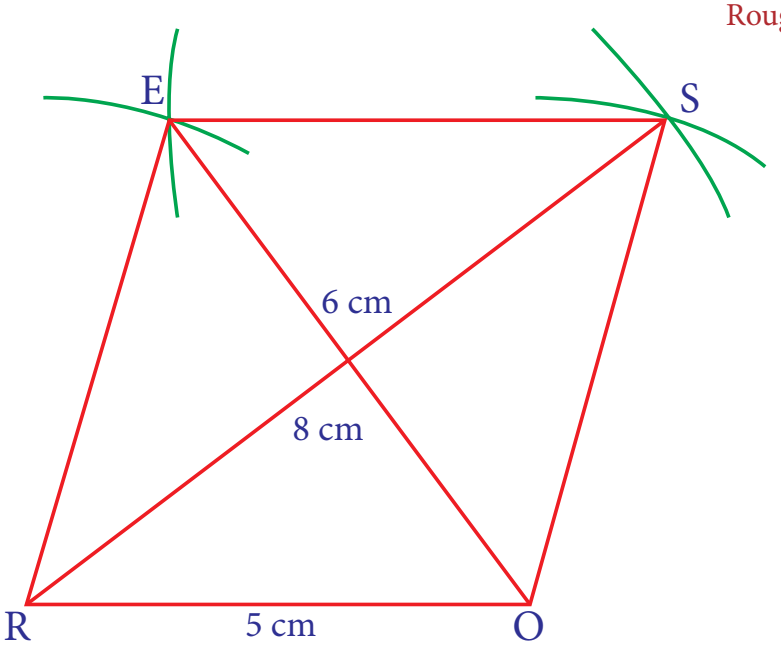


Fig. 5.53

Steps:

1. Draw a line segment RO = 5 cm.
2. With R and O as centres, draw arcs of radii 8 cm and 5 cm respectively and let them cut at S.
3. Join RS and OS.
4. With R and S as centres, draw arcs of radius 5 cm each and let them cut at E.
5. Join RE and SE.
6. ROSE is the required rhombus.

Calculation of Area:

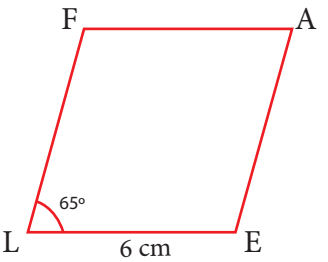
Area of rhombus ROSE $= \frac{1}{2} \times d_1 \times d_2 \; sq.units$

$= \frac{1}{2} \times 8 \times 6 = 24 sq.cm$

5.14.2 Construction of a rhombus when one side and one angle are given

Example 5.35

Construct a rhombus LEAF with LE = 6 cm and $\angle L = 65°$. Also find its area.



Rough diagram

Solution:

Given: LE = 6 cm and $\angle L = 65°$

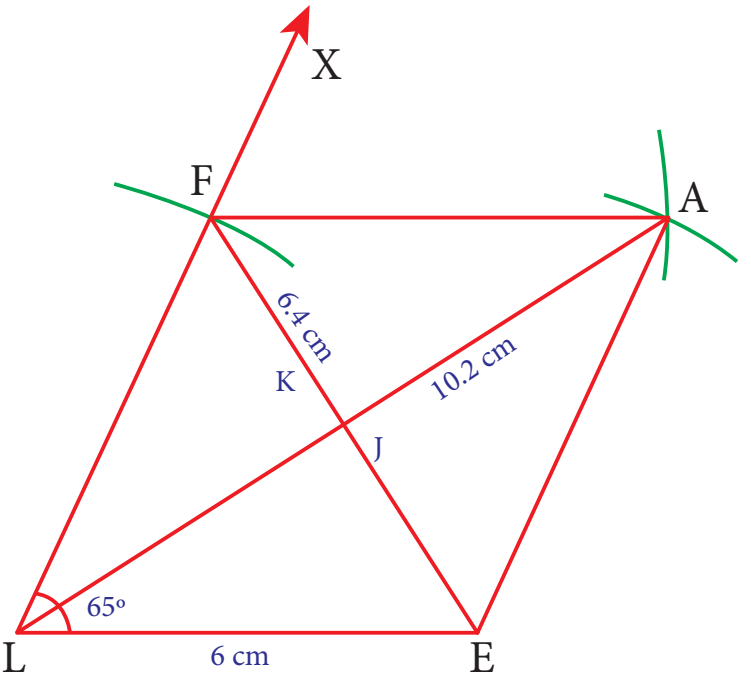


Fig. 5.54

Steps:

1. Draw a line segment LE = 6 cm.
2. At L on LE, make $\angle ELX = 65°$.
3. With L as centre draw an arc of radius 6 cm. Let it cut LX at F.
4. With E and F as centres, draw arcs of radius 6 cm each and let them cut at A.
5. Join EA and AF.
6. LEAF is the required rhombus.

Calculation of Area:

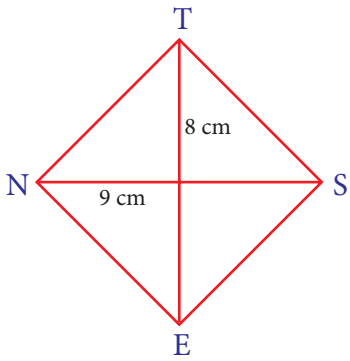
Area of rhombus LEAF $= \frac{1}{2} \times d_1 \times d_2 \; sq.units$

$= \frac{1}{2} \times 6.4 \times 10.2 = 32.64 \; sq.cm$

5.14.3 Construction of a rhombus when two diagonals are given

Example 5.36

Construct a rhombus NEST with NS = 9 cm and ET = 8 cm. Also find its area.



Rough diagram

Solution:

Given: NS = 9 cm and ET = 8 cm

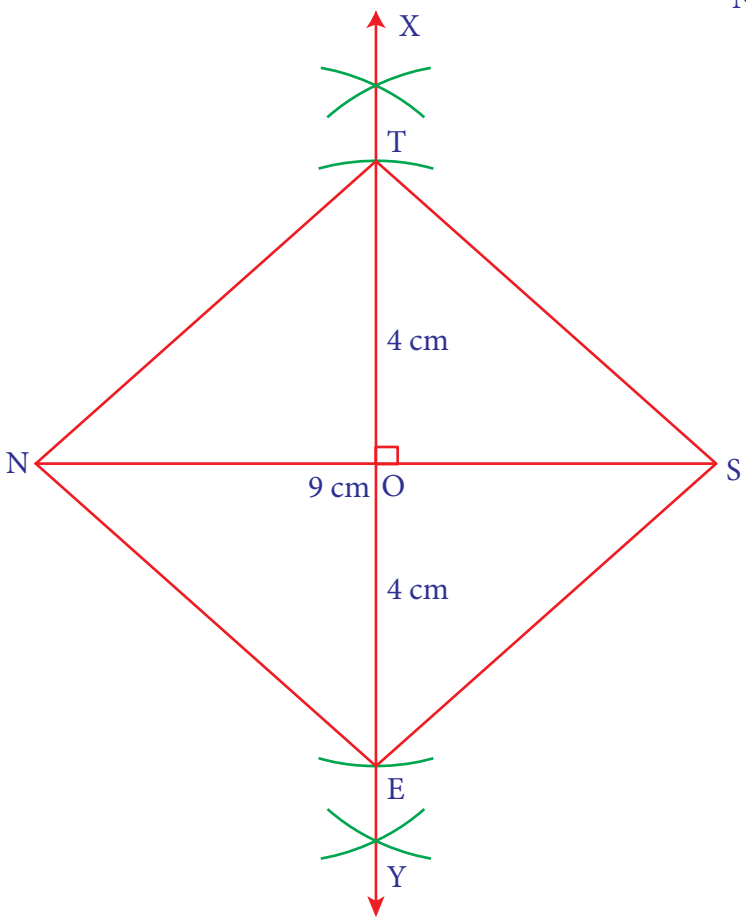


Fig. 5.55

Steps:

1. Draw a line segment NS = 9 cm.
2. Draw the perpendicular bisector XY to NS. Let it cut NS at O.
3. With O as centre, draw arcs of radius 4 cm on either side of O which cut OX at T and OY at E.
4. Join NE, ES, ST and TN.
5. NEST is the required rhombus.

Calculation of Area:

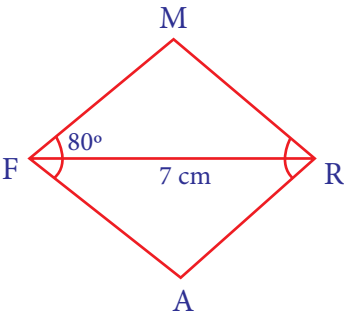
Area of rhombus NEST $= \frac{1}{2} \times d_1 \times d_2 \; sq.units$

$= \frac{1}{2} \times 9 \times 8 = 36 \; sq.cm$

5.14.4 Construction of a rhombus when one diagonal and one angle are given

Example 5.37

Construct a rhombus FARM with FR = 7 cm and $\angle F = 80°$. Also find its area.



Rough diagram

Solution:

Given: FR = 7 cm and $\angle F = 80°$

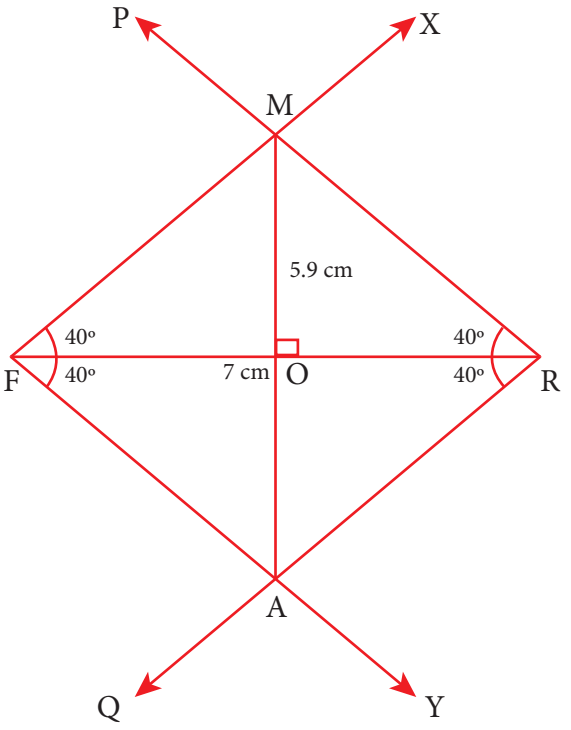


Fig. 5.56

Steps:

1. Draw a line segment FR = 7 cm.
2. At F, make $\angle RFX = \angle RFY = 40°$ on either side of FR.
3. At R, make $\angle FRP = \angle FRQ = 40°$ on either side of FR.
4. Let FX and RP cut at M and FY and RQ cut at A.
5. FARM is the required rhombus.

Calculation of Area:

Area of rhombus FARM $= \frac{1}{2} \times d_1 \times d_2 \; sq.units$

$= \frac{1}{2} \times 7 \times 5.9 = 20.65 \; sq.cm$

5.15 Construction of a Rectangle

Let us now construct a rectangle with the given measurements

(i) length and breadth

(ii) a side and a diagonal

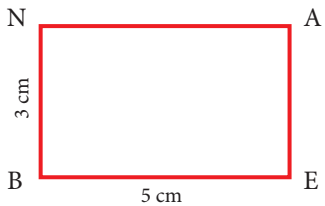
5.15.1 Construction of a rectangle when its length and breadth are given

Example 5.38

Construct a rectangle BEAN with BE = 5 cm and BN = 3 cm. Also find its area.

Solution:

Given: BE = 5 cm and BN = 3 cm



Rough diagram

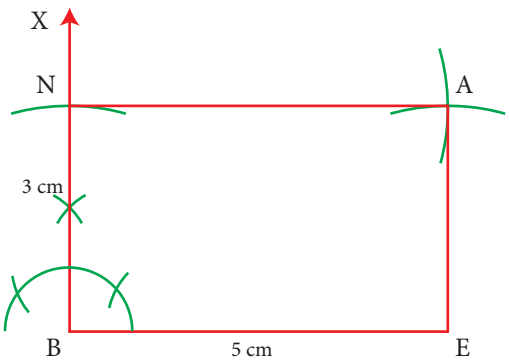


Fig. 5.57

Steps:

1. Draw a line segment BE = 5 cm.
2. At B, construct $BX \perp BE$.
3. With B as centre, draw an arc of radius 3 cm and let it cut BX at N.
4. With E and N as centres, draw arcs of radii 3 cm and 5 cm respectively and let them cut at A.
5. Join EA and NA.
6. BEAN is the required rectangle.

Calculation of Area:

Area of rectangle BEAN $= l \times b \, sq.units$

$= 5 \times 3 = 15 \, sq.cm$

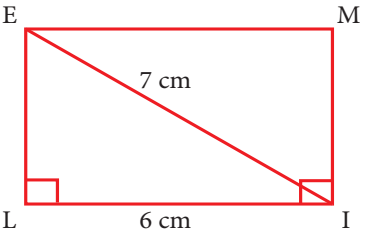
5.15.2 Construction of a rectangle when a side and a diagonal are given

Example 5.39

Construct a rectangle LIME with LI = 6 cm and IE = 7 cm. Also find its area.

Solution:

Given: LI = 6 cm and IE = 7 cm



Rough diagram

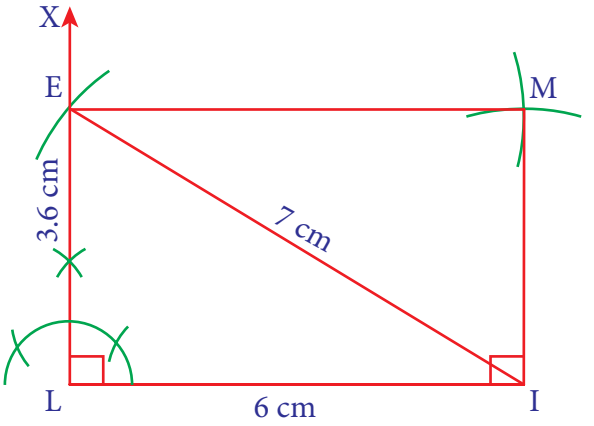


Fig. 5.58

Steps:

1. Draw a line segment LI = 6 cm.
2. At L, construct $LX \perp LI$.
3. With I as centre, draw an arc of radius 7 cm and let it cut LX at E.
4. With I as centre and LE as radius draw an arc. Also, with E as centre and LI as radius draw an another arc. Let them cut at M.
5. Join IM and EM.
6. LIME is the required rectangle.

Calculation of Area:

Area of rectangle LIME $= l \times b \; sq.units$

$= 6 \times 3.6 = 21.6 \; sq.cm$

5.16 Construction of a Square

Let us now construct a square when (i) its side is given and (ii) its diagonal is given

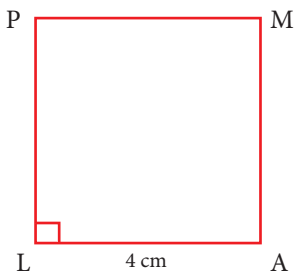
5.16.1 Construction of a square when its side is given

Example 5.40

Construct a square LAMP of side 4 cm. Also find its area.

Solution:

Given: side = 4 cm



Rough diagram

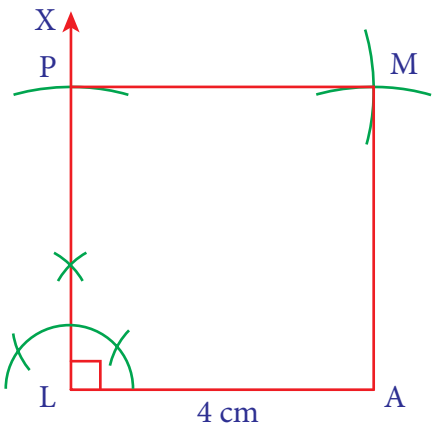


Fig. 5.59

Steps:

1. Draw a line segment LA = 4 cm.
2. At L, construct $LX \perp LA$.
3. With L as centre, draw an arc of radius 4 cm and let it cut LX at P.
4. With A and P as centres, draw arcs of radius 4 cm each and let them cut at M.
5. Join AM and PM. LAMP is the required square.

Calculation of Area:

Area of square LAMP $= a^2$ *sq.units*

$= 4 \times 4 = 16$ *sq.cm*

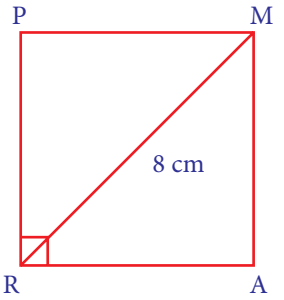
5.16.2 Construction of a square when its diagonal is given

Example 5.41

Construct a square RAMP of a diagonal 8 cm. Also find its area.

Solution:

Given: diagonal = 8 cm



Rough diagram

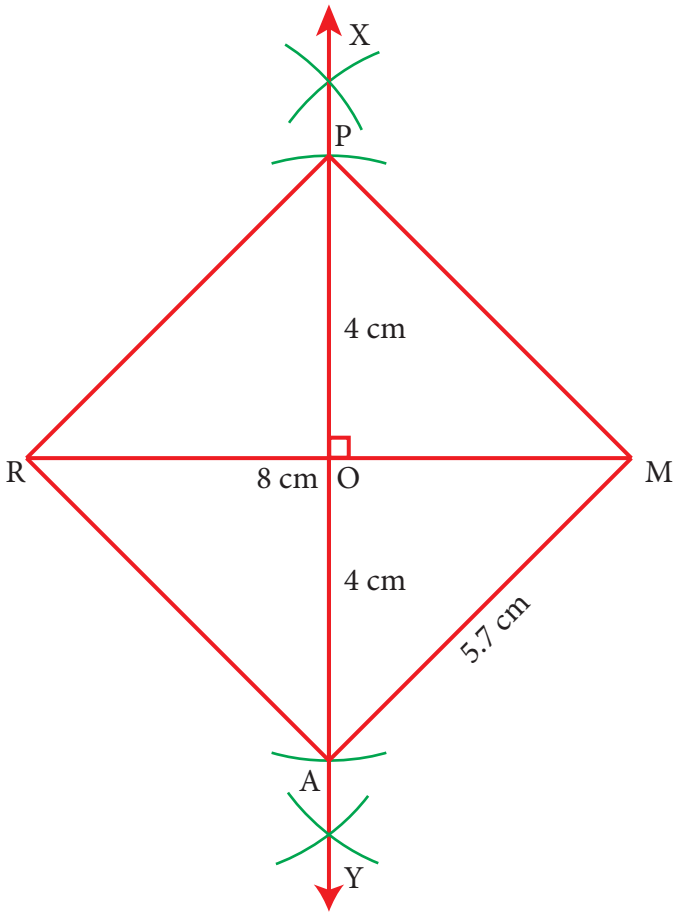


Fig. 5.60

Steps:

1. Draw a line segment RM = 8 cm.
2. Draw the perpendicular bisector XY to RM. Let it bisect RM at O.
3. With O as centre, draw arcs of radius 4 cm on either side of O which cut OX at P and OY at A.
4. Join RA, AM, MP and PR.
5. RAMP is the required square.

Calculation of Area:

Area of square RAMP $= a^2$ *sq.units*

$= 5.7 \times 5.7 = 32.49$ *sq.cm*

Exercise 5.5

I. Construct the following parallelograms with the given measurements and find their area.

1. ARTS, AR=6*cm*, RT=5*cm* and $\angle ART = 70°$.
2. CAMP, CA=6*cm*, AP=8*cm* and CP=5.5*cm*.
3. EARN, ER=10*cm*, AN=7*cm* and $\angle EOA = 110°$ where $\overline{ER}$ and $\overline{AN}$ intersect at O.
4. GAIN, GA=7.5*cm*, GI=9*cm* and $\angle GAI = 100°$.

II. Construct the following rhombuses with the given measurements and also find their area.

(i) FACE, FA= 6 cm and FC = 8 cm

(ii) CAKE, CA=5 cm and ∠A = 65°

(iii) LUCK, LC = 7.8 cm and UK = 6 cm

(iv) PARK, PR = 9 cm and ∠P =70°

III. Construct the following rectangles with the given measurements and also find their area.

(i) HAND, HA = 7 cm and AN = 4 cm

(ii) LAND, LA = 8 cm and AD = 10 cm

IV. Construct the following squares with the given measurements and also find their area.

(i) EAST, EA = 6.5 cm

(ii) WEST, WS = 7.5 cm

SUMMARY

- Congruent figures are exactly the same in shape and size.
- Similar figures have the same shape but different sizes.
- In a right angled triangle, the square on the hypotenuse is equal to the sum of the squares on the other two sides. This is known as Pythagoras theorem.
- The three medians of any triangle are concurrent. The point of concurrence of the three medians in a triangle is called its Centroid, denoted by the letter G.
- The three altitudes of any triangle are concurrent. The point of concurrence of the three altitudes of a triangle is called as its Orthocentre, denoted by the letter H.
- The three perpendicular bisectors of the sides of any triangle are concurrent. The point of concurrence of the three perpendicular bisectors of a triangle is called as its Circumcentre, denoted by the letter S.
- The three angle bisectors of any triangle are concurrent. The point of concurrence of the three angle bisectors of a triangle is called as its Incentre, denoted by the letter I.
- A trapezium is a quadrilateral in which a pair of opposite sides are parallel.
- A parallelogram is a quadrilateral in which the opposite sides are parallel.
- Rhombus is a parallelogram in which all its sides are congruent.
- Rectangle is a parallelogram whose all its angles are right angles.
- Square is a parallelogram in which all its sides and angles are equal.

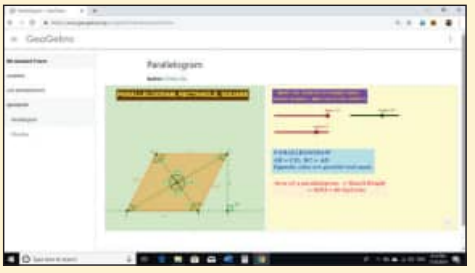
ICT CORNER

Step-1 Open the Browser type the URL Link given below (or) Scan the QR Code. GeoGebra work book named "GEOMETRY" will open. Click on the worksheet named "Parallelogram".

Step-2 In the given worksheet you can move the sliders Base, Height and the angle. Check for what value(s) the parallelogram becomes rectangle and square. Study the properties.

Expected Outcome

Step 1



Step 2

LCZFK

Browse in the link

Geometry:

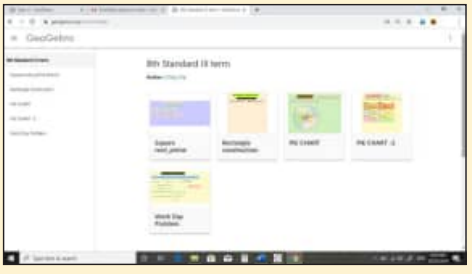
https://www.geogebra.org/m/fqxbd7rz#chapter/409576 or Scan the QR Code.

ICT CORNER

Step-1 Open the Browser type the URL Link given below (or) Scan the QR Code. GeoGebra work sheet named "8th Standard III term" will open. Select the work sheet named "Rectangle Construction".

Step-2 Move the sliders on left side to change the length and breadth of the rectangle. Drag the slider step by step on right side to see the steps for construction.

Expected Outcome



Step 1

Step 2

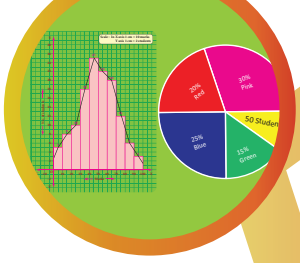
TSUWZ

Browse in the link

Geometry:

https://www.geogebra.org/m/xmm5kj9r or Scan the QR Code.

6 STATISTICS



Learning Objectives

7X2SM

- To recall the formation of frequency tables.
- To construct simple Pie-charts for the given data.
- To know how to draw Histogram and Frequency Polygon for grouped data.

6.1 Introduction

Before we learn on Pie charts, Histograms and Frequency Polygons, let us recall what we have studied in the previous classes like data (primary and secondary) and frequency tables for ungrouped data.

Kamaraj! Go and collect II-term Math marks of all the students from our class.

Geetha! You go and note down the heights of all the students from the cumulative record.

Students, here the marks collected by Kamaraj and heights noted by Geetha are called 'Data'. Data is a collection of facts such as numbers, words, measurements and observations.

For example: Staff's age in a company 27, 51, 19, 21, 46, 35, 52, 25, 57, 29.

6.1.1 Data:

Primary data:

These are the data that are collected in person for the first time for a specific purpose. Here, Kamaraj has collected the data of math marks from the students in person. It is called primary data.

Also, (i) Census in a village

(ii) Collection of colours which the students like in a class are some examples of primary data.

Secondary data:

These are the data that are sourced from some places that has originally collected it. This kind of data has already been collected by some other persons. The statistical operation may have been performed on them already. Here, Geetha also collected the data but she took it from a record which had already collected them. This is called secondary data.

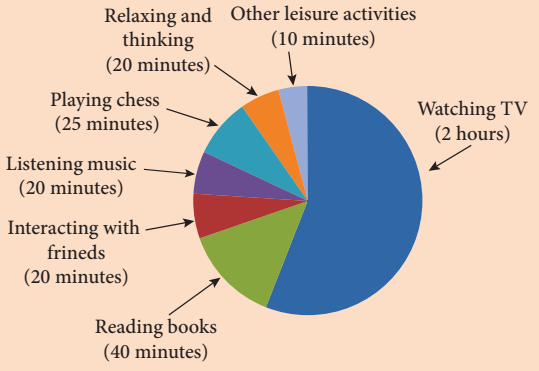
Also, (i) The details of 'PATTA' for a land can be had from the registration office.

(ii) Birth–Death details data can be got from concern office are some examples of secondary data.

From these primary and secondary data , Sometimes we can't get any specific or required information directly like, how many students have got more than 50 marks? how many students got marks between 30 and 40? how many of them are with height 125 cm? If we need answer for these questions, we have to tabulate the data.

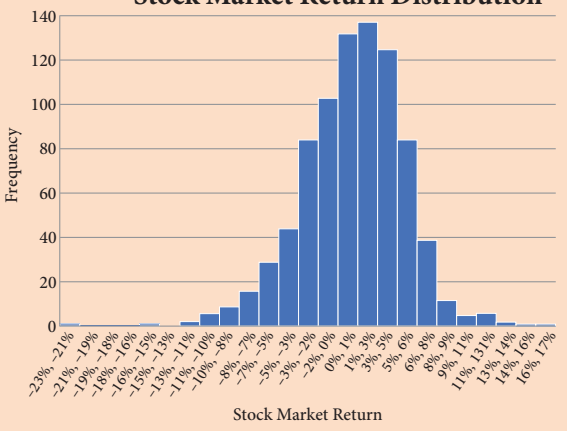
MATHEMATICS ALIVE – STATISTICS IN REAL LIFE

Leisure time on an average day


Relaxing and thinking (20 minutes)
Other leisure activities (10 minutes)
Playing chess (25 minutes)
Watching TV (2 hours)
Listening music (20 minutes)
Interacting with frineds (20 minutes)
Reading books (40 minutes)

Pie charts are used in our daily life activities

Stock Market Return Distribution



Histograms are used in stock market to represent the returns of stock buyer.

6.2 Frequency Distribution Table

Frequency distribution:

A frequency distribution is the arrangement of the given data in the form of the table showing frequency with which each variable occurs.

There are two types of distribution table namely

(i) frequency distribution table for ungrouped data and

(ii) frequency distribution table for grouped data.

Note

Range: The difference between the largest and the smallest values of the data given. If 5, 15, 10, 7, 20, 18 are the data then,
Range = 20–5=15

Try these

1. Arrange the given data in ascending and descending order:
   9,34,4,13,42,10,25,7,31,4,40
2. Find the range of the given data : 53, 42, 61, 9, 39, 63, 14, 20, 06, 26, 31, 4, 57

6.2.1 Construction of frequency distribution table for ungrouped data

Ungrouped data or Discrete Data:

An ungrouped data can assume only whole numbers and exact measurement. These are the data that cannot have a range of values. A usual way to represent this is by using Bar graphs.

Examples: 1. The number of teachers in a school.

2. The number of players in a game.

Example 6.1

Form an ungrouped frequency distribution table for the weight of 25 students in STD IV given below and answer the following questions.

25, 24, 20, 25, 16, 15, 18, 20, 25, 16, 20, 16, 15, 18, 25, 16, 24, 18, 25, 15, 27, 20, 20, 27, 25.

(i) Find the range of the weights.

(ii) How many of the students has the highest weight in the class?

(iii) What is the weight to which more number of students belong to?

(iv) How many of them belong to the least weight?

Solution:

To form a distribution table, arrange the given data in ascending order under Weight column then, put a vertical mark against each variable under Tally marks column and count the number of tally marks against the variable and enter it in Frequency column as given below. Hence, the distribution table is

Weight	Tally Marks	Frequency
15	|||	3
16	||||	4
18	|||	3
20	𝍸	5
24	||	2
25	𝍸 |	6
27	||	2
	Total	25

(i) The range of the given data is the difference between the largest and the smallest value. Here, the range = 27–15 =12**.**

(ii) From this table, two of the students have the highest weight of 27 kg.

(iii) 6 students belong to 25 kg weight.

(iv) 3 students belong to the least weight of 15 kg.

So, when we tabulate the given data, it is easy to get the information at a glance, Isn't it?

Activity

1. Collect the blood group of your classmates. Complete the table and analyse.

Blood group	Tally marks	No. of students
A+		
B+		
AB+		
O+		
A-		
B-		
AB-		
O-		

2. Observe the last alphabet in the name of your classmates, tabulate them and answer the following questions.

Alphabet	Tally marks	No. of students (f)

1. In which letter do the names end the most?
2. In which letter do the names end the least?
3. What are the letters in which the names do not end with?
4. Girl names mostly end with _____ letter(s).
5. Boy names mostly end with _____ letter(s).

6.2.2 Construction of frequency distribution table for grouped data

Grouped data or Continuous Data:

A grouped data is any value within a certain interval. The data can take values between certain range with the highest and the lowest value. Continuous data can be tabulated in what is called as frequency distribution. They can be graphically represented using Histograms.

Example: 1. The age of persons in a village.
2. The height and the weight of the students of your class.

Now, we will consider a situation, if we collect data of marks for 50 students, it becomes very difficult to put tally for each and every marks of all the 50 students. Because if we arrange the marks in a table, it will be very large in length and not understandable at once. In this case, we use class intervals. In this table, consider the groups of data in the form of class intervals to tally the frequency for the given data.

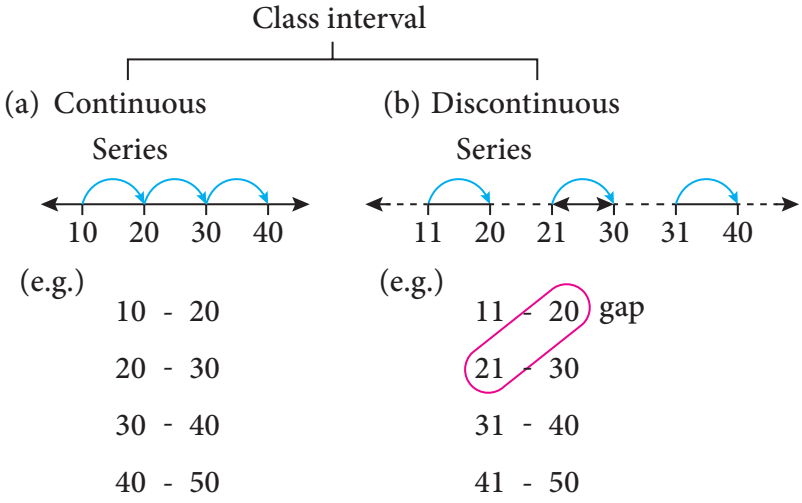
Class Interval:

The range of the variable is grouped into number of classes, and each group is known as class interval (C.I). The difference between the upper limit (U) and the lower limit (L) of the class is known as class size.

i.e. C.I = Upper limit – Lower limit

For example,

Marks for the C.I 10 to 20 can be written as 10-20, whose class size is 20–10=10



(a) While distributing the frequency, we follow the counting as given below. Suppose the classes are 10-20, 20-30, 30-40, 40-50 ..... This represent a continuous series. Here, 20 is included in the class 20-30 and 30 is included in 30-40, likewise for the other classes also.

(b) In case the given series has a gap between the limits of any two adjacent classes, this gap may be filled up by extending the two limits of each class by taking half of the value of the gap. Half of the gap is called the adjustment factor.

Conversion of a discontinuous series into continuous series:

In case the given series is a discontinuous, we can make it as continuous as follows,

Illustration 1:

11 - 20 gap

21 - 30

31 - 40

41 - 50

difference in the gap = 21 – 20
= 1

Lower boundary = lower limit –half of the gap

$$= 11 - \frac{1}{2}(1)$$

= 11 – 0.5 = 10.5

Upper boundary = upper limit + half of the gap

$$= 20 + \frac{1}{2}(1)$$

= 20 + 0.5

= 20.5 and so on for other classes too.

Therefore, the class interval can be changed into a continuous one as given in the following table,

Discontinuous series	Continuous series
11-20 (-0.5 | +0.5)	10.5-20.5
21-30	20.5-30.5
31-40	30.5-40.5
41-50	40.5-50.5

Note

Inclusive series:

In the class-intervals, if the upper limit and lower limit are included in that class interval then it is called inclusive series. For example, 11-20, 21-30 , 31-40, 41-50 etc is an inclusive series.

Here, the data 11 and 20 are included in the class (11-20) and so on. Clearly, it is a discontinuous series.

Exclusive series:

In the class intervals, if the upper limit of one class interval is the lower limit of the next class interval then it is called exclusive series. For example, 10-15, 15-20, 20-25, 25-30 etc., is an exclusive series.

Here, 15 is included in the class 15-20 and 20 is included in 20-30. Clearly, it is a continuous series.

6.2.2 (i) Construction of grouped frequency distribution table – Continuous series

Example 6.2

The EB bill(in ₹) of each of the 26 houses in a village are given below. Construct the frequency table.

215	200	120	350	800	600	350	400	180	210	170	305	204
220	425	540	315	640	700	790	340	586	660	785	290	300

Solution:

Maximum bill amount = ₹ 800

Minimum bill amount = ₹ 120

Range = largest value – smallest value

Range = 800 – 120 = ₹ 680

Suppose if we want to take class size as 100, then

The number of possible class intervals = $\frac{Range}{Class\ size} = \frac{680}{100} = 6.8 \simeq 7$

Class Intervals	Tally Marks	Frequency
100-200	III	3
200-300	𝍸 I	6
300-400	𝍸 I	6
400-500	II	2
500-600	II	2
600-700	III	3
700-800	IIII	4
	Total	26

6.2.2 (ii) Construction of grouped frequency distribution table - Discontinuous series.

Example 6.3

Convert the given discontinuous series into a continuous series.

Class	0-5	6-11	12-17	18-23	24-29
Frequency(f)	7	10	9	5	12

Solution:

As told above, first we should fill the gap by extending the two limits of each class by half of the value of the gap. Here the gap is 1, so subtracting and adding half of the gap i.e 0.5 to the lower and the upper limit of each class makes it as a continuous series.

Class	-0.5-5.5	5.5-11.5	11.5-17.5	17.5-23.5	23.5-29.5
Frequency(f)	7	10	9	5	12

Try these

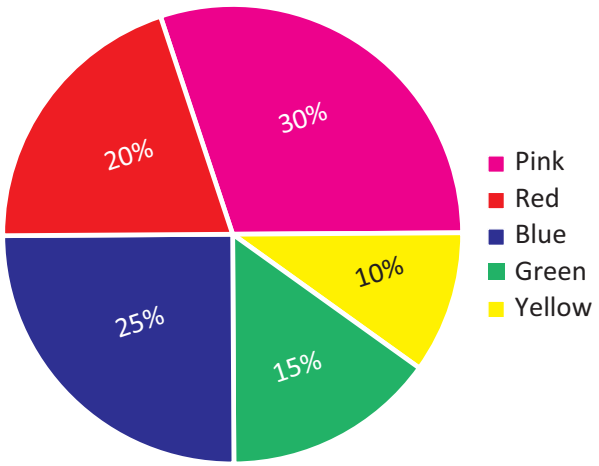
1. Prepare a frequency table for the data :
   3, 4, 2, 4, 5, 6, 1, 3, 2, 1, 5, 3, 6, 2, 1, 3, 2, 4
2. Prepare a grouped frequency table for the data :
   10, 9, 3, 29, 17, 34, 23, 20, 39, 42, 5, 12, 19, 47, 18, 19, 27, 7, 13, 40, 38, 24, 34, 15, 40

6.3 Graphical Representation of the Frequency Distribution for Ungrouped Data

A graphical representation is the geometrical image of a set of data. It is a mathematical picture. It enables us to think about a statistical problem in visual terms. A picture is said to be more effective than words for describing a particular thing. The graphical representation of data is more effective for understanding. In the previous classes, we have studied some graphical representations of ungrouped data such as Line graph, Bar graph, and Pictograph. Now, we are going to represent the given ungrouped data in the circular form namely the pie diagram or the pie chart.

6.3.1 Pie chart (or) Pie diagram

A pie chart is a circular graph which shows the total value with its components. The circle is divided into sectors and the area of the sectors is proportional to the information given. The area of a circle represents the total value and the different sectors of the circle represent the different components. In the 'pie chart' the data are mostly expressed in percentage. Each component is expressed as percentage of the total value.



The Pie diagram is so called because the entire graph looks like an American food 'pie' and the components like slices cut from 'pie'.

American food 'pie'

6.3.2 Method of constructing a pie chart:

In a pie chart, we know that the various components are represented by the sectors of a circle and the whole circle represents the sum of the value of all the components. Therefore, the total angle of 360° at the centre of the circle is divided into different sectors according to the value of the components.

$$\textbf{The central angle of a component} = \frac{value\ of\ the\ component}{total\ value} \times 360^\circ$$

Sometimes, the value of the components are expressed in percentage. In such cases,

$$\textbf{The central angle of a component} = \frac{percentage\ value\ of\ the\ component}{100} \times 360^\circ$$

Steps for construction of the pie chart:

1) Calculate the central angle for each component using the above formula and tabulate it.
2) Draw a circle of convenient radius and mark one horizontal radius in it.
3) Draw radius making central angle of first component with horizontal radius. This sector represents the first component. From this radius, draw next radius with central angle of the second component and so on, until the completion of all components.
4) For identification of each sector, shade with different colours.
5) Label each sector.

Here are given some examples, let us draw the pie chart for the given data.

Example 6.4

Draw a pie diagram to represent the following data, which shows the expenditure of paddy cultivation in 2 acres of land.

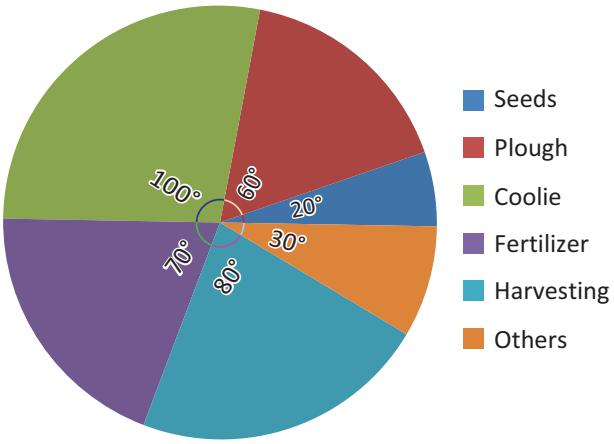
Particulars	Seeds	Ploughing	Wages	Fertilizer	Harvest	Others
Expenses (₹)	2000	6000	10000	7000	8000	3000

Also, 1. Find the percentage of the head in which more money had been spent?

2. What percentage of money was spent for seeds?

Solution:

Expenditure of paddy cultivation in 2 acres.



Particulars	Central angle $= \frac{\text{Value}}{\text{Total}} \times 360°$
Seeds	$\frac{2000}{36000} \times 360° = 20°$
Plough	$\frac{6000}{36000} \times 360° = 60°$
Coolie	$\frac{10000}{36000} \times 360° = 100°$
Fertilizer	$\frac{7000}{36000} \times 360° = 70°$
Harvesting	$\frac{8000}{36000} \times 360° = 80°$
Others	$\frac{3000}{36000} \times 360° = 30°$
Total	360°

1. More money had been spent for wages ₹10,000. Converting into percentage, We have

   $$\text{Wages} = \frac{10000}{36000} \times 100\% = 27.7\%$$

2. ₹2000 was spent for seeds. Converting into percentage, We have,

   $$\text{Seeds} = \frac{2000}{36000} \times 100\% = 5.55\%$$

Example 6.5

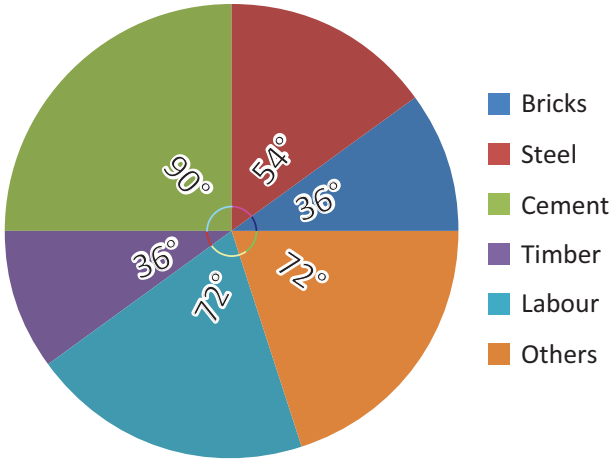
Draw a suitable pie chart for the following data relating to the cost of construction of a house.

Particulars	Bricks	Steel	Cement	Timber	Labour	Others
Expenses:	10%	15%	25%	10%	20%	20%

Also, find how much was spent on labour if ₹55000 was spent for cement.

Solution:

Cost of construction of a house.



Particulars	Central angle
Bricks	$\frac{10}{100} \times 360° = 36°$
Steel	$\frac{15}{100} \times 360° = 54°$
Cement	$\frac{25}{100} \times 360° = 90°$
Timber	$\frac{10}{100} \times 360° = 36°$
Labour	$\frac{20}{100} \times 360° = 72°$
Others	$\frac{20}{100} \times 360° = 72°$
Total	360°

If the expenses on cement is ₹ 55000 then, it represents 25 % and he spent 20 % on labour

Therefore, the expense on Labour $= \frac{20}{25} \times 55000$

$= ₹\ 44{,}000$

%	Expenses
25 ↓	55000 ↓
20	?
Direct proportion	

Note

Uses of pie chart:

1. Pie charts are widely used by the business and the media people.
2. With the help of Pie charts, one can show how the expenditure of the Government or Industry is distributed over different heads.
3. Research people use these type of charts to show their results.

Exercise 6.1

1. **Fill in the blanks:**
   (i) Data has already been collected by some other person is _____ data.
   (ii) The upper limit of the class interval (25-35) is _____.
   (iii) The range of the data 200, 15, 20, 103, 3, 196, is _____.
   (iv) If a class size is 10 and range is 80 then the number of classes are _____.
   (v) Pie chart is a _____ graph.

2. **Say True or False:**
   (i) Inclusive series is a continuous series.
   (ii) Comparison of parts of a whole may be done by a pie chart.
   (iii) Media and business people use pie charts.
   (iv) A pie diagram is a circle broken down into component sectors.

MY80D

3. Represent the following data in ungrouped frequency table which gives the number of children in 25 families.
   1, 3, 0, 2, 5, 2, 3, 4, 1, 0, 5, 4, 3, 1, 3, 2, 5, 2, 1, 1, 2, 6, 2, 1, 4

4. Form a continuous frequency distribution table for the marks obtained by 30 students in a X std public examination.
   328, 470, 405, 375, 298, 326, 276, 362, 410, 255, 391, 370, 455, 229, 300, 183, 283, 366, 400, 495, 215, 157, 374, 306, 280, 409, 321, 269, 398, 200.

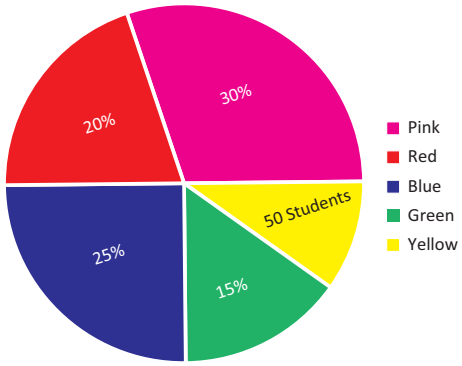
5. A paint company asked a group of students about their favourite colours and made a pie chart of their findings. Use the information to answer the following questions.
   (i) What percentage of the students like red colour?
   (ii) How many students liked green colour?

(iii) What fraction of the students liked blue?

(iv) How many students did not like red colour?

(v) How many students liked pink or blue?

(vi) How many students were asked about their favourite colours?



6. A survey gives the following information of food items preferred by people. Draw a Pie chart.

Items	Vegetables	Meat	Salad	Fruits	Sprouts	Bread
No.of people	160	90	80	50	30	40

7. Income from various sources for Government of India from a rupee is given below. Draw a pie chart.

Source	Corporation tax	Income tax	Customs	Excise duties	Service Tax	Others
Income (in paise)	19	16	9	14	10	32

8. Monthly expenditure of Kumaran's family is given below. Draw a suitable Pie chart.

Particulars	Food	Education	Rent	Transport	Miscellaneous
Expenses (in %)	50 %	20 %	15 %	5 %	10 %

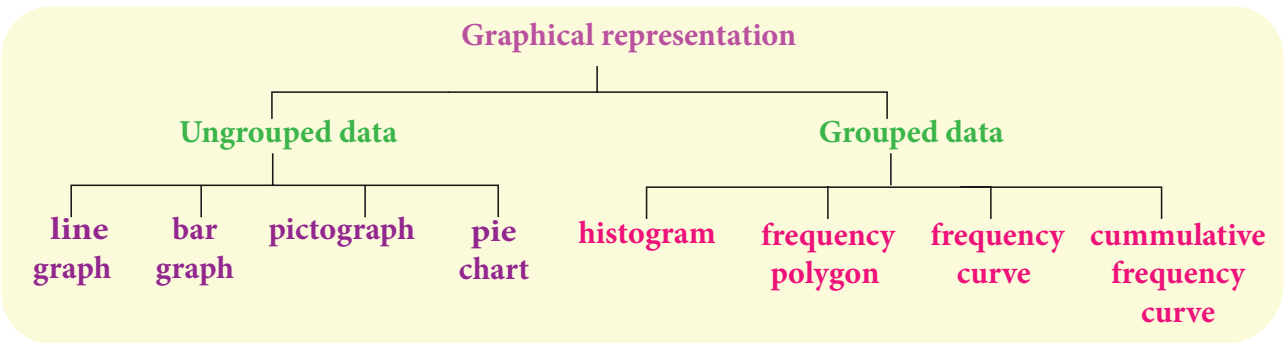
Also

1. Find the amount spent for education if Kumaran spends ₹6000 for Rent.
2. What is the total salary of Kumaran?
3. How much did he spend more for food than education?

6.4 Graphical Representation of the Frequency Distribution for Grouped Data

The Line graph, Bar graph, Pictograph and the Pie chart are the graphical representations of the frequency for ungrouped data. Histogram, Frequency polygon, Frequency curve, Cumulative frequency curves (Ogives) are some of the graphical representations of the frequency distribution for grouped data.

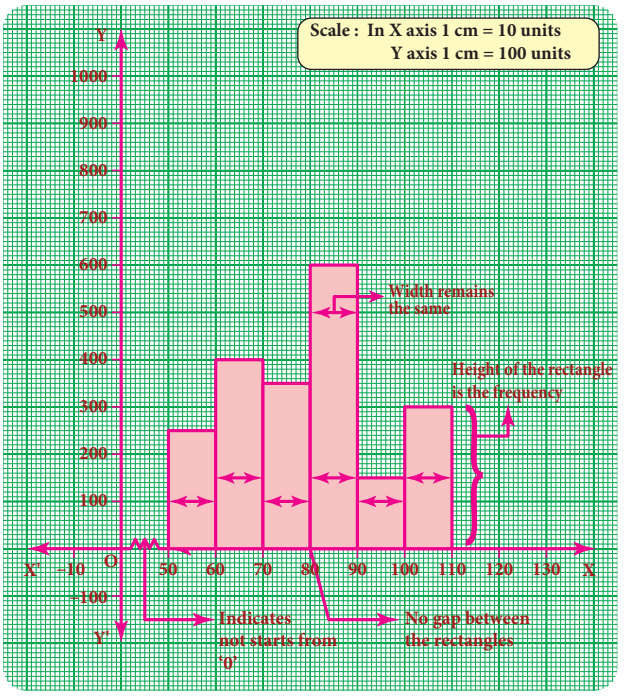
In this class, we are going to represent the grouped data frequency by Histogram and Frequency polygon only. You will learn the other type of representations in the higher classes.



6.4.1 Histogram

A histogram is a graph of a continuous frequency distribution. Histogram contains a set of rectangles, the base of which is the length of the class interval and the frequency in each class interval is its height. i.e the class intervals are represented on the horizontal axis (x- axis) and the frequencies are represented on the vertical axis (y-axis).

The area of each rectangle is proportional to the frequency in the respective class interval and the total area of the histogram is proportional to the total frequency. Because of the continuous frequency distribution, the rectangles are placed continuously side by side with no gap between adjacent rectangles.



Steps to construct a Histogram:

1. Represent the data in the continuous form (exclusive form) if it is in discontinuous form (inclusive form) by converting it using the adjustment factor.
2. Select the appropriate units along the x-axis and y-axis.
3. Plot the lower limits of all class interval on the x –axis.
4. Plot the frequencies of the distribution on the y – axis.
5. Construct the rectangles with class intervals as bases and corresponding frequencies as heights. Each class has lower and upper values. This gives us two equal vertical lines representing the frequencies. The upper ends of the lines are joined together and this process will give us rectangles.

Note

Differences between a Bar graph and a Histogram

	Bar graph	Histogram
1	Used for Ungrouped data	Used for Grouped data
2	Gap between the bars	No gap between rectangles
3	Height of each bar is important and not its width	Height and width of each rectangle are equally important

6.4.1 (i) Construction of a histogram for continuous frequency distribution:

Example 6.6

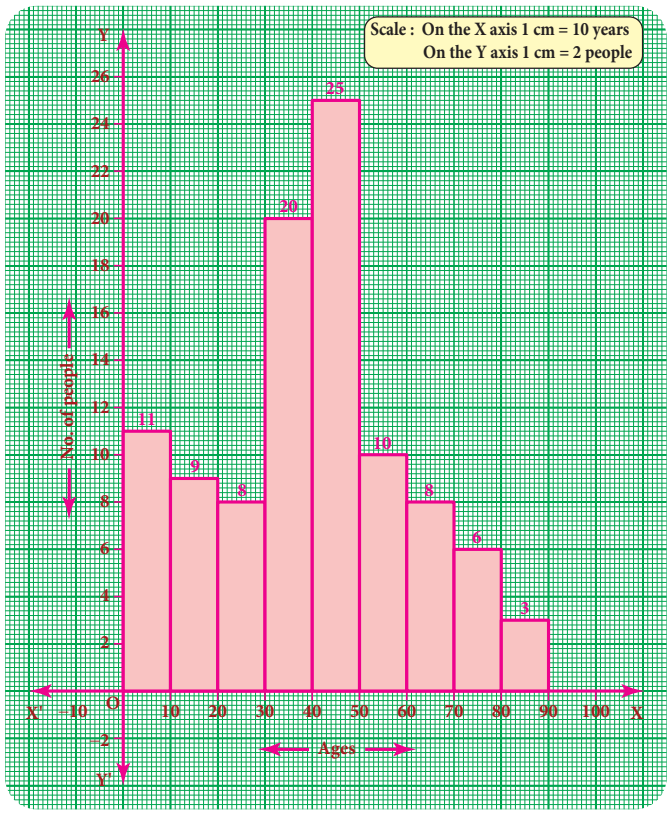
Draw a histogram for the following table which represents the age groups from 100 people in a village.

Ages	0-10	10-20	20-30	30-40	40-50	50-60	60-70	70-80	80-90
Number of people	11	9	8	20	25	10	8	6	3

Solution:

The given data is a continuous frequency distribution. The class intervals are drawn on x-axis and their respective frequencies on y-axis. Classes (ages) and its frequencies (number of people) are taken together to form a rectangle.

The histogram is constructed as given below.



Note

If class intervals do not start from '0' then, it is indicated by drawing a kink (Zig-Zag) mark (⩙) on the x-axis near the origin. If necessary, the kink mark (⩙) may be made on y-axis or on both the axes. i.e it indicates that we do not have data starting from the origin (O)

6.4.1 (ii) Construction of histogram for discontinuous frequency distribution:

Example 6.7

The following table gives the number of literate females in the age group 10 to 45 years in a town.

Age group	10-15	16-21	22-27	28-33	34-39	40-45
No. of females	350	920	850	480	230	200

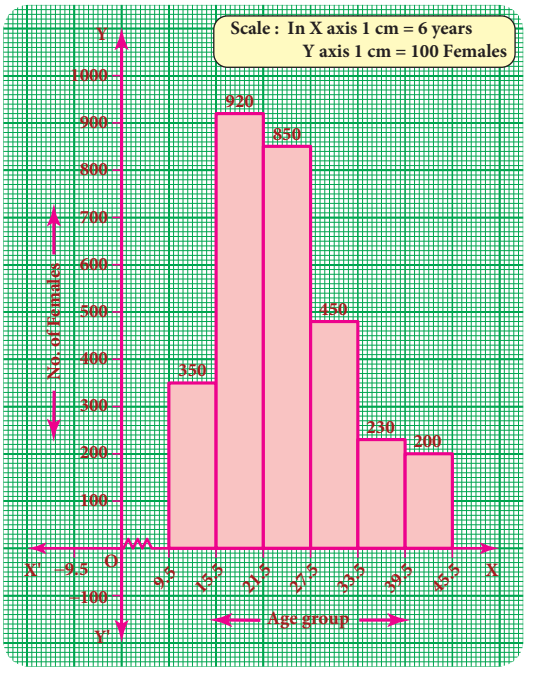
Draw a histogram to represent the above data

Solution:

The given distribution is discontinuous. If we represent the given data as it is by a graph we shall get a bar graph, as there will be gaps in between the classes. So, convert this into a continuous distribution using the adjustment factor 0.5.

The first class interval can be written as 9.5-15.5 and the remaining class intervals are changed in the same way. There are no changes in frequencies.

The new continuous frequency table is



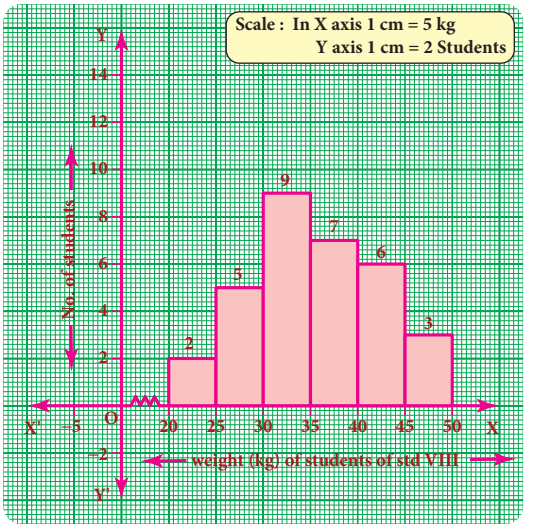
Age group	9.5-15.5	15.5-21.5	21.5-27.5	27.5-33.5	33.5-39.5	39.5-45.5
No of females	350	920	830	480	230	200

Example 6.8

Observe the given histogram and answer the following questions

Hint: Under weight: less than 30 *kg;* Normal weight: 30 to 45 *kg;* Obese: More than 45 *kg*

1. What information does the histogram represent?
2. Which group has maximum number of students?
3. How many of them are under weight?
4. How many students are obese?
5. How many students are in the weight group of 30-40 kg?



Solution:

1. The histogram represents the collection of weight from std VIII.
2. There are maximum 9 students in 30-35 kg weight.
3. There are $7(=2+5)$ students who are under weight.
4. There are 3 students who are obese.
5. There are $16(=9+7)$ students in the 30-40 kg weight group.

6.4.2 Frequency Polygon

A frequency polygon is a line graph for the graphical representation of the frequency distribution. If we mark the midpoints on the top of the rectangles in a histogram and join them by straight lines, the figure so formed is called a frequency polygon. It is called a polygon as it consists of a number of lines as the sides of a polygon.

A frequency polygon is useful in comparing two or more frequency distributions. A frequency polygon for a grouped frequency distribution can be constructed in two ways.

i) Using a histogram
ii) Without using a histogram

6.4.2 (i) To construct a frequency polygon using a histogram:

1. Draw a histogram from the given data.
2. Join the consecutive midpoints of the upper sides of the adjacent rectangles of the histogram by the line segments.
3. It is assumed that the class interval preceding the first rectangle and the class interval succeeding the last rectangle exists in the histogram and the frequency of each extreme class interval is zero. These class intervals are known as imagined class intervals.
4. To get frequency polygon, join the midpoints of these imagined classes with the corresponding midpoints of the upper sides of the first and last rectangles of the histogram.

Example 6.9

The following is the distribution of time spent in the library by students in a school.

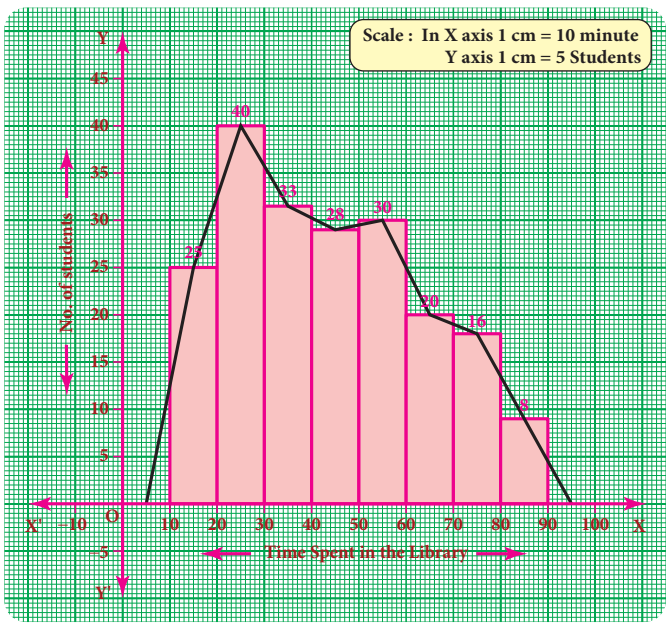
Time spent (in minutes)	10-20	20-30	30-40	40-50	50-60	60-70	70-80	80-90
Number of Students	25	40	33	28	30	20	16	8

Draw a frequency polygon using histogram.

Solution:

Represent the time spent in the library along x- axis and number of students along the y–axis.

Draw a histogram for the given data. Now, mark the midpoints of the upper sides of the consecutive rectangles. Also mark the midpoints of two imagined class intervals 0-10 and 90-100 whose frequency is 0 on x- axis. Now, join all the midpoints with the help of ruler. We get a frequency polygon imposed on the histogram.



Note

Sometimes imagined class intervals do not exist. For example, in case of marks obtained by the students in a test, we cannot go below zero and beyond maximum marks on the two sides. In such cases, the extreme line segments meet at the mid points of the vertical left and right sides of first and last rectangles respectively.

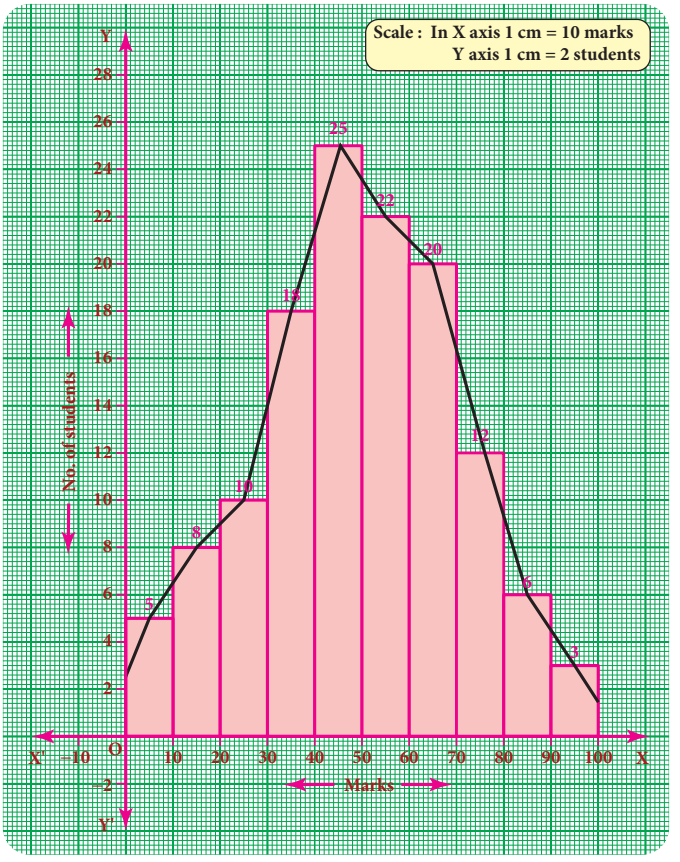
Example 6.10

Draw a frequency polygon for the following data using histogram.

Marks	0-10	10-20	20-30	30-40	40-50	50-60	60-70	70-80	80-90	90-100
Number of students	5	8	10	18	25	22	20	13	6	3

Solution:

Mark the class intervals along the x-axis and the number of students along the y-axis . Draw a histogram for the given data and mark the midpoints of the rectangles and join them by lines. We get frequency polygon. Note that the first and last edges of the frequency polygon meet at the mid points of the left and right vertical edges of first and last rectangles. Because imagined class intervals do not exist in the marks (refer the above note).



6.4.2 (ii) To draw a frequency polygon without using a histogram:

(1) Find the midpoints of the class intervals and tabulate it.

(2) Mark the midpoints of the class intervals on x-axis and frequencies on y-axis.

(3) Plot the points corresponding to the frequencies at each midpoints.

(4) Join the points using a ruler, to get the frequency polygon.

Example 6.11

Draw a frequency polygon for the following data without using histogram.

Class interval (Marks)	10-20	20-30	30-40	40-50	50-60	60-70	70-80	80-90
Frequency	4	6	8	12	10	14	5	7

Solution:

Find the midpoint of the class intervals and tabulate it.

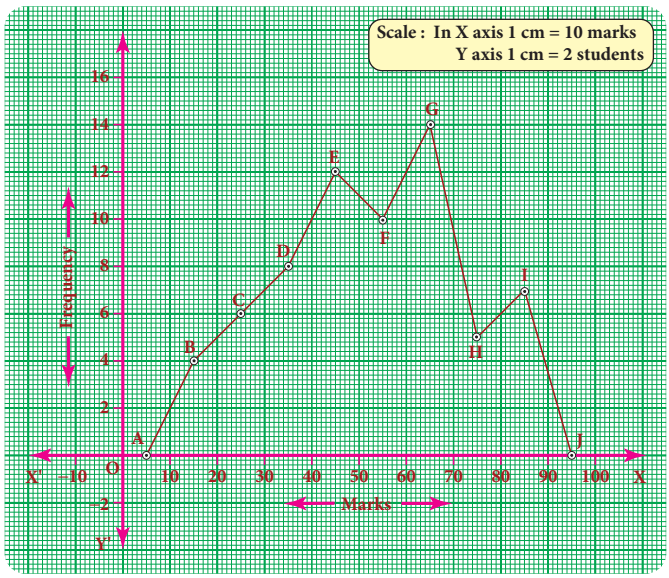
Class interval (C.I)	Mid point (x)	Frequency (f)
10 – 20	15	4
20 – 30	25	6
30 – 40	35	8
40 – 50	45	12
50 – 60	55	10
60 – 70	65	14
70 – 80	75	5
80 – 90	85	7

The points are A(5,0) B(15,4) C(25,6) D(35,8) E(45,12) F(55,10) G(65,14) H(75,5) I(85,7) J(95,0).

In the graph sheet, mark the midpoints along the x- axis and the frequency along the y- axis.

We take the imagined class as 0 – 10 at the beginning and 90 – 100 at the end , each with frequency 'zero'.

From the table, plot the points. We draw the line segments AB, BC, CD, DE, EF, FG, GH, HI, IJ to obtain the required frequency polygon ABCDEFGHIJ.

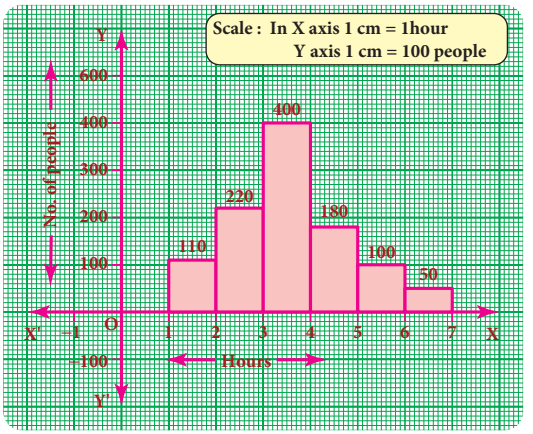


Exercise 6.2

1. Which of the following data can be represented in a histogram?
   (i) The number of mountain climbers in the age group 20 to 60 in TamilNadu.
   (ii) Production of cycles in different years.
   (iii) The number of students in each class of a school.
   (iv) The number votes polled from 7 am to 6 pm in an election.
   (v) The wickets fallen from 1 over to 50th over in a one day cricket match.

2. **Fill in the blanks:**
   (i) The total area of the histogram is _____ to the total frequency of the given data.
   (ii) A graph that displays data that changes continuously over the periods of time is _____.
   (iii) Histogram is a graphical representation of _____ data.

3. In a village, there are 570 people who have cell phones. An NGO survey their cell phone usage. Based on this survey a histogram is drawn. Answer the following questions.
   (i) How many people use the cell phone for less than 3 hours?
   (ii) How many of them use the cell phone for more than 5 hours?
   (iii) Are people using cell phone for less than 1 hour?



4. Draw a histogram for the following data.

Class Interval	0-10	10-20	20-30	30-40	40-50	50-60
No. of students	5	15	23	20	10	7

5. Construct a histogram from the following distribution of total marks of 40 students in a class.

Marks	90-110	110-130	130-150	150-170	170-190	190-210
No. of Students	9	5	10	7	4	6

6. The distribution of heights (in cm) of 100 people is given below. Construct a histogram and the frequency polygon imposed on it.

Height (in cm)	125-135	136-146	147-157	158-168	169-179	180-190	191-201
Frequency	12	22	18	24	15	7	2

7. In a study of dental problem, the following data were obtained.

Ages	0-10	10-20	20-30	30-40	40-50	50-60	60-70	70-80
No. of patients	5	13	25	14	30	35	43	50

Represent the above data by a frequency polygon.

8. The marks obtained by 50 students in Mathematics are given below (i) Make a frequency distribution table taking a class size of 10 marks (ii) Draw a histogram and a frequency polygon.

52	33	56	52	44	59	47	61	49	61
47	52	67	39	89	57	64	58	63	65
32	64	50	54	42	48	22	37	59	63
36	35	48	48	55	62	74	43	41	51
08	71	30	18	43	28	20	40	58	49

Objective Type Questions

9. Data is a collection of _____

(A) numbers (B) words (C) measurements (D) all the three

10. The number of times an observation occurs in the given data is called _____

(A) tally marks (B) data (C) frequency (D) none of these

11. The difference between the largest value and the smallest value of the given data is _____

(A) range (B) frequency (C) variable (D) none of these

12. The data that can take values between a certain range is called_____

(A) ungrouped (B) grouped (C) frequency (D) none of these

13. Inclusive series is a _____series.

(A) continuous (B) discontinuous (C) both (D) none of these

14. In a class interval the upper limit of one class is the lower limit of the other class. This is _____series.

(A) Inclusive (B) exclusive (C) ungrouped (D) none of these

15. The graphical representation of ungrouped data is _____

(A) histogram (B) frequency polygon (C)pie chart (D) all the three

16. Histogram is a graph of a _____ frequency distribution.

(A) continuous (B) discontinuous (C) discrete (D) none of these

17. A _____ is a line graph for the graphical representation of the continuous frequency distribution.

(A) frequency polygon (B) histogram (C) pie chart (D) bar graph

18. The graphical representation of grouped data is _____

(A) bar graph (B) pictograph (C) pie chart (D) histogram

Exercise 6.3

Miscellaneous Practice Problems

1. Draw a pie chart for the given table.

Continent	Asia	Africa	North America	South America	Europe	Australia	Antarctica
Area	30 %	20 %	16 %	12 %	7 %	6 %	9 %

2. The data on modes of transport used by the students to come to school are given below. Draw a pie chart for the data.

Mode of transport	Bus	Cycle	Walking	Scooter	Car
Percentage of students	40 %	30 %	15 %	10 %	5 %

3. Draw a histogram for the given frequency distribution.

Age	41-45	46-50	51-55	56-60	61-65	66-70	71-75
Frequency	4	9	17	25	15	8	2

4. Draw a histogram and the frequency polygon in the same diagram to represent the following data.

Weight (in kg)	50-55	56-61	62-67	68-73	74-79	80-85	86-91
No.of persons	15	8	12	17	9	10	6

Challenging problems

5. Form a continuous frequency distribution table and draw histogram from the following data.

Age (in years)	No. of persons
Under 5	1
Under 10	12
Under 15	19
Under 20	26
Under 25	27
Under 30	35
Under 35	38
Under 40	45
Under 45	48
Under 50	53

6. A rupee spent in a cloth manufacturing company is distributed as follows. Represent this in a pie chart.

Particulars	Paise
Farmer	20
Spinner	35
Dyer	15
Weaver	15
Printer	05
Salary	10

7. Draw a histogram for the following data.

Mid Value (x)	15	25	35	45	55	65	75
Frequency (f)	12	24	30	18	26	10	8

SUMMARY

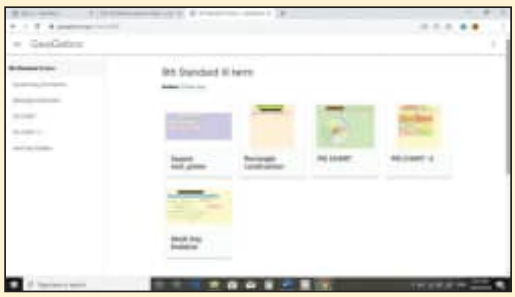
- Data is a collection of facts such as numbers, words, measurements and observations.
- A frequency distribution is the arrangement of the given data in the form of the table showing frequency with which each variable occurs.
- In the class-intervals, if the upper limit and lower limit are included in that class interval then it is called inclusive series.
- In the class intervals, if the upper limit of one class interval is the lower limit of the next class interval then it is called exclusive series.
- A pie chart is a circular graph which shows the total value with its components.
- A histogram is a graph of a continuous frequency distribution.
- A frequency polygon is a line graph for the graphical representation of the frequency distribution.

ICT CORNER

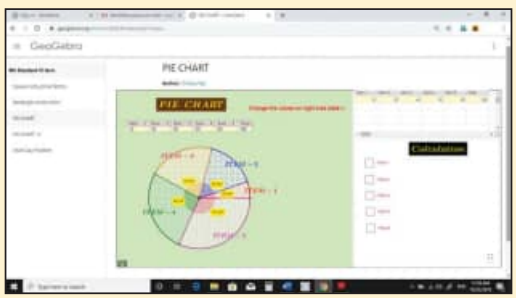
Step-1 Open the Browser type the URL Link given below (or) Scan the QR Code. GeoGebra work sheet named "8th Standard III term" will open. Select the work sheet named "PIE CHART"

Step-2 Type your values in the check box on right side. You can observe the change in the pie chart. Click on the check boxes to see respective calculations.

Expected Outcome



Step 1



Step 2

Browse in the link
Statistics:
https://www.geogebra.org/m/xmm5kj9r or Scan the QR Code.

7 INFORMATION PROCESSING

Learning Objectives

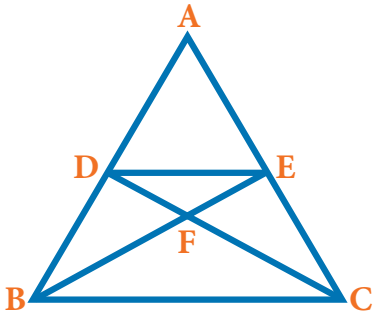
- To determine the number of possible orderings of an arbitrary number of objects following certain procedures.
- To learn a SET game for developing logical thinking .
- To investigate the role of map colouring in representing and modelling of mathematical ideas.
- To observe the Fibonacci pattern in physical and biological phenomena.
- To choose the best method in finding the HCF of numbers.
- To understand how information can be processed in, encryption and decryption.
- To consider alternatives in shopping before making a purchase, calculate the unit price for each items and make purchase in limited budget.
- To understand how to pack things efficiently in a given space and find the optimal solution.

Recap

Before to learn, we recall the concepts like listing, counting, pattern of Fibonacci series and calculate the unit price of the products by answering the following questions

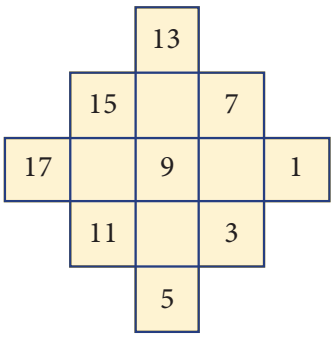
1. Find the number of all possible triangles that can be formed from the triangle given below.

ZF7E1

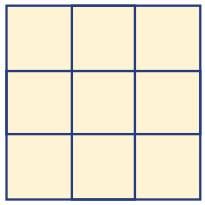


Ans:_____

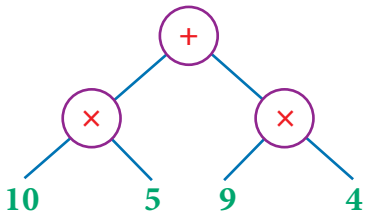
2. Use the numbers given in the figure to form a 3 x 3 magic square.



Ans:

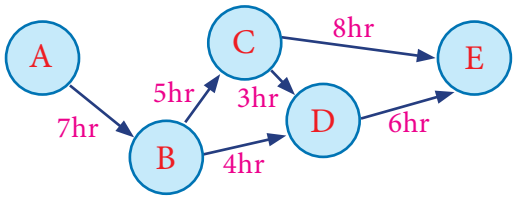


3. Convert the tree diagram into a numeric expression.



Ans:_____

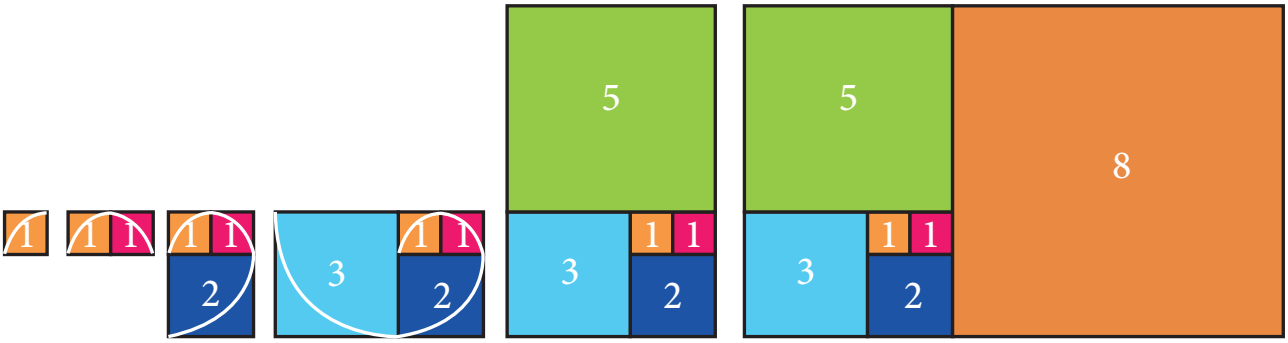
4. (i) Find the total time taken by the bus to reach from A to E via B , C and D.
(ii) Find which is the shortest route from A to E.



(i) .....hrs.

(ii) A → ... → ... → E

5. Connect the Fibonacci squares through diagonals by curve from corner to corner across each square to draw a Golden Spiral.

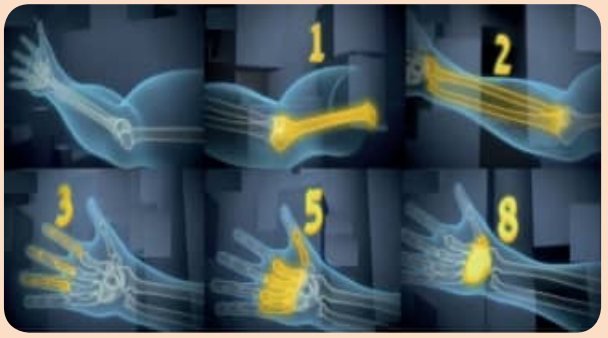


FIBONACCI SQUARES

6. When you plan to buy a shirt, one shop offers a discount of ₹200 on MRP ₹1000 and another shop offers 15% discount on the same MRP. Where would you buy?
7. Amazing park is offers a package deal of 5 entrance passes for ₹130. If one entrance pass normally costs ₹30, how much will you save by taking advantage of this special deal?

7.1 Introduction

Success in Mathematics depends on the development of the number sense, logical thinking and cognitive skills. In this class, we find practical solutions for day to day life situations like selection procedures from various forms, colour the given map with minimum number of colours. We are also going to see Fibonacci number pattern in physical and biological phenomena and also discuss some cipher for the higher order thinking which is to do well in the competitive exams. We will also discuss how will you be a wise consumer while shopping and packing. Meanwhile we will play a game to support your mind blooming activities. All the above discussions are very helpful to you, to explore other chapters and improve your interest in Mathematics.



Biological example for the Fibonacci numbers

Mason packing bricks for constructing a wall between two pillars

7.2 Principles of Counting

There are some basic counting techniques which will be useful in determining the number through different ways of arranging or selecting objects. The basic counting principles are given below.

7.2.1 Addition principle

If there are two selections such that they can be done **independently** in **m** ways and **n** ways respectively, then either of the two selections can be done in (**m** + **n**) ways.

Let us learn about this addition principle of counting as given below :

Situation:

In class VIII, there are 16 boys and 9 girls. The teacher wants to select either a boy or a girl as the class leader. Let us see, in how many ways can the teacher select the class leader.

Fig. 7.1

The teacher can select the class leader in any one of the following ways.

(i) In the first choice, the teacher can select **a boy among 16 boys in 16 ways** (who ever may be of the 16 boys).

(ii) In the second choice, the teacher can select **a girl among 9 girls in 9 ways** (who ever may be of the 9 girls).

Hence, the teacher can select the class leader who is **a boy** or **a girl** in **25 different ways (16 boys + 9 girls).**

Thus, we come to know, if a selection **A** can occur in ***m*** ways and another selection **B** can occur in ***n*** ways, and suppose that **both cannot occur together**, then **A** or **B** can occur in (***m*** + ***n***) ways. Let us see an example.

Example 7.1

If you are going to a hotel to have food and the hotel offers different food items as shown in Fig 7.2. Find how many ways are possible to have either tiffin or meals?

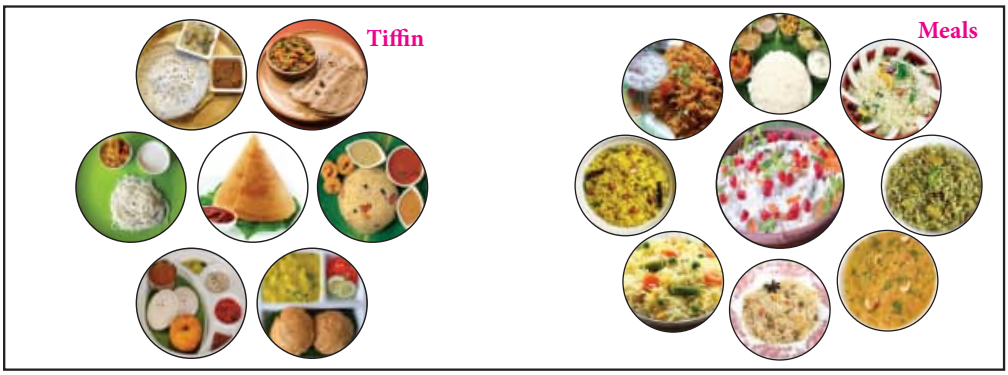


Fig. 7.2

Solution:

From the above Fig. 7.2, we come to know

(i) For tiffin, we can choose one among **7 items** in 7 ways

(ii) For meals, we can choose one among **9 items** in 9 ways.

Therefore, there are **16** (**7 tiffin items + 9 meals items**) **different ways** by which we can choose any one food.

7.2.2 Multiplication principle

If a selection can be performed in ***m*** ways, following which another selection can be performed in ***n*** ways, and both the selections are **dependent** on each other then, the two selections together can be performed in exactly ($m \times n$) different ways.

Now, we shall learn about multiplication principle of counting from the following situation.

Situation:

There are 3 places in a city namely A,B and C. There are 3 routes **a1, a2 and a3** from A to B. There are 2 different routes, **b1 and b2** from B to C as shown in the Fig 7.3.

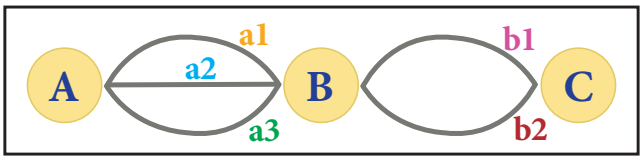


Fig. 7.3

Suppose a person wants to travel from A to C via B. Lets us see the number of ways he can go from place A to C via B.

(i) In the first way, he can go from **A to B in 3 routes** and

(ii) In the second way, he can go from **B to C in 2 different routes.**

Therefore, the total number of ways in which he can travel is **6** (**3 × 2**) routes as shown in Fig 7.4.

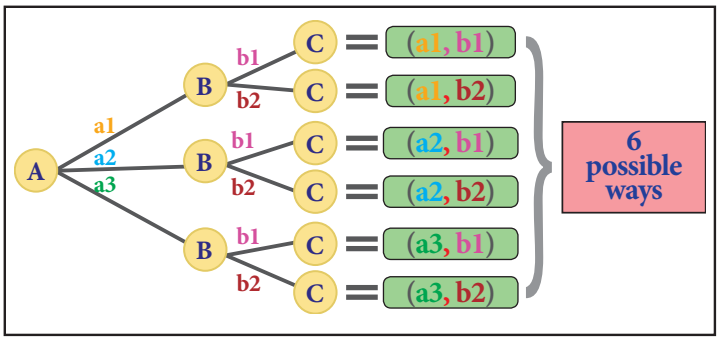


Fig. 7.4

Thus, we come to know, if a selection **A** can occur in ***m*** ways and another selection **B** can occur in ***n*** ways, and both the selections are **dependent** on each other then, the two selections can be performed in exactly ($m \times n$) different ways. Let as learn more about from the following examples.

Example 7.2

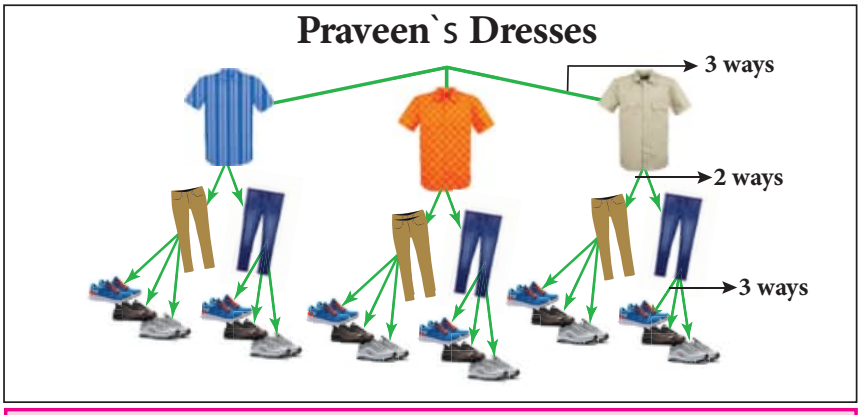
Praveen bought **3 shirts, 2 jeans and 3 pairs of shoes** for his birthday. In Fig. 7.5 shows the different ways of wearing the dress. In how many different ways can Praveen wear a dress on his birthday?

Fig. 7.5

Solution:

Here, Praveen has 3 shirts , 2 jeans and 3 pairs of shoes.

He can wear a dress either this way or he can have the choices as shown in the Fig. 7.6



3 shirts × 2 jeans × 3 pairs of shoes = 18 ways

Fig. 7.6

Therefore, Praveen can wear his dress in **18** **(3×2×3)** **different possible ways** on his birthday.

Example 7.3

In class VIII, a math club has four members **M,A,T** and **H.** Find the number of different ways, the club can elect

(i) a leader,

(ii) a leader and an assistant leader.

Solution:

(i) To elect a leader

In class VIII, a math club has four members namely **M, A, T** and **H.**

Therefore, there are **4** **(4 × 1)** **different ways** by which they can be elected a leader.

(ii) To elect a leader and an assistant leader

In the Fig. 7.7, the **red** shaded boxes show that same member comes twice. As, one person cannot have two leadership, therefore, the red shaded boxes cannot be counted. So there are only **12 ((4 × 1 × 4) - 4) different ways** (shown in yellow boxes and green boxes) to choose a leader and an assistant leader for the math club.

LEADER \ ASSISTANT LEADER	M	A	T	H
M	MM	MA	MT	MH
A	AM	AA	AT	AH
T	TM	TA	TT	TH
H	HM	HA	HT	HH

(1 (M or A or T or H) × 4 (Leader) × 4 (Assistant Leader)) – 4 (Leader and Assistant Leader) = 12 ways

Fig. 7.7

Activity

1. Determine the number of two digit numbers that can be formed using the digits 1, 3 and 5 with repetition of digits allowed.

The activity consists of two parts

(i) Choose a one's digit.

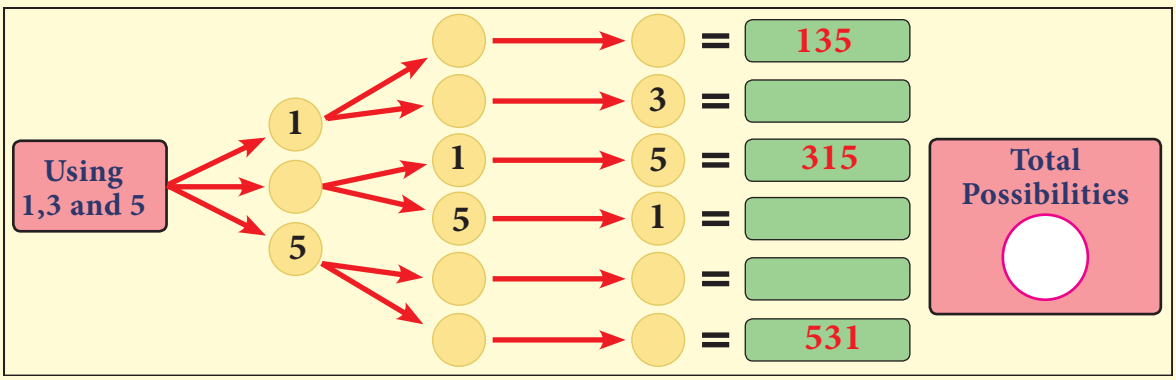
(ii) Choose a ten's digit.

Complete the table given beside

Ten's Digit \ One's Digit	1	3	5
1			15
3		33	
5	51		

2. Find the three digit numbers that can be formed using the digits 1, 3 and 5 without repetition of digits.

Complete the tree diagram given below to the numbers



DO YOU KNOW?

A **password** using 6 characters is created where the first 2 characters are any of the **alphabets**, the third character is any one special character **like @, #, $, %, &, _,+,~, * or -** and the last 3 characters are any of the numbers from **0 to 9**. For that, there are **26 × 26 × 10 × 10 × 10 × 10 = 67,60,000** number of different ways possible to create that password.

Example 7.4

In how many ways, can the students answer 3 true or false type questions in a slip test?

Solution:

(i) Assuming that the question Q_1 is answered True, questions Q_2 and Q_3 can be answered as **TT, TF, FT** and **FF** in 4 ways.

(ii) Assuming that the question Q_1 is answered False, Q_2 and Q_3 can also be answered as **TT, TF, FT,** and **FF** in 4 ways.

Thus, as each question has only two options (True or False) in 2 ways, the number of ways of answering these 3 questions in a slip test is **8 (2 x 2 x 2) possible ways** as shown in Fig. 7.8.

Number of ways of answering the questions

	Questions		
	Q_1	Q_2	Q_3
	T/F	T/F	T/F
Answer1	T	T	T
Answer2	T	T	F
Answer3	T	F	T
Answer4	T	F	F
Answer5	F	T	T
Answer6	F	T	F
Answer7	F	F	T
Answer8	F	F	F

$[(1\ (Q_1) \times 2\ (T/F)) \times (1\ (Q_2) \times 2\ (T/F)) \times (1\ (Q_3) \times 2\ (T/F))] = 8$ ways

Fig. 7.8

Example 7.5

Madhan wants to a buy a new car. The following choices are available for him.

- There are **2 types** of cars as shown in the Fig. 7.9
- There are **5 colours** available in each type as shown in Fig. 7.9.
- There are **3 models** available in each colour
  - (i) **GL** (standard model)
  - (ii) **SS** (sports model)
  - (iii) **SL** (luxury model)

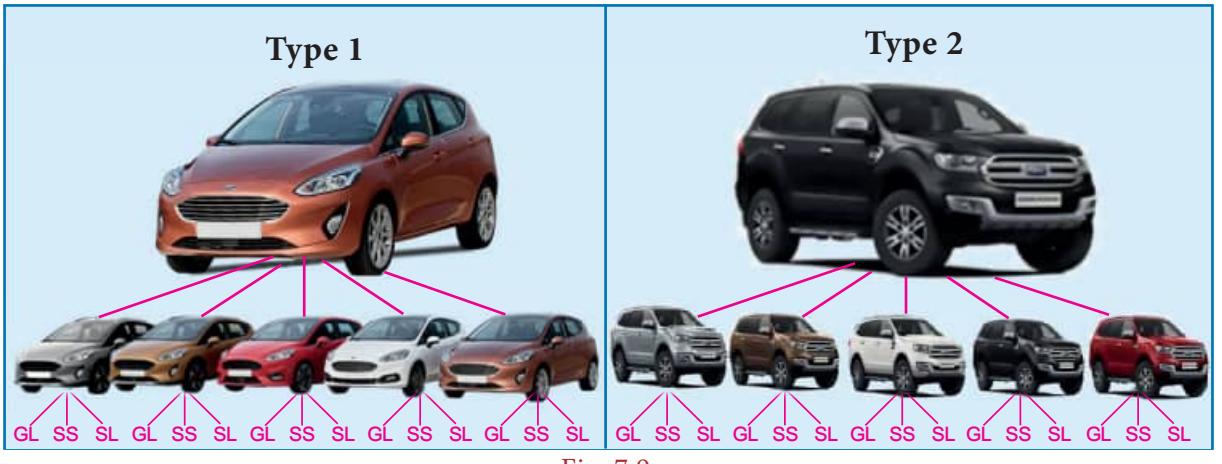


Fig. 7.9

(i) In how many different ways can Madhan buy any one of the new car?

(ii) If the **white colour** is not available in **Type 2**, then in how many ways can Madan buy a new car among the given option?

Solution:

(i) To buy any one of the new car from the given choices

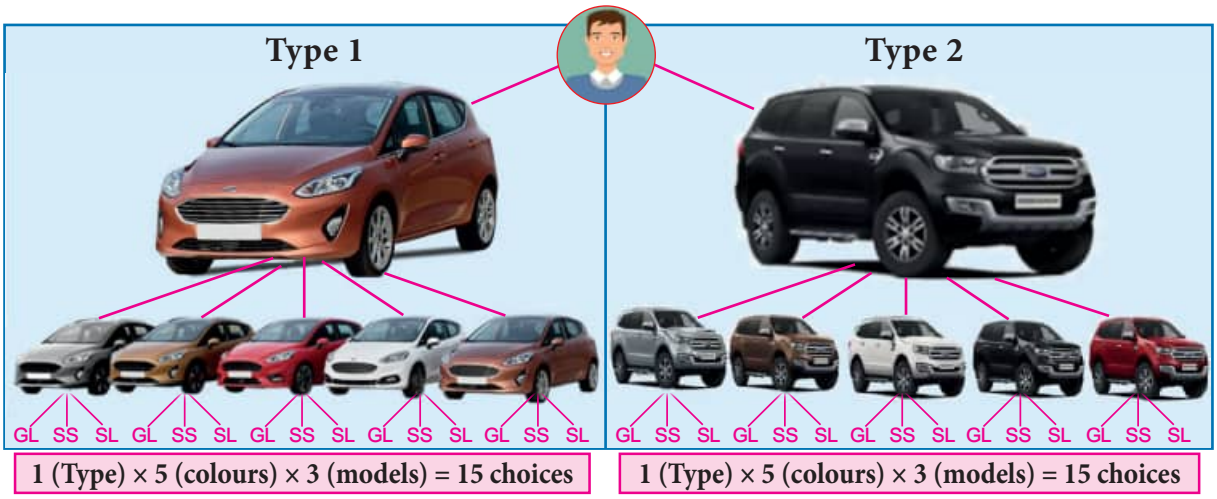

Type 1
Type 2
GL SS SL GL SS SL GL SS SL GL SS SL GL SS SL
GL SS SL GL SS SL GL SS SL GL SS SL GL SS SL
1 (Type) × 5 (colours) × 3 (models) = 15 choices
1 (Type) × 5 (colours) × 3 (models) = 15 choices

Fig. 7.10

Here, we have **2 types** of car with **5 different colours** and **3 models** in each colour. Therefore, there are **30** [**2 (1 × 5 × 3)**] **different ways** to buy a new car by Madhan.

(ii) If the **white colour** is not available in **Type 2,** then...

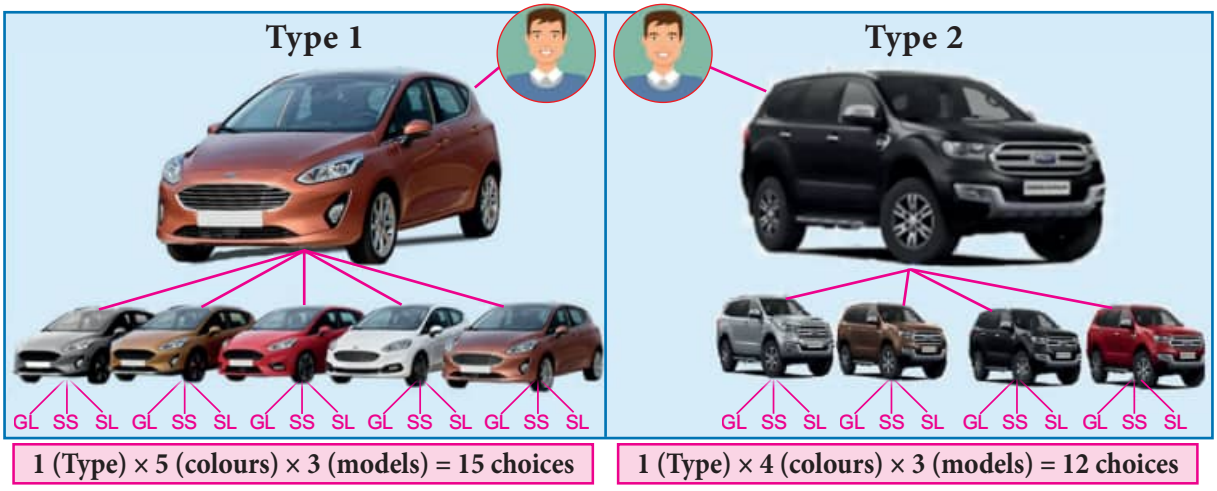

Type 1
Type 2
GL SS SL GL SS SL GL SS SL GL SS SL GL SS SL
GL SS SL GL SS SL GL SS SL GL SS SL
1 (Type) × 5 (colours) × 3 (models) = 15 choices
1 (Type) × 4 (colours) × 3 (models) = 12 choices

Fig. 7.11

(i) For **Type 1,** we have 5 colours and 3 models and hence there are **1 × 5 × 3 = 15 choices.**

(ii) For **Type 2,** we have only 4 colours and 3 models and hence there are **1 × 4 × 3 = 12 choices.**

Therefore, there are **27 (15 + 12) different ways** to buy a new car by Madhan

The above example illustrates both the addition and multiplication principles.

7.3 SET - Game

Any game that uses features can be used to stimulate logical thinking and it provides an interesting and challenging context for exploring ideas in discrete mathematics.

Now, let us learn about a SET game

A **SET** game proves to be an excellent extension for activities that involve organizing the objects by attributes. The **SET** game builds the cognitive, logical, spatial reasoning as well as visual perception skills.

The **SET** game is a puzzle that uses cards which have four features on them. They are shapes, colours, shades and the number of shapes.

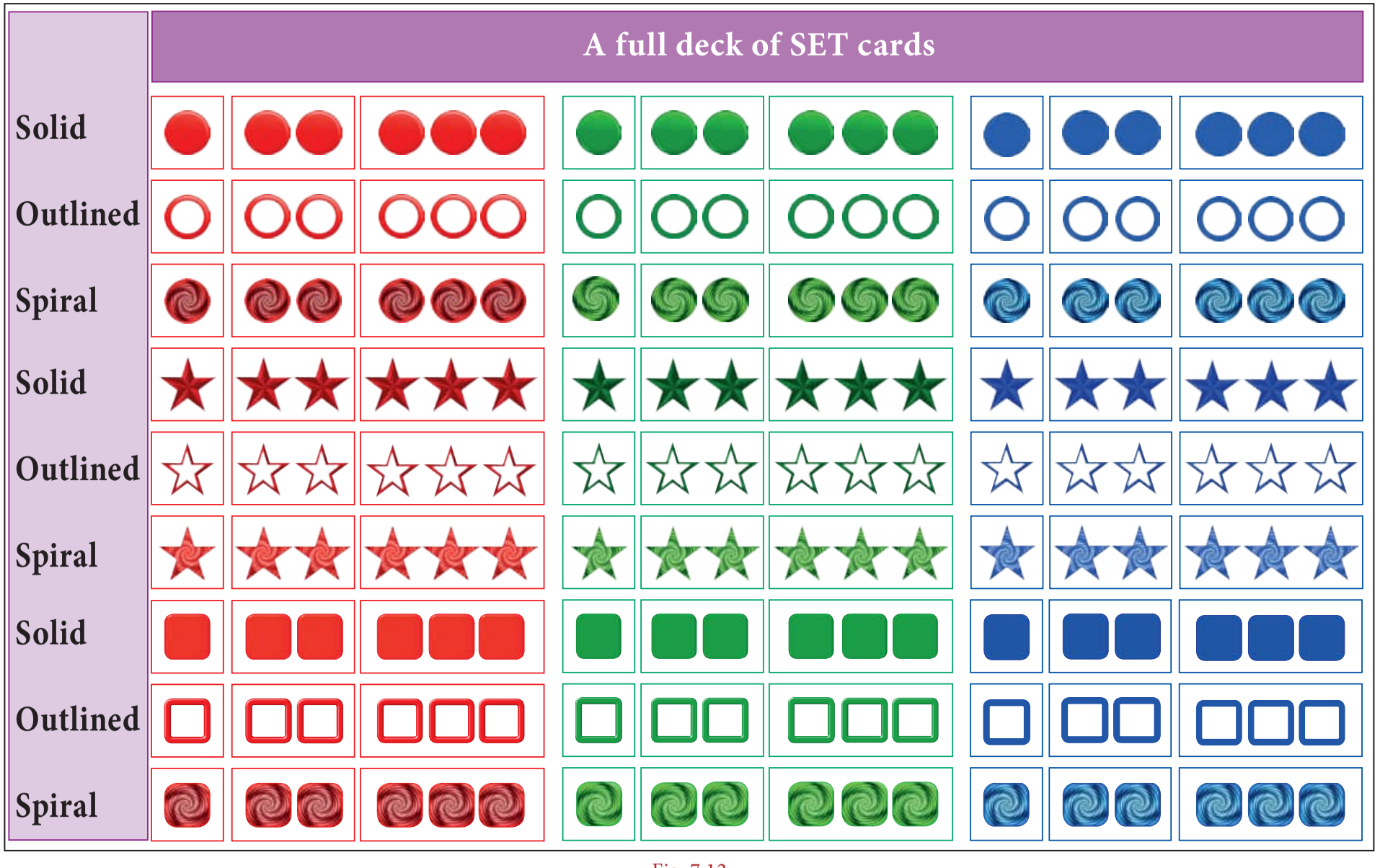

A full deck of SET cards
Solid
Outlined
Spiral
Solid
Outlined
Spiral
Solid
Outlined
Spiral

Fig. 7.12

In one full deck of **SET** cards, there are **3 different shapes**: circle, star, square and in **3 different colours**: red, green, blue. **3** Each of these 9 cards **(3 shapes × 3 colours)** have **3 different shades:** solid, outlined, spiral and also they can be paired **3 with different numbers:** one, two and three. So there are totally **81 cards (3 shapes × 3 colours × 3 shades × 3numbers)** in the deck as shown in Fig. 7.12.

A **SET** which consists of **three cards** should satisfy all the **four** following conditions:

(i) All the three cards have the **same shape** or have **three different shapes**.
(ii) All the three cards have the **same colour** or have **three different colours**.
(iii) All the three cards have the **same shade** or have **three different shades**.
(iv) All the three cards have the **same number** or have **three different numbers**.

Situation:

The teacher displays **12 cards** as shown in Fig. 7.13 and explains how to form a **SET** using these 2 cards and taken from them. Now, follow the step by step procedure to figure out the third card to complete this **SET** is as follows.

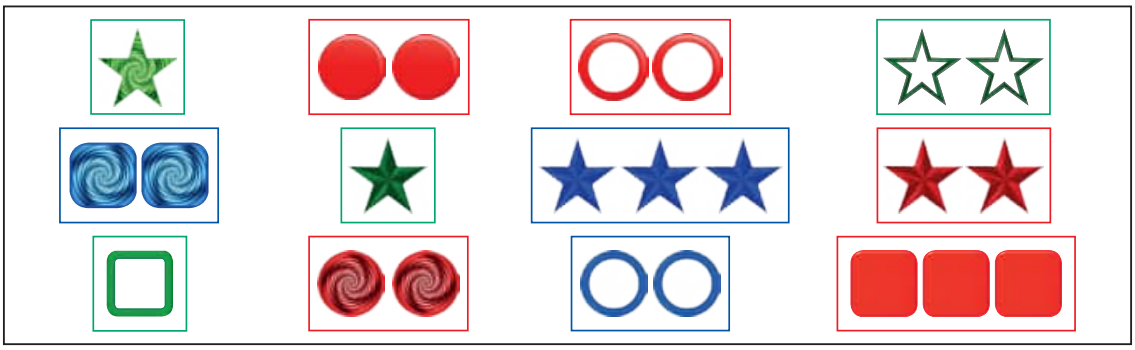


Fig. 7.13

Remember, a **SET** consists of 3 set of cards.

STEP 1: If you look at the **shape** then, **one is star** and the other one is also **star**. These two cards have the **same shape**. So, the last card also should have the **same shape**.

STEP 2: If you look at the **colour** then, **one is green** and the **other is red** These two cards have **different colours**. So, the last card also should have a **different colour** that is **blue**.

STEP 3: If you look at the **shade** then, **one is solid** and the other is also **solid** . These two cards have the **same shade**. So, the last card also should have the **same shade**.

STEP 4: If you look at the **number** then, **one star card is green solid** and the other has **two star red solid cards** . So, the last card also should have a **different number three blue cards**. Therefore these three set of cards have different numbers and different colours with same shape and shade.

Finally,

Shape : All same ✓ **Colour** : All different ✓

Shade : All same ✓ **Number** : All different ✓

Now, this completes the rules for a SET.

Now, the teacher asks the students to find two more **SET**s from Fig.7.13. Let us check the **SET** again.

1.

Shape : All same ✓ **Colour** : All same ✓

Shade : All different ✓ **Number** : All same ✓

Yes, This is a SET.

Think

For the given set of **12 cards** from Fig.7.13 **7 SETs** are possible. Explore. (repetition of cards allowed).

2.

Shape : All different ✓ **Colour** : All different ✓

Shade : All different ✓ **Number** : All different ✓

Yes, This is a SET.

Again , the teacher makes an arrangement of cards and asks them to check **whether, it forms a SET?**

3.

Shape : All same or all different ✗ **Colour** : All same or all different ✗

Shade : All same or all different ✗ **Number** : All same or all different ✗

No, this is not a SET.

Since, the above **SET** does not complete the **SET** rule.

Thus,we come to know a **SET** consists of **3 set of cards** in which each individual feature is either **all same** on each card or **all different** on each card.

Activity

Choose the correct card to complete the perfect **SET**. One is done for you.

1.

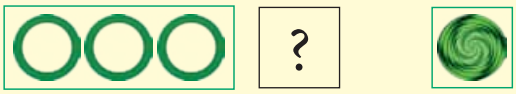
(i) (ii) (iii) Ans:

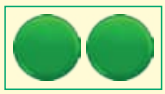
2.

(i) (ii) (iii)

3.

(i) (ii) (iii)

4. 

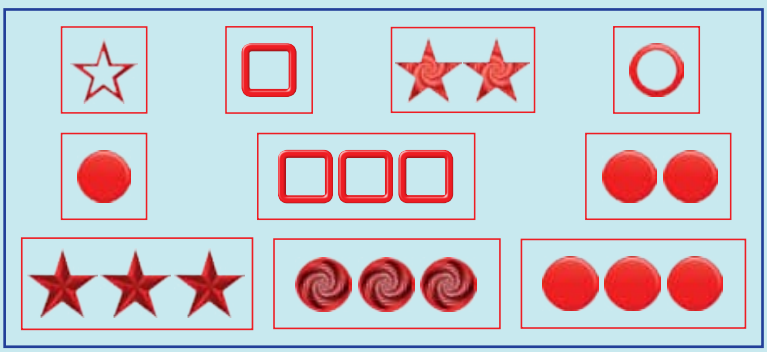
(i)  (ii) (iii)

5.

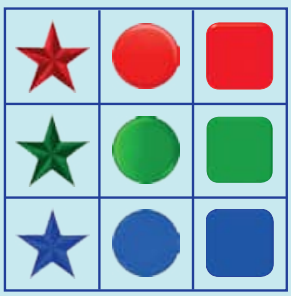
(i) (ii) (iii)

Try these

1. Find any five **SET**s among these set of cards (repetition of cards allowed).



2. This is an example for a magic square in **SET**s. Can you make another two?



7.4 Map Colouring

Map colouring is the act of assigning different colours to different features on a map. In Mathematics, the problem is to determine the minimum number of colours required to colour a map,so that no two adjacent regions have the same colour. Let us learn about the role of map colouring.

Situation:

The teacher divides the class into two groups and instructs one group to use as many colours possible and another group use minimum number of colours for the given patterns.

The only rule is that, shaded regions cannot share the same colour at edges, although they are allowed to meet at a corner.

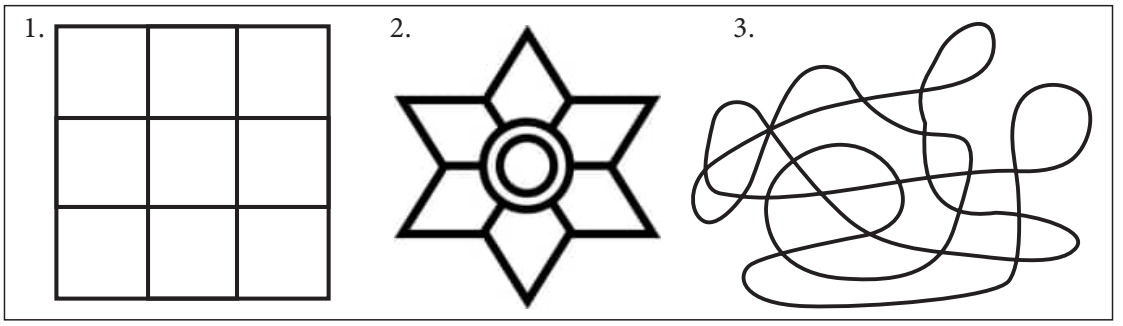


Fig. 7.14

Fig. 7.15 , shows how each group is coloured

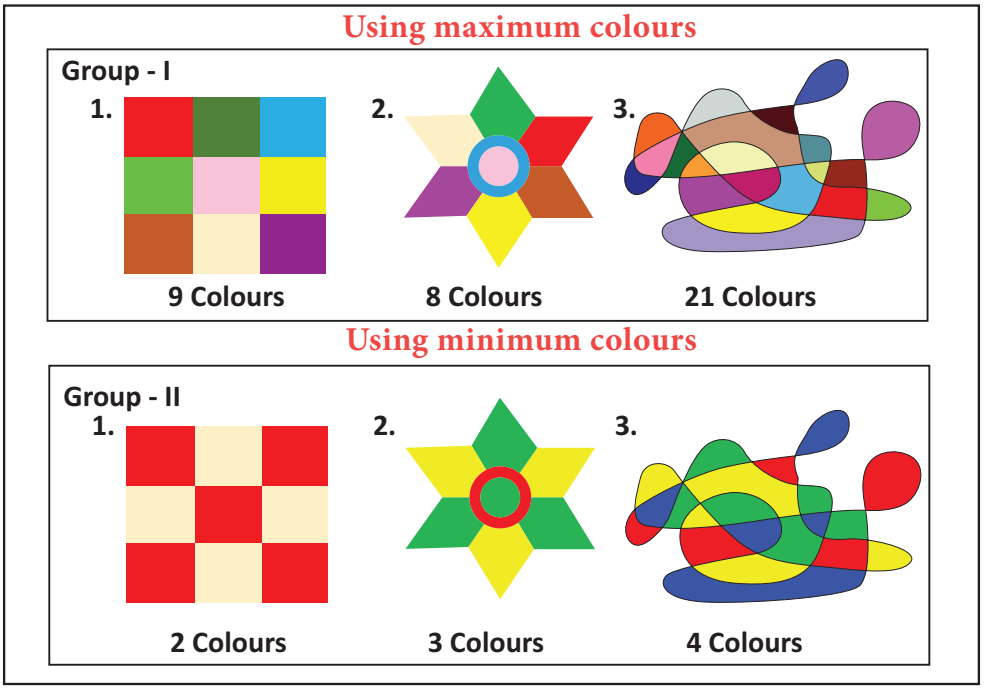


Fig. 7.15

From this investigation, we will get results. Which could be more interesting , if we want to colour a map.

Map colouring is about the colours that must be chosen for regions in a map, which make bordering regions with different colours. Let us learn more from the following examples.

Example 7.6

Colour a map of South India (Fig. 7.16) with the fewest number of colours.

Solution:

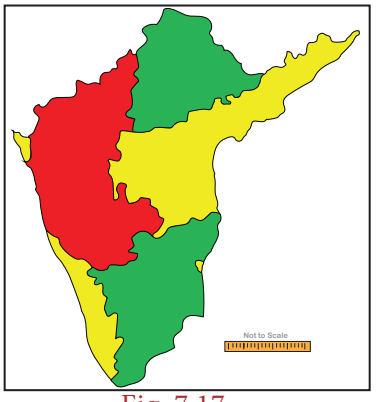


Fig. 7.17

This is one of the solutions with minimum number of colours . Try for more solutions.

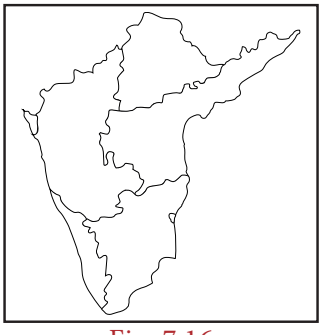


Fig. 7.16

Activity

Try to colour the INDIA-States map with the fewest number of colours.

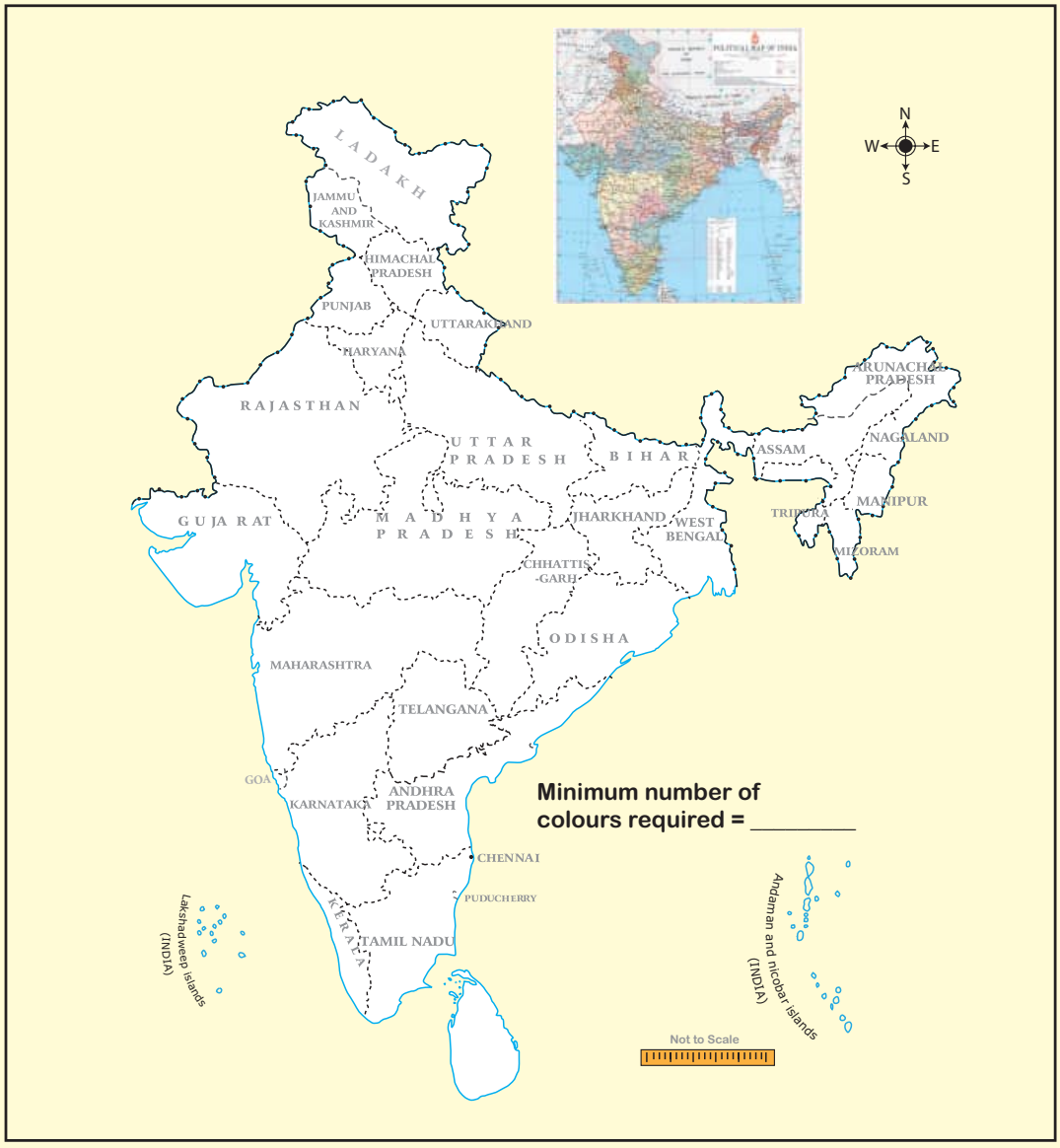


Fig. 7.18

Try these

Draw your school building map showing the HM room, class rooms, staff room, science lab, PET room, computer lab, office room etc., and use map colouring to detetermine the minimum number of colours that can be used to colour the map of your school.

Exercise 7.1

1. You want to have an ice cream or a cake. There are three flavours (chocolate, strawberry and vanilla) in ice creams, and two flavours (orange and red velvet) in the cakes. In how many possible ways can you choose an ice cream or a cake?
2. Shanthi has 5 chudithar sets and 4 frocks. In how many possible ways, can she wear either a chudithar or a frock ?

3. In a Higher Secondary School, the following groups are available in XI standard

I. Science Group:

(i) Physics, Chemistry, Biology and Mathematics

(ii) Physics, Chemistry, Mathematics and Computer Science

(iii) Physics, Chemistry, Biology and Home Science

G89NL

II. Arts Group:

(i) Accountancy, Commerce, Economics and Business Maths

(ii) Accountancy, Commerce, Economics and Computer Science

(iii) History, Geography, Economics and Commerce

III. Vocational Group:

(i) Biology, Nursing Theory, Nursing Practical I and Nursing Practical II

(ii) Home Science, Textiles and Dress Designing Theory, Textiles and Dress Designing Practical I and Textiles and Dress Designing Practical II

In how many possible ways, can a student choose a group?

4. If you have 2 school bags and 3 water bottles then, in how many different ways can you choose each one of them, while going to school ?

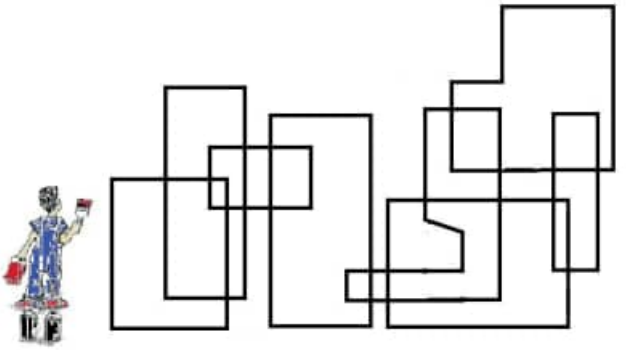
5. Roll numbers are created with a letter followed by 3 digits in it, from the letters A, B, C, D and E and any 3 digits from 0 to 9. In how many possible ways can the roll numbers be generated? (except A000, B000, C000, D000 and E000)

6. A safety locker in a jewel shop requires a 4 digit unique code. The code has the digits from 0 to 9. How many unique codes are possible ?

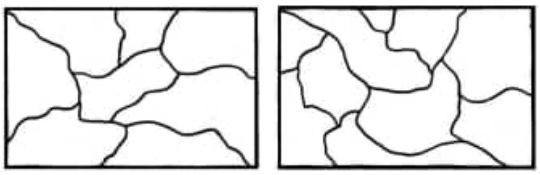
7. An examination paper has 3 sections, each with five questions and students are instructed to answer one question from each section. In how many different ways of can the questions be answered?

8. The given spinner is spun twice and the two numbers got are used to form a 2 digit number. How many different 2 digits numbers are possible?

9. Ramya wants to paint a pattern in her living room wall with minimum budget. Help her to colour the pattern with 2 colours but make sure that no two adjacent boxes are the same colour. The pattern is shown in the picture.



10. Colour the regions in the maps with few colours as possible but make sure that no two adjacent countries are of the same colour.



Objective Type Questions

11. In a class there are 26 boys and 15 girls. The teacher wants to select a boy or a girl to represent a quiz competition. In how many ways can the teacher make this selection?
(A) 41 (B) 26 (C) 15 (D) 390

12. How many outcomes can you get when you toss three coins once?
(A) 6 (B) 8 (C) 3 (D) 2

13. In how many ways can you answer 3 multiple choice questions, with the choices A,B,C and D?
(A) 4 (B) 3 (C) 12 (D) 64

14. How many 2 digit numbers contain the number 7 ?
(A) 10 (B) 18 (C) 19 (D) 20

7.5 Fibonacci Numbers

We have learnt in earlier classes, on how all beautiful things in nature as well as man made things are connected with Mathematics. Now, we just refresh everyone's memory and show how Math can be beautiful when seen in physical and biological things everywhere around us.

Fibonacci (real name Leonardo Bonacci) was an Italian mathematician who developed the Fibonacci Sequence. Remember the pattern of the Fibonacci sequence we already studied in standard VI it looks like this: 1, 1, 2, 3, 5, 8, 13, 21, 34... and it goes on.

Let us tabulate the Fibonacci sequence and find a rule.

Term (n)	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	...
F (n)	1	1	2	3	5	8	13	21	34	55	89	144	233	377	610	...

We observe that the **3rd** term of the Fibonacci sequence is the sum of **2nd term** and the **1st term**.

That is, **F(3) = F(2) + F(1)** and so we can extend and write the rule is

F(1)	F(2)	F(3)	F(4)	F(5)	F(6)	F(7)	...
1	1	2	3	5	8	13	...

+ =

F(3) = F(2)+F(1)

F(n)= F(n–1) + F(n–2)

where **F(n)** is the **nth** term

F(n–1) is the previous term to the **nth** term

F(n–2) is the term before the **(n–1)th** term

This is how the Fibonacci Sequence is obtained. Let us learn more from the following real life examples.

Situation:

Let us look at the family tree of a male drone bee and a female bee as shown Fig 7.19.

Here, female bees have 2 parents, male (drone) bees have just one parent, a female. (Male bee (drone) are produced by the queen's unfertilized eggs, so male(drone) bees only have a mother but no father!)

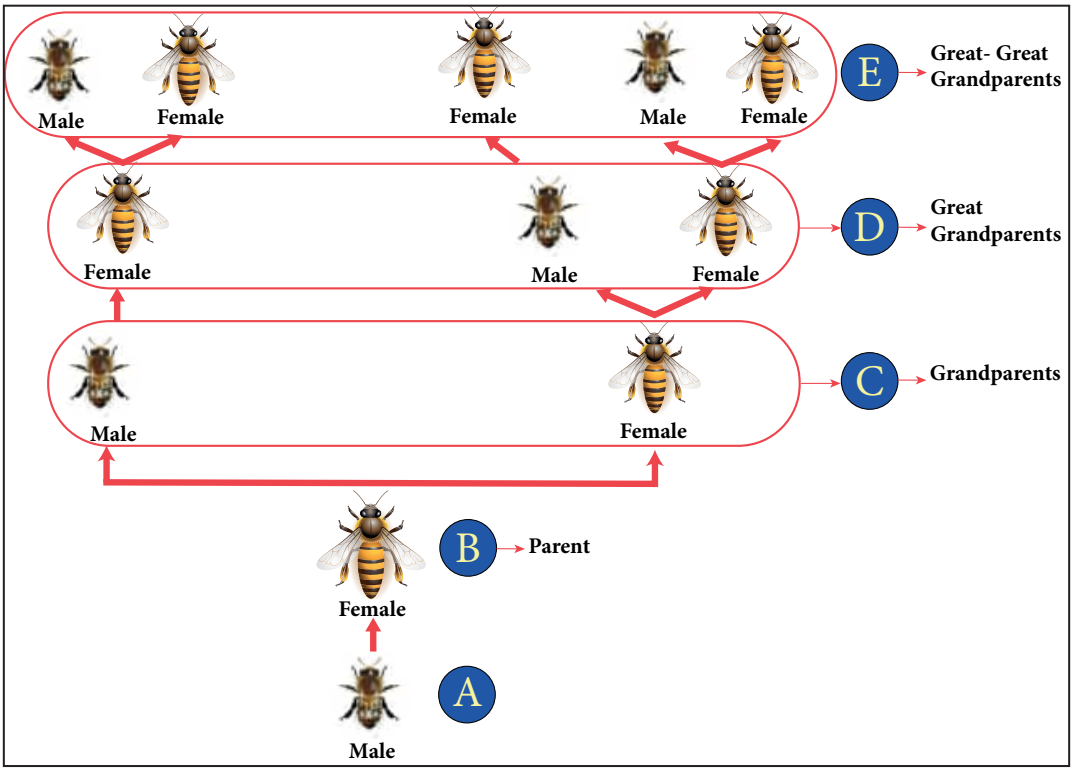


Fig. 7.19

From the picture the following points are noted:

1. The male Ⓐ has 1 parent, a female Ⓑ.
2. The male Ⓐ has **2** grandparents Ⓒ, since his mother had parents Ⓒ, a male and a female.
3. The male Ⓐ has **3** great-grandparents Ⓓ: since his grandmother has two parents but his grandfather has only one.

Now, answer, how many great-great-grandparents did the male Ⓐ have?

Let us try to find the relationship among the pattern of bees family by representing in the tabular form given below,

Number of Ⓐ	Parents Ⓑ	Grandparents Ⓒ	Great Grandparents Ⓓ	Great- Great Grandparents Ⓔ	Great- Great-Great Grandparents Ⓕ
a Male bee (1)	1	2	3	5	8
a Female bee (1)	2	3	5	8	13

We see the Fibonacci numbers 1, 1, 2, 3, 5, 8, 13… in the above table.

Note

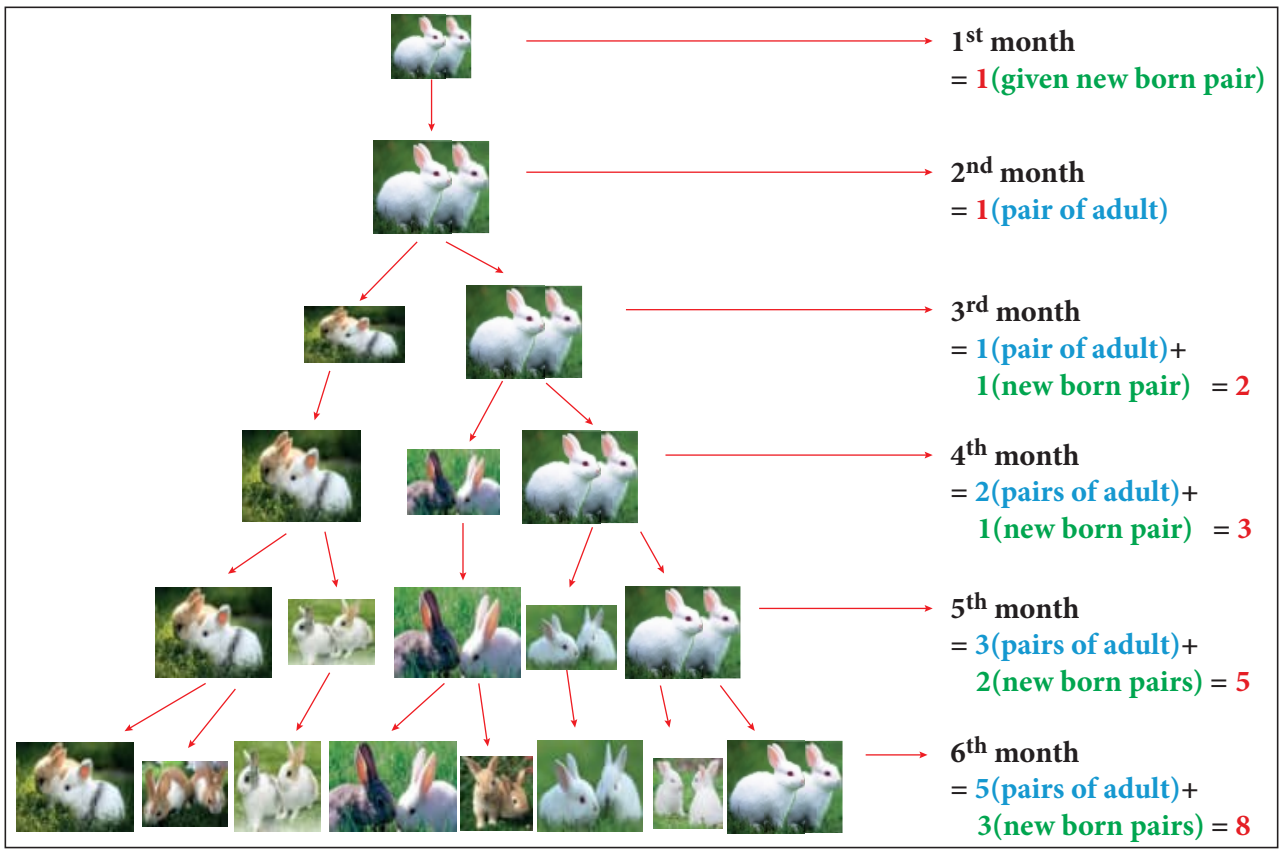
The difference between two consecutive numbers of the Fibonacci sequence increase very quickly. ((For example F(5) – F(4) = 5 – 3 = 2 ; F(10) – F(9) = 55 – 34 = 21 ; F(15) – F(14) = 610 – 377 = 233)(1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987, 1597, 2584, 4181, 6765, 10946 ...))

Example 7.7

Given that one pair of new born rabbits they produce a new pair each month and from the second month, each new pair can breed themselves. Find how many pairs of rabbits are bred from one pair in a year, and find the relationship between the number of months and the number of pairs of rabbits by tabulation (a pair means (a male and a female)).

Solution:

The below picture clearly forms the sequence is **1,1,2,3,5,8...** Here, we find the pattern in which each number is in the Fibonacci sequence, obtained by adding together with previous two. Going on like this to find subsequent numbers at the **twelfth month,** we will get **144 pairs of rabbits.** In the other words, twelfth Fibonacci number is 144.



Number of months	1	2	3	4	5	6	7	8	9	10	11	12
Number of pairs of rabbits	1	1	2	3	5	8	13	21	34	55	89	144

Activity

Using the given Table I, find the pattern, answer the following questions and colour the values in the given Table II. One is done for you.

Table I

Term(n)	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	...
F(n)	1	1	2	3	5	8	13	21	34	55	89	144	233	377	610	

1. Where are the even Fibonacci Numbers? Colour both the term n and where **F(n) is even** in **yellow.** Do you find any pattern? Every **Third** Fibonacci number is **a multiple of 2(even).**
2. Where there are Fibonacci numbers which are **multiple of 3 ?**
   Colour both the term **n and** where **F(n) is multiple of 3** in **red.**
   Write down the pattern you find
   Every .....Fibonacci number is .....
3. What about the **multiple of 5**? Colour both the term **n** and where **F(n) is multiple of 5** in **blue.** Write down the pattern you find.
   Every .....i.e. .....
4. What about the **multiple of 8?** Colour both the term **n** where **F(n) is multiple of 8** in **green.** Write down the pattern you find.
   Every .....i.e. .....

Table II

Term(n)	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	...
F(n)	1	1	2	3	5	8	13	21	34	55	89	144	233	377	610	

From the above activity, we conclude that
Every Fibonacci number is a factor of (a term number of) Fibonacci numbers in miltiples.

	n	3	4	5	6	7	8	9	10	11	12	13	14	15	...
	F(n)	2	3	5	8	13	21	34	55	89	144	233	377	610	...
Factors	2=F(3)	✓	×	×	✓	×	×	✓	×	×	✓	×	×	✓	Every 3rd Fib number
	3=F(4)	×	✓	×	×	×	✓	×	×	×	✓	×	×	×	Every 4th Fib number
	5=F(5)	×	×	✓	×	×	×	×	✓	×	×	×	×	✓	Every 5th Fib number
	8=F(6)	×	×	×	✓	×	×	×	×	×	✓	×	×	×	Every 6th Fib number
	F(k)														Every Kth Fib number

From the above table, we get a general rule as **Every k^{th} Fibonacci number is a multiple of F(k).**

7.6 Highest Common Factor

We have learnt in class VI that iteration is a process wherein a set of instructions or structures are repeated in a sequence for a specified number of times or until a condition is met. Here, we are going to learn to find HCF by listing all factors and find the biggest, then to find HCF by repeated subtraction and see to how much faster the iteration goes (and how in fewer steps you get the HCF) and then how to improve further by repeated division and remainder and that both lead to the same solution but one is faster than the other.

We know that, HCF is used in simplifying or reducing fractions. To understand how this concept applies in real life, imagine the following situation.

7.6.1 Methods to find HCF (Highest Common Factor):

1. Factorisation Method:

Situation:

Let us assume that you have 20 mangoes and 15 apples. You want to donate them together equally among the orphan children. How many orphan children can you help at the maximum?

Here, basically question demands finding HCF of two numbers. HCF is Highest Common Factor, also known as GCD (Greatest Common Divisor). HCF of two or more than two numbers is such that, it is the largest possible number which divides given numbers completely.

Here, let us find the HCF of 20 mangoes and 15 apples.

Factors of 20 = 1,2,4,(5),10,20

Factors of 15 = 1,3,(5),15

So, the HCF of 20 and 15 is 5. That is, you can a help maximum of 5 orphan children.

So that for **5 children** you can give **4 mangoes** (20 ÷ 5 = 4) and **3 apples** (15 ÷ 5 = 3) to each of them. In this way you can distribute equally the mangoes and the apples to each child.

2. Prime Factorisation Method:

Situation:

Suppose there are 18 students in Class VII and 27 in Class VIII and each class is divided into teams to prepare for an upcoming sports tournament, with the winning teams from each class play each other in the final. What would be the biggest possible team size that you could divide both these classes such that each team has exactly the same number of students and that no one is left behind.

The problem here is to find the HCF of 18 and 27.

Prime Factors of 18 = 2 x **3** x **3**

Prime Factors of 27 = 3 x **3** x **3**

Common prime factors of 18 and 27 = 3 x 3 = 9

So, the HCF of 18 and 27 is **9.**

Now, let us learn some more methods of finding the HCF. The largest team member each group is **9**, Std VII has 2 teams and Std VIII has 3 teams.

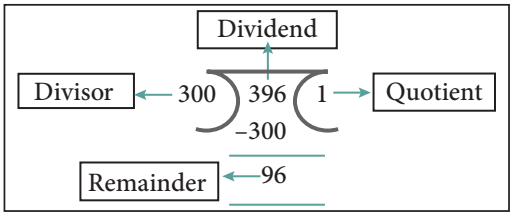
3. Repeated Division Method:

The above methods are easy to finding HCF, but for larger numbers these methods are tedious to find factors of the given numbers. In that case, alternatively we have some more methods to find HCF. Let us learn more about the other methods of finding the HCF.

For the above Situation, what if the Class VII had 396 students and Class VIII had 300 students? Then, what would be the biggest possible team size? Well, the above said two methods may not help us quickly. So, we can use continuous division method for finding the highest common factor.

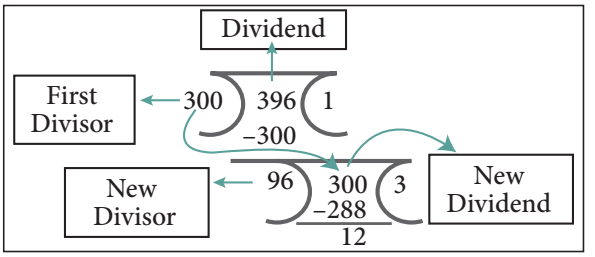
STEP 1: Divide the larger number by the smaller number.

Here, 360 is the larger number. So, we divide 360 (Dividend) by 300 (Divisor). We get the Remainder as 96.



STEP 2: The remainder from Step 1 becomes the new divisor, and divisor of Step 1 becomes the new dividend.

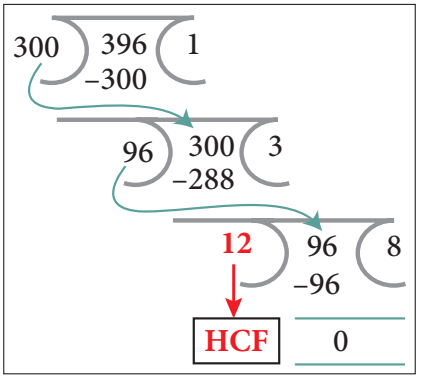
From the step 1, we got 96 as remainder. So, in the second step 96 becomes the new divisor and 300 becomes the new dividend.



STEP 3: Repeat this division process till remainder becomes zero. The divisor of the last division (when remainder is zero) is the HCF.

From step 2, we got 12 as the new remainder which will become the new divisor. In the third step 12 becomes the new divisor and 96 becomes the new dividend. **Now, the remainder is zero when 12 is the last divisor of the division. Therefore,12 is the required HCF.**

Hence, the **HCF of 396 and 300 is 12.** So each team would be **12 students.**



4. Repeated Subtraction Method:

To find the HCF for the given two numbers say **m** and **n** we do the subtraction continuously until m and n are equal. For example,

Find the HCF of 144 and 120

STEP 1: Check whether m = n

Here, take **m** = 144 and **n** = 120

Check whether **m = n** or **m > n** or **m < n** Here **m > n (144 > 120).**

STEP 2: m > n perform m – n repeat the process till m = n or m < n perform n – m repeat the process till m = n

If m is greater than **n,** then we perform **m – n** and assign the result (the difference) as **m.** Again we check whether **m and n are equal or not** and repeat the process. If **m** is less than **n,** then we perform **n – m** and we assign the result (the difference) as **n**. Again we check whether **m and n are equal or not** and the process is repeated.

Subtract **120(n)** from **144(m)** till **m = n.**

First	144 – 120 = 24	Repeat	120 – 24 = 96	Repeat	96 – 24 = 72
Repeat	72 – 24 = 48	Repeat	48 – 24 = 24	Repeat	24 – 24 = 0

STEP 3: When m and n values are equal then that equal value will be the HCF (m, n).

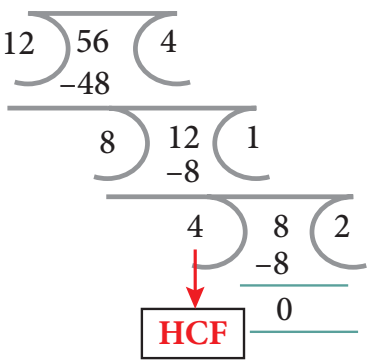
Now m = n, Hence, we conclude that the **HCF of 144 and 120 is 24.**

Comparing both the repeated division and repeated subtraction methods, in finding the HCF, we can conclude that the repeated subtraction, in one way is easier and gives the HCF faster that the repeated division and also that one would want to easily do subtraction rather than division. Isn't it ?

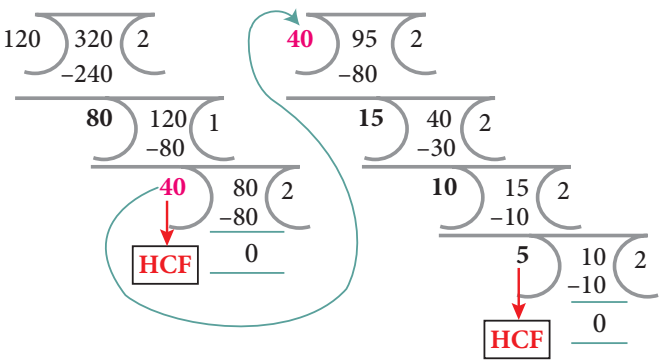
Exercise 7.2

1. Using repeated division method, find the HCF of the following:
   (i) 455 and 26 (ii) 392 and 256 (iii) 6765 and 610 (iv) 184, 230 and 276
2. Using repeated subtraction method, find the HCF of the following:
   (i) 42 and 70 (ii) 36 and 80 (iii) 280 and 420 (iv) 1014 and 654
3. Do the given problems by repeated subtraction method and verify the result.

(i) 56 and 12



(ii) 320, 120 and 95



4. Kalai wants to cut identical squares as big as she can, from a piece of paper measuring 168mm and by 196mm. What is the length of the side of the biggest square? (To find HCF using repeated subtraction method)

Objective Type Questions

5. What is the eleventh Fibonacci number?
   (a) 55 (b) 77 (c) 89 (d) 144

6. If F(n) is a Fibonacci number and n =8, which of the following is true?
   (a) F(8) = F(9)+F(10) (b) F(8) = F(7)+F(6) (c) F(8) = F(10)×F(9) (d) F(8) = F(7)–F(6)

7. Every 3rd number of the Fibonacci sequence is a multiple of _____
   (a) 2 (b) 3 (c) 5 (d) 8

8. Every _____ number of the Fibonacci sequence is a multiple of 8
   (a) 2nd (b) 4th (c) 6th (d) 8th

9. The difference between the 18th and 17th Fibonacci number is
   (a) 233 (b) 377 (c) 610 (d) 987

10. Common prime factors of 30 and 250 are
    (a) 2 x 5 (b) 3 x 5 (c) 2 x 3 x 5 (d) 5 x 5

11. Common prime factors of 36, 60 and 72 are
    (a) 2 x 2 (b) 2 x 3 (c) 3 x 3 (d) 3 x 2 x 2

12. Two numbers are said to be co-prime numbers if their HCF is
    (a) 2 (b) 3 (c) 0 (d) 1

7.7 Cryptology

In today's world, security in information is a fundamental necessity not only for military and political departments but also for private communication. Today's world of communication has increased the importance of financial data exchange, image processing, biometrics and e-commerce transaction which in turn has made data security an important issue. Cryptology is defined as the science which is concerned with communication in secured form.

7.7.1 Cryptology – Some technical details

Plain text: The original message is called plain text.

Cipher text or Cipher number: The encrypted output (converted message into code) is called Cipher text or Cipher number. Cipher text is written in capital letters, while plain text is usually written in lowercase. A secret key is to use something to generate the Cipher text from the plain text.

Encryption and Decryption: The process of converting the plain text to the Cipher text is called encryption and the vice versa is called decryption.

Let us try to create some Cipher text that we use in the form of coded message at some point in our real life.

7.7.2 Examples of Cipher Code

1. Shifting Cypher Text

Ceasar Cipher

The Ceasar Cipher is one of the earliest known and simplest ciphers. It is a type of substitution cipher in which each letter in the text is **"shifted"** a certain number of places down the alphabets.

To pass an encrypted message from one person to another, it is first necessary that both parties have the **'key'** for the cipher, so that the sender may encrypt it and the receiver may decrypt it. For the Caesar Cipher, the **"key"** is the number of characters to shift the cipher alphabet. So, we have to know how big the switch is to break the code.

Let us know about more Ciphers from the following examples and situations.

Example 7.8

Use Ceasar Cipher table set **+4** and try to solve the given secret sentence.

fvieo mr gshiw ger fi xvmgoc

Solution:

Let us make Ceasar Cipher table first. Here, we have to set to **+4** table.

For that, we have to start letter **e** to set as **A**, **f** as **B** … likewise **d** as **Z.**

Now, the **+4** Ceasar Cipher table looks like

Plain Text	a	b	c	d	e	f	g	h	i	j	k	l	m	n	o	p	q	r	s	t	u	v	w	x	y	z
Cipher Text	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V

The given plain text is

fvieo mr gshiw ger fi xvmgoc

To crack this secret code, follow the steps given below.

Step 1: Using Ceasar Cipher table, let us first match the most repeated letters. This will help us to progress faster.

fvieo mr gshiw ger fi xvmgoc

Plain Text	a	b	c	d	e	f	g	h	i	j	k	l	m	n	o	p	q	r	s	t	u	v	w	x	y	z
Cipher Text	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V

f v i e o m r g s h i w g e r f i x v m g o c

B R E A K I N C E C A N B E R I C K

Step 2: Then, let us find remaining letters to complete the code.

Plain Text	a	b	c	d	e	f	g	h	i	j	k	l	m	n	o	p	q	r	s	t	u	v	w	x	y	z
Cipher Text	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V

f v i e o m r g s h i w g e r f i x v m g o c

B R E A K I N C O D E S C A N B E T R I C K Y

Thus, the secret sentence is decoded as, **BREAK IN CODES CAN BE TRICKY**

2. Substituting Cypher Text

Each letter in a text is "substituted" by certain pictures and symbols, a key message, group of words, letters or a combination of these can be used to encode or decode the information. Let us learn more from the following situation.

Situation:

The teacher divides the class into two teams and displays a worksheet given in the figure below for students to complete. Play a game, teams take turns suggesting letter substitutions and someone entering your suggestions in the table. Teams earn points for correct letter guesses.

Worksheet 1 :- Additive Cipher [key = 5]

Convert a given plain text into Additive Cipher text (number) code.

" mathematics is a unique symbolic language in which the whole world works and acts accordingly."

Tips for cracking additive ciphers:

- Especially for additive Ciphers, you only need **key number**. You can complete the cipher table by filling in the rest of the numbers in order.
- If you can, find and make **frequency table for alphabets**, it is help you to get started.
- Match the most **repeated letters** first and fill the Cipher number.
- Look for familiar one-letter words like **a** or **i**. common two and three letter words like **of, to, in, it, at, the, and, for, you …..**
- Look for **consecutive numbers** in the Cipher text and match them with possible **consecutive letters** in the plain text.

Now, to start converting a plain text into Cipher number, first we have to make a cipher table as shown below. Here there **key is 5**. As per key number, we have to start and fill **a = 05, b = 06 …….. z = 04** respectively.

Plain Text	**a**	**b**	**c**	**d**	**e**	**f**	**g**	**h**	**i**	**j**	**k**	**l**	**m**	**n**	**o**	**p**	**q**	**r**	**s**	**t**	**u**	**v**	**w**	**x**	**y**	**z**
Numbers	**00**	**01**	**02**	**03**	**04**	**05**	**06**	**07**	**08**	**09**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**
Cipher Numbers	**05**	**06**	**07**	**08**	**09**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	**00**	**01**	**02**	**03**	**04**

To start encoding the text, let us count the frequency of alphabets and frame frequency table as shown below.

Plain Text	a	b	c	d	e	f	g	h	i	j	k	l	m	n	o	p	q	r	s	t	u	v	w	x	y	z
Frequency of alphabets in the plain text	8	1	6	3	5	0	3	5	7	0	1	5	3	5	5	0	1	3	5	4	3	0	4	0	2	0

With the help of Cipher table and frequency table, let us fill the Cipher numbers for the plain text. Let us first match the most repeated letters (5 times and above), then 2 to 4 times repeated letters and finally remaining letters step by step as shown in given figure.

Step 1:- Cipher number for most repeated letters(5 times and above)

Plain text	m	a	t	h	e	m	a	t	i	c	s		i	s		a		u	n	i	q	u	e		s	y	m	b	o	l	i	c
Cipher Numbers		05		12	09		05		13	07	23		13	23		05			18	13			09		23				19	16	13	07

Plain text	l	a	n	g	u	a	g	e		i	n		w	h	i	c	h		t	h	e		w	h	o	l	e
Cipher Numbers	16	05	18			05		09		13	18			12	13	07	12			12	09			12	19	16	09

Plain text	w	o	r	l	d		w	o	r	k	s		a	n	d		a	c	t	s		a	c	c	o	r	d	i	n	g	l	y
Cipher Numbers		19		16				19			23		05	18			05	07		23		05	07	07	19			13	18		16	

Step 2: Cipher number for most repeated letters (2 times and above)

Plain text	m	a	t	h	e	m	a	t	i	c	s		i	s		a		u	n	i	q	u	e		s	y	m	b	o	l	i	c
Cipher Numbers	17	05	24	12	09	17	05	24	13	07	23		13	23		05		25	18	13		25	09		23	03	17		19	16	13	07

Plain text	l	a	n	g	u	a	g	e		i	n		w	h	i	c	h		t	h	e		w	h	o	l	e
Cipher Numbers	16	05	18	11	24	05	11	09		13	18		01	12	13	07	12		24	12	09		01	12	19	16	09

Plain text	w	o	r	l	d		w	o	r	k	s		a	n	d		a	c	t	s		a	c	c	o	r	d	i	n	g	l	y
Cipher Numbers	01	19	22	16	08		01	19	22		23		05	18	08		05	07	24	23		05	07	07	19	22	08	13	18	11	16	03

Step 3: Cipher numbers for non-repeated letters

Plain text	m	a	t	h	e	m	a	t	i	c	s		i	s		a		u	n	i	q	u	e		s	y	m	b	o	l	i	c
Cipher Numbers	17	05	24	12	09	17	05	24	13	07	23		13	23		05		25	18	13	21	25	09		23	03	17	06	19	16	13	07

Plain text	l	a	n	g	u	a	g	e		i	n		w	h	i	c	h		t	h	e		w	h	o	l	e
Cipher Numbers	16	05	18	11	25	05	11	09		13	18		01	12	13	07	12		24	12	09		01	12	19	16	09

Plain text	w	o	r	l	d		w	o	r	k	s		a	n	d		a	c	t	s		a	c	c	o	r	d	i	n	g	l	y
Cipher Numbers	01	19	22	16	08		01	19	22	15	23		05	18	08		05	07	24	23		05	07	07	19	22	08	13	18	11	16	03

Thus, the Additive Cipher text for plain text is as follows:

"17 05 24 12 09 17 05 24 13 07 23 13 23 05 25 18 13 21 25 09 23 03 17 06 19 16 13 07 16 05 18 11 25 05 11 09 13 08 01 12 13 07 12 24 12 09 01 12 19 16 09 01 19 22 16 08 01 19 22 15 23 05 18 08 05 07 24 23 05 07 07 19 22 08 13 18 11 16 03"

Activity

TREASURE HUNT

Treasure in the Mathematics Club room

The teacher divides the students into four groups and gives each group a code and a clue and then asks them to cracking the clues to find

(i) the identity of the treasure (ii) the place of the treasure

(iii) the room in which the treasure is present.

You may take notes on this piece of paper as you proceed through the search

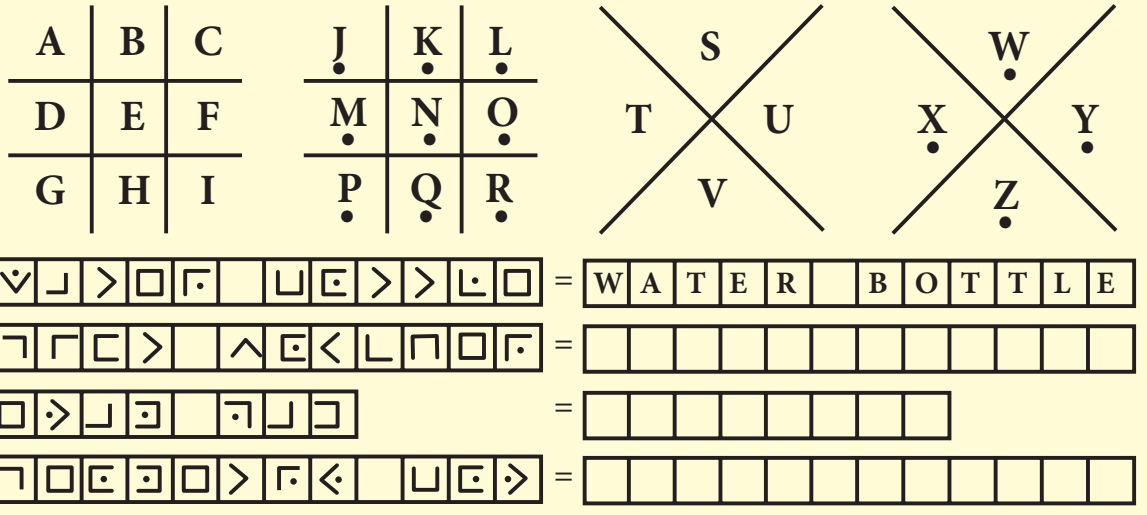
Code 1: Pigpen

Message:- If you decode the clue here you can get the four expected treasure names

I. Fill in the blank boxes and decode

The **Pigpen code** looks like meaningless writing, but it is quite easy to catch on to. Each letter is represented by the part of the "Pigpen" that surrounds it.

The first code uses the following **key**. To complete the code, you need to work out how to use the key to decode the message.



Code 2: Polybius Square Cipher

Message:- If you decode this you can get the clue to identify the name of the treasure name

II. Fill in the blanks

A Polybius Square is a table that allows someone to convert letters into numbers. Use the Polybius square rows and column values to find the code

5	A	B	C	D	E
4	F	G	H	I/J	K
3	L	M	N	O	P
2	Q	R	S	T	U
1	V	W	X	Y	Z
	1	2	3	4	5

(4,2) (3,4) (5,5) (3,3) (1,5) (2,3) (5,5) (4,3) (1,4)

T H E _____ O F

(4,2) (2,2) (5,5) (1,5) (3,2) (5,2) (2,2) (5,5) (4,5) (4,3) (5,5) (3,2)

_____ D O E S

(3,3) (4,3) (4,2) (3,4) (1,5) (1,1) (5,5) "(2,5)" (1,5) (3,3) (4,5) "(4,5)"

_____ _____ ____ A N D _____

Code 3: Atbash Cipher

Message:- If you decode this you can get the clue room number where the treasure is present (It may be 24 or 25 or 26 or 27 or 28 or 29 or 30)

III. Find the code using the key as shown in given figure:

Atbash cipher is a substitution cipher with just one specific key where all the letters are reversed that is **A to Z** and **Z to A**.

Plain Text	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
Cipher Text	Z	Y	X	W	V	U	T	S	R	Q	P	O	N	M	L	K	J	I	H	G	F	E	D	C	B	A

GSV ILLN MFNYVI RH Z NFOGRKOV LU ULFI ZMW HVEVM

THE ____ _____ _____ ___ _ _____ ___ ____ ____ _____.

Code 4: Using a Key – Reflection Table

Message:- If you decode this clue you can get the possible place where the treasure is located (It used to sit)

Use the reflection table which is given below and find the correct word by using a reflected alphabet.

A	B	C	D	E	F	G	H	I	J	K	L	M
↕	↕	↕	↕	↕	↕	↕	↕	↕	↕	↕	↕	↕
N	O	P	Q	R	S	T	U	V	W	X	Y	Z

J V A Q B J - _____

P B Z C H G R E G N O Y R - _____

P U N V E - _____

P H O O B N E Q - _____

After finding the codes, the teacher then asks students to rearrange the clues one by one

CLUES

i. _____

ii. _____

iii. _____

iv. _____

RESULT

(i) The room in which the treasure took place :- _____.

(ii) The place of the treasure :- _____.

(iii) The identity of the treasure :- _____.

(Hint:- If you answered the question number 6 in Exercise 4.3, you can compare and verify your results) The gift voucher contains (20 full marks awarded).

Try these

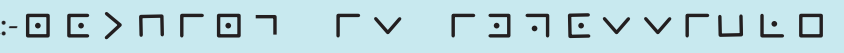
1. Use Pigpen Cipher code and write the code for your name _____and your chapter names

   (i) LIFE MATHEMATICS (ii) ALGEBRA

   (iii) GEOMETRY (iv) INFORMATION PROCESSING

2. Decode the following Shifting and Substituting secret codes given below. Which one is easier for you?

   (i) Shifting method:- M N S G H M F H R H L O N R R H A K D

   (ii) Substituting method:- 

3. Write a short message to a friend and get him to decipher it.(use either shifting or substituting code method)

Exercise 7.3

1. Fill in the blanks **(Use Atbash Cipher that is given in code 3)**

   (i) G Z N R O = _____

   (ii) V M T O R H S = _____

   (iii) N Z G S V N Z G R X H = _____

   (iv) H X R V M X V = _____

   (v) H L X R Z O H X R V M X V = _____

2. Match the following **(a = 00..... Z= 25).**

   (i) mathematics - (a) 18 20 01 19 17 00 02 19 08 14 13

   (ii) addition - (b) 03 08 21 08 18 08 14 13

   (iii) subtraction - (c) 12 00 19 07 04 12 0019 08 02 18

   (iv) multiplication - (d) 00 03 03 08 19 08 14 13

   (v) division - (e) 12 20 11 19 08 15 11 15 02 00 19 08 14 13

   AS6Y8

3. Frame Additive cipher table **(key = 4).**

4. A message like **"Good Morning"** written in reverse would instead be **"Doog Gninrom".** In the same way decode the sentence given below:

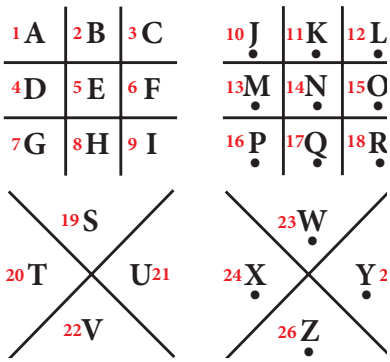
   "Ot dnatsrednu taht scitamehtam nac eb decneirepxe erehwreve ni erutan dna laer efil."

5. Decode the given **Pigpen Cipher text** and compare your answer to get the **Activity 3 result**.

I. The room number in which the treasure took place:

II. Place of the treasure :

III. The name of the treasure :



6. Praveen recently got the registration number for his new two-wheeler. Here, the number is given in the form of mirror-image. Encode the image and find the correct registration number of praveen's two-wheeler.

TN12H2589

(a) 6852H21NT (b) TNIZHSZ8ϑ

(c) ϑ8ZSHZINT (d) 9852H21NT

Objective Type Questions

7. In questions **(i)** and **(ii)**, there are four groups of letters in each set. Three of these sets are alike in some way while one is different. Find the one which is different.

(i). (A) C R D T (B) A P B Q (C) E U F V (D) G W H X

(ii). (A) H K N Q (B) I L O R (C) J M P S (D) A D G J

8. A group of letters are given. A numerical code has been given to each letter. These letters have to be unscrambled into a meaningful word. Find out the code for the word so formed from the 4 answers given.

L	I	N	C	P	E
1	2	3	4	5	6

(A) 2 3 4 1 5 6 (B) 5 6 3 4 2 1 (C) 6 1 3 5 2 4 (D) 4 2 1 3 5 6

9. Questions **(iii)** and **(iv)** are based on code language. Find the correct answer from the four alternatives given.

(iii) In a certain code, '**M E D I C I N E**' is coded as '**E O J D J E F M**', then how is '**C O M P U T E R**' written in the same code ?

(A) C N P R V U F Q (B) C M N Q T U D R

(C) R F U V Q N P C (D) R N V F T U D Q

(iv) If the word '**P H O N E**' is coded as '**S K R Q H**', how will '**R A D I O**' be coded ?

(A) S C G N H (B) V R G N G (C) U D G L R (D) S D H K Q

7.8 Shopping Comparison

Students, I hope you all have an experience in shopping. can you share a few of your experiences? I would like to raise some questions on your shopping experience. How will you shop the things you need? By (i) attractive colour or (ii) best price or (iii) big in size or (iv) on seeing. Whatever it may be, among all these things, one important point should be noted. What is that? Yes, that is the expiry date. Have you ever noticed the expiry date on all the packed goods? It is very important to see that and another way of shopping is to compare goods by means of its price, quality, quantity, offers, discount and other considerable things.

Before spending your money to shop any item from a market or a departmental store, consider the best prices, the best quality and other reliable things. This is what we call as wise shopping.

Here, we will learn how to be a wise consumer before shopping a product from the following situation.

Situation:

Imagine that the teacher maker you and your friend to be incharge of the fruit section of your school canteen for a week. She also instructs the following steps and she can help you when needed.

- Now you have to buy fruits for 2 days as per your the shopping list.
- One of you should go to the market and the other should go to the departmental store to know the cost of the fruits before shopping.
- Estimate yourselves about which place will give you the best deal.

Shopping list

1. 20 kg apples
2. 20 kg of guavas
3. 30 boxes of strawberries
4. 20 dozens of bananas

After that,

- Check your shopping list to see how much fruits you require.
- Compare the weight and price for each item from both places.
- Select the best deal for all items in only one place .
- Discuss and compare the price list so that you decide where to buy the required list of fruits.

For example, the collected model price list from both shops is given in the table below:

S. No	Fruit name	Departmental store		Market price	
		Quantity	Price (₹)	Quantity	Price (₹)
1	Apple	1 kg	120	1 kg	110
2	Guava	1 kg	50	1 kg	40
3	Strawberry	1 box	80	1 box	85
4	Banana	1 dozen	60	1 kg	50

Now, we will calculate the total price of the required and quantity of fruits from both the departmental store and market.

Calculating the total price from the Departmental store:

Fruit Name	Cost of the required fruits	Total Price (₹)
Apple	Cost of 1 kg of apples = ₹120 Cost of 20 kg of apples = 20 × 120 = ₹2400	2400
Guava	Cost of 1 kg guavas = ₹50 Cost of 20 kg guavas = 20 × 50 = ₹1000	1000
Strawberry	Cost of 1 box of strawberries = ₹80 Cost of 30 boxes of strawberries = 30 × 80= ₹2400	2400
Banana	Cost of 1dozen of bananas = ₹60 Cost of 20 kg of bananas = 20 × 60 = ₹1200	1200

Calculating the total price from the Market Price:

Fruit Name	Cost of the required fruits	Total Price (₹)
Apple	Cost of 1 kg of apples = ₹110 Cost of 20 kg of apples = 20 × 110 = ₹2200	2200
Guava	Cost of 1 kg of guavas = ₹40 Cost of 20 kg of guavas = 20 × 40 = ₹800	800
Strawberry	Cost of 1 box of strawberries = ₹85 Cost of 30 boxes of strawberries = 30 × 85 = ₹2550	2550
Banana	Cost of 1 kg of bananas = ₹50 Cost of 20 kg of bananas = 20 × 50 = ₹1000	1000

Now, let us compare the shopping price of the Departmental store to that of the Market shop.

Fruits	Cost of items as per your requirement (₹)	
	Departmental Store	Market
20 kg of Apples	2400	2200
20 kg of Guavas	1000	800
30 boxes of Strawberries	2400	2550
20 dozens of Bananas	1200	1000
Total cost of shopping	**7000**	**6550**

From the above comparison, we find that shopping made at the Market shop is the best deal quantity wise as well as in price wise and hence it is wise to shop in the Market.

Activity

Consider that you are going to a store with your total budget of ₹220 to buy things without changing the quantity of the items given in the list below with the following conditions.

Conditions:

(i) First you have to complete the price list given.

(ii) You have to buy three items as per the given price list but within your budget ₹**220**.

(iii) You won't carry exceeding **5kg** because you have to walk home carrying them, so they cannot be bulky.

Now, answer the following questions:

1. In how many ways can you buy your items? Complete the price lists given below. One is done for you.
2. Which one is the best purchase price list and why?

Price List

S.No.	Description	Price / 1 kg (₹)	Quantity kg	Amount (₹)
1	Rice	37.50	2.50	
2	Toor Dal	62.00	1.00	
3	Sugar	32.50	1.50	
4	Wheat	26.50	1.00	
Total Bill Amount				

Price List

S.No.	Description	Price / 1 kg (₹)	Quantity kg	Amount (₹)
1	Rice	37.50	2.50	93.75
2	Toor Dal	62.00	1.00	62.00
3	Wheat	26.50	1.00	26.50
Total Bill Amount				182.25

Price List

S. No.	Description	Price / 1 kg (₹)	Quantity kg	Amount (₹)
Total Bill Amount				

Price List

S. No.	Description	Price / 1 kg (₹)	Quantity kg	Amount (₹)
Total Bill Amount				

Price List

S. No.	Description	Price / 1 kg (₹)	Quantity kg	Amount (₹)
Total Bill Amount				

7.8.1 Comparing containers of different size

Many times, items are packed in different sized of containers.

- Sometimes, shoppers save money by selecting a larger container of the same item. For example, 1 litre of milk often costs less than 5 units of 200ml pack of milk.
- Sometimes, a store has two prices for the same item. One price is for buying a single item, while the other price is for buying more than one of the same item. For example, groundnut oil may cost ₹135 for 1 litre bottle and ₹240 for 2 litre bottles. In this case, if you buy two 1 litre bottles, you will pay more. Sometimes, buying in quantity also saves money.
- Sometimes, the consumer may not be able to use up the larger size of an item before it becomes stale or outdated. To find out which size container is the best to buy, you will need to know the price of single pack of the contents.

Activity

Consider that you want to buy 12 litres of the same quality of edible oil at your budget price of ₹250 per litre. In a supermarket, there are a lot of offers on various oil brands. Some of the offers are given below. Complete the table and find which one is the best offer for you and how much you will save for your total purchase.

Which one is the best deal?

Product (Edible oil)	Size (in litres)	Regular Price (₹)	Offer	Special Price (₹)	Saving Price (₹)	Cost of 1 litre (₹)	Cost of 12 litres(₹)
	1	293	₹ 50 off	243		243	
	2	850	1 *l* +1 *l* combo	499	351 (850-499)	249.50	
	5+1 = 6	2000	Buy 5 *l* get 1 *l* free	1500			3000
	2+2 = 4	1486	Buy 1 get 1 free	743		185.75	
	1+1= 2	850	Spl. offer 1 *l* pack of 2 ₹ 390	390		195	
	12 (1) = 12	5100	1 *l* pack of 12	1650	3450		

Best offer price for you _____

Amount that you saved for your total purchase _____

Try these

The teacher divides the class into four groups and sets up a mock market in the class room and ask the students to involve in role play as two groups of businessmen and two groups of consumers. Consumers have to buy products at different shops and prepare a price list.

The two supermarkets in which the two groups can buy are Star Food Mart and Super Provisions. This week they each have got a special deal on some products. At Star Food Mart, you can buy items at discount prices. At Super Provisions, there are some "**BUY ONE GET ONE**" deals. Have a look at their deal:

Star Food Mart

Chocolate Biscuits worth ₹114 per packet at ₹30 off

Peanut candies worth ₹90 now at ₹20 off

Protein milk worth ₹60 now at ₹20 off

Badam nuts worth ₹450 now at ₹150 off

Super Provisions

Chocolate Biscuits worth ₹180 per packet Buy one get one free

Peanut candies worth ₹150 Buy one get one free

Protein milk worth ₹80 Buy one get one free

Badam nuts worth ₹580 Buy one get one free

Now, answer the following questions.

I. Here is your shopping list:

(i) 4 bottles of Protein Milk (200 ml size) **(ii) 2 packets of Peanut candies(200 gm)**

(iii) 1 packet of Chocolate biscuits **(iv) 1 packet of Badam nuts (500 gm)**

(i) If you buy all the items in one shop, where will you get the best price?

(ii) If you buy the items from the two shops, how will you do it to spend the least amount of money?

II. You have ₹1000/- to spend to buy the following shopping list:

(i) 6 bottles of Protein Milk (200 ml size) **(ii) 3 packets of Peanut candies(200 gm)**

(iii) 3 packets of Chocolate biscuits **(iv) 1 packet of Badam nuts (250 gm).**

(i) How can you do this so that you don't go over your budget amount ₹1000?

(ii) Which shop offers you the best value for money on each item?

(iii) Is the "**BUY ONE GET ONE**" deal at Super Provisions the same as "**50% off**" deal?

7.9 Packing

When we are packing something in a box or a suitcase or a cupboard, first we have to decide how we are going to pack. How many items can be fitted into that fixed space? A good example of this is, before you go to school, you try to pack everything you need (like your books, notebooks, geometry box, sports equipment, food and water bottle) into your school bag. At that time, you are very clear that your books should not be damaged and you are able to carry everything yourself. Think! The same rules apply to posting a package to a friend or family member or others.

Apart from these, the packaging methods are used in many cases such as cutting of sheets, glass, paper, wood, cloth or other materials and room allotment, seating arrangement in the particular space provided, parking vehicles with proper lanes and saving data in hard disk, CD, pen drive and so on.

Using some of the packing methods from the following situations and examples, let us try to understand how best to fit the items into the space in the containers or in rooms or in boxes etc.,

7.9.1 Packing Approaches - Fractional method

Here, when we fill items in bags or in containers, we determine the weight, value and number of each item with the condition that the total weight of the container is less than or equal to a given limit and the total value is as large as possible. Fractional method uses the technique of buying things fractionally and admits buying of more items within a given budget. Let us learn more about this approach from the following situation.

Situation 1:

Suppose that you want to buy some vegetables and fruits that are given in the Fig. 7.20 with their weights and price and that you have a bag that capacity of carrying 15 kg. The objective is to buy more items within your budget of ₹550 and also that weight should not exceed 15 kg.

Fig. 7.20

You cannot buy all the items because if you calculate the total weight of all the items, then it would be greater than 15 kg (maximum capacity of your bag is 15 kg). So, let us try some approaches, to find how you can select more items so that you will buy them with maximum price within your budget of ₹550. For that let us tabulate the items with their weights and price you want to buy.

Items							
Weight (kg)	1	3	5	4	1	3	2
Price (₹)	60	105	150	70	80	90	40

Approach I - Selecting items with Maximum Price

In this approach, we select the items according to the maximum price. Here the maximum price in the table is ₹150/-. Now, let us tabulate to find the total price and how much can we buy vegetables and fruits within our budget and not exceeding 15kg.

Items	Weight (kg)	Remaining weight to buy	Price (₹)
	5	15−5=10	150.00
	3	10−3=7	105.00

	3	7−3=4	90.00
	1	4−1=3	80.00
	3	3−3=0	$70 \times \frac{3}{4} = 52.50$
Total price	**15 kg**		**472.50**

Here, 3 kg of papaya is enough as the total weight reaches 15 kg. So, 3 kg of papaya costs ₹52.50. Hence, in this approach we will spend maximum ₹472.50 to buy 15 kg of vegetables and fruits.

Approach II - Selecting items with Minimum Weight

In this approach, we select the items according to the minimum weight. Here, we can select more and more items. Now, let us tabulate to find the total price and how much can we buy vegetables and fruits within our budget and not exceeding 15kg.

Items	Weight(kg)	Remaining weight to buy	Price (₹)
	1	15−1=14	60.00
	1	14−1=13	80.00
	2	13−2=11	40.00
	3	11−3=8	105.00
	3	8−3=5	90.00

	4	5−4=1	70.00
	1	1−1=0	$\mathbf{150 \times \frac{1}{5} = 30.00}$
Total price	**15 kg**		**475.00**

Here, 1 kg of Sapotta is enough to complete 15 kg with minimum price of ₹30 per kg. Hence in this approach, we will spend maximum ₹475 to buy 15 kg of vegetables and fruits.

Approach III - Finding the Maximum price to Weight Ratio.

In this approach, we select the items according to the maximum price to weight ratio (find the rate of 1kg). Now, let us tabulate to find the total price and how much we can buy vegetables and fruits within our budget and not exceeding 15 kg.

Items	Weight (kg)	Price of 1kg	Remaining weight to buy	Price (₹)
	1	80.00	15−1=14	80.00
	1	60.00	14−1=13	60.00
	3	35.00	13−3=10	105.00
	5	30.00	10−5=5	150.00
	3	30.00	5−3=2	90.00
	2	20.00	3−2=0	40.00
Total price	**15 kg**			**520.00**

In this approach, we can buy all vegetables and fruits except papaya as we need with maximum price within our budget and not exceeding 15 kg. Comparatively, in the II approach we can buy more items but spend minimum amount only. So, we can say third approach is best one. Isn't it?

Exercise 7.4

1. Find the best buy of the following purchases:
   (i) A pack of 5 chocolate bars for ₹175 or 3 chocolate bars for ₹114?
   (ii) Basker buy 1½ dozen of eggs for ₹81 and Aruna buy 15 eggs for ₹64.50?

2. Using the given picture find the total special offer price of fresh sweets and bakery products to buy ½ kg laddu, 1 kg cake, 6 pockets of bread.

Fresh sweets and Bakery Products 20% OFF

Laddu (1 kg) ₹245

Chocolate Cake (1 kg) ₹550

Healthy Sliced Bread ₹20

3. Using the given picture prepare a price list.

Suppose you plan to buy 1½ kg of apple, 2 kg of pomegranate, 2 kg of banana, 3 kg of mango, ½ kg of papaya, 3 kg of onion, 1½ kg of tomato, and 1 kg of carrot in shop 1, how much will you save compared to shop 2.

Shop 1

Freshly Picked Fruits and Vegetables

Flat 15% offer on all items

Apple Simla (1 kg) ₹168

Carrot (1 kg) ₹19

Tomato (1 kg) ₹46

Onion (1 kg) ₹22

Mango Langda (1 kg) ₹39

Pomegranate (1 kg) ₹82

Banana (1 kg) ₹45

Papaya (1 kg) ₹36

Potato (1 kg) ₹21

Broccoli (250 g) ₹45

Shop 2

Farm fresh Fruits and Vegetables

Apple Simla (1 kg) ~~₹168~~ ₹148

Carrot (1 kg) ~~₹19~~ ₹17

Tomato (1 kg) ~~₹45~~ ₹38

Onion (1 kg) ~~₹22~~ ₹21

Mango Langda (1 kg) ~~₹39~~ ₹35

Pomegranate (1 kg) ~~₹82~~ ₹75

Banana (1 kg) ~~₹45~~ ₹43

Papaya (1 kg) ~~₹36~~ ₹30

Potato (1 kg) ~~₹21~~ ₹18

Broccoli (250 g) ~~₹45~~ ₹37

4. You want to buy some grocery items as per your shopping list that are given in the picture with their price. Also you have a bag that capacity of carrying 7 kg. Using weight ratio approach tabulate to find the total price and how much can you buy more grocery items within your budget of ₹1000 and not exceeding 7 kg.

Shopping list

1. 2 kg of red chilli
2. 2 kg of coriander
3. 1 kg of garlic
4. 1 kg of tamarind
5. 2 kg of toor dal

Red chilli	Coriander	Garlic	Tamarind	Toor dal
₹145/1 kg.	₹130/1 kg.	₹82/1 kg.	₹99/1 kg.	₹78/1 kg.

Objective Type Questions

5. Online or television advertisements influence people on spending decisions by
   (a) using special music
   (b) making them think that they need the item
   (c) using attractive pictures
   (d) all the above

6. When I go shopping, I will buy
   (a) something that looks attractive
   (b) something my friend has
   (c) something that I need to purchase
   (d) the first thing I see in the store

7. The best shopping choice is to
   (a) shop at brand name stores always buy
   (b) compare the choices before buying
   (c) the same thing my friends bought
   (d) buy at a regular shop always

ANSWERS

1. NUMBERS

Exercise 1.1

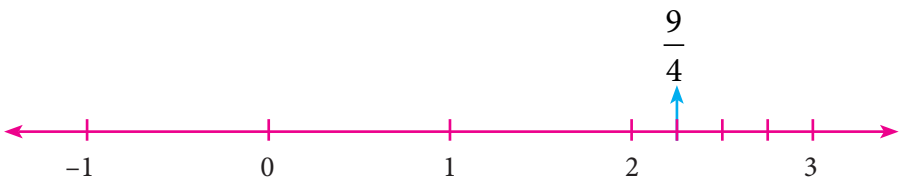
1. (i) -4 and -3 (ii) -3.75 (iii) 0 (iv) $\frac{30}{-48}$ (v) $\frac{-29}{39}$

2. (i) False (ii) True (iii) False (iv) True (v) True

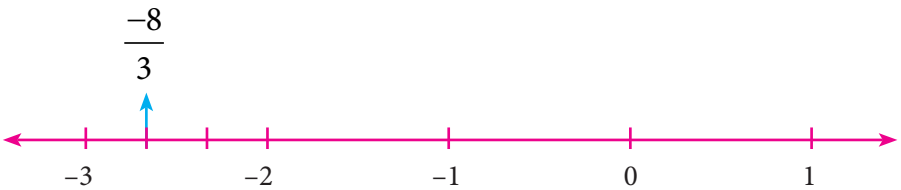
3. (i) $-\frac{11}{3}$ (ii) $\frac{-2}{5}$ (iii) $\frac{7}{4}$ 4. $Y=\frac{-5}{3}, N=\frac{-4}{3}, A=\frac{9}{4}, T=\frac{10}{4}, I=\frac{11}{4}$

5.

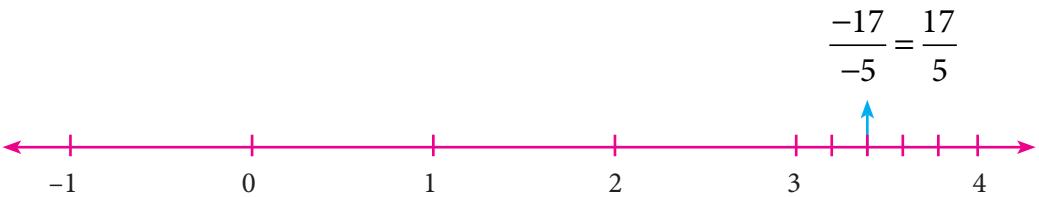
(i)



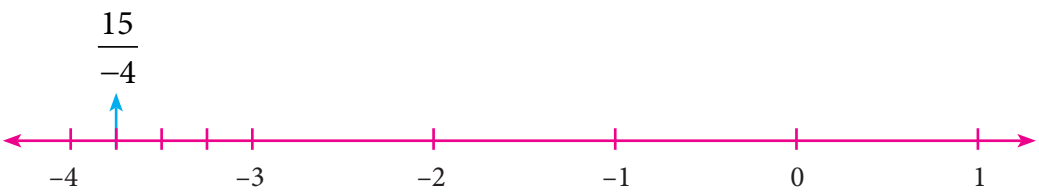
(ii)



(iii)



(iv)



6. (i) 0.0909... (ii) 3.25 (iii) -2.5714285714 (iv) 1.4 (v) -3.5

7. (i) $-2,\ 0 \rightarrow \frac{-19}{10}, \frac{-18}{10}, \frac{-7}{10}, \frac{-6}{10}, \frac{-5}{10}$ (ii) $\frac{-1}{2}, \frac{3}{5} \rightarrow \frac{-3}{10}, \frac{-1}{10}, 0, \frac{1}{10}, \frac{2}{10}, \frac{5}{10}$

(iii) $0.25,\ 0.35 \rightarrow \frac{26}{100}, \frac{27}{100}, \frac{30}{100}, \frac{32}{100}, \frac{33}{100}$ (iv) $-1.2,\ -2.3 \rightarrow \frac{-21}{10}, \frac{-20}{10}, \frac{-15}{10}, \frac{-14}{10}, \frac{-13}{10}$

8. $\frac{61}{15}$ and $\frac{103}{30}$, many answers possible 9. (i) $\frac{-11}{5} > \frac{-21}{8}$ (ii) $\frac{3}{-4} < \frac{-1}{2}$ (iii) $\frac{2}{3} < \frac{4}{5}$

10. (i) Ascending Order: $\frac{-11}{8}, \frac{-15}{24}, \frac{-5}{12}, \frac{12}{36}, \frac{-7}{-9}$ Descending Order: $\frac{-7}{-9}, \frac{12}{36}, \frac{-5}{12}, \frac{-15}{24}, \frac{-11}{8}$

(ii) Ascending Order: $\frac{-17}{10}, \frac{-7}{5}, \frac{-19}{20}, \frac{-2}{4}, 0$ Descending Order: $0, \frac{-2}{4}, \frac{-19}{20}, \frac{-7}{5}, \frac{-17}{10}$

11. (B) $\frac{-142}{99}$ 12. (B) $\frac{16}{-30}, \frac{-8}{15}$ 13. (C) -1 and -2 14. (A) $\frac{-17}{24}$ 15. (C) 6

Exercise 1.2

1. (i) $\frac{1}{20}$ (ii) 1 (iii) 1 (iv) 0 (v) -1

2. (i) True (ii) False (iii) False (iv) True (v) False

3. (i) 2 (ii) $\frac{74}{35}$ (iii) $\frac{4}{15}$ (iv) $2\frac{3}{4}$ 4. $\frac{-15}{11}$ 5. (i) $\frac{-1}{4}$ (ii) $\frac{8}{45}$

6. (i) -6 (ii) $\frac{1}{13}$ (iii) 5 7. (i) -7 (ii) $\frac{7}{11}$

8. $\frac{23}{2}\left(=11\frac{1}{2}\right)$ and hence lies between 11 and 12. 9. (i) $\frac{133}{60}$ (ii) $\frac{-5}{2}$ 10. 120

11. (A) 1 12. (C) $\frac{5}{8}$ 13. (B) $\frac{2}{3}$ 14. (D) $\frac{15}{16}$ 15. (D) all of these

Exercise 1.3

Verify yourself for Qns 1 to 6.

7. (C) 0 8. (D) associative 9. (A) $\frac{1}{8} - \frac{1}{8} = 0$ 10. (B) subtraction

Exercise 1.4

1. (i) 9 (ii) 48 (iii) 5 (iv) 3 (v) 13, 14

2. (i) True (ii) True (iii) False (iv) False (v) True

3. (i) 289 (ii) 41209 (iii) 1205604 4. (i) No (ii) No (iii) Yes (iv) Yes

5. (i) 12 (ii) 16 (iii) 28 (iv) 34 (v) 69 (vi) 95

6. (i) 42 (ii) 83 (iii) 105 (iv) 134 (v) 647 7. (i) 21 (ii) 28 (iii) 32

8. (i) 1.7 (ii) 8.2 (iii) 1.42 (iv) $\frac{12}{15}$ (v) $2\frac{5}{7}$ 9. 105, 81 10. 2, 60

11. (A) 9 12. (D) 7^2 13. (C) 7 14. (D) $\sqrt{32}$ 15. (B) 5

Exercise 1.5

1. (i) 7 (ii) 6 (iii) 42 (iv) 30 (v) 0.017

2. (i) True (ii) True (iii) False (iv) True (v) True 4. 5 5. 5 6. 120

7. 9,19 8. $\sqrt{36} = 6$ 9. $\sqrt{3} = 1.732$ 10. 4,16

Exercise 1.6

1. (i) 1 (ii) 1 (iii) 20^{-3} (iv) $\frac{-1}{128}$ (v) -243

2. (i) True (ii) False (iii) False (iv) True (v) False

3. (i) $\frac{1}{8}$ (ii) 32 (iii) $\frac{-216}{125}$ (iv) 16 (v) 6

4. (i) $\frac{64}{15625}$ (ii) $\frac{4}{5}$ (iii) 1024 5. (i) $\frac{21}{2}$ (ii) 5 (iii) 1

6. (i) 1 (ii) $\frac{3}{2}$ (iii) 72 7. (i) $x = 5$ (ii) $x = 6$

8. (i) $6 \times 10^3 + 5 \times 10^1 + 4 \times 10^0 + 3 \times 10^{-1} + 2 \times 10^{-2} + 1 \times 10^{-3}$
(ii) $8 \times 10^2 + 9 \times 10^1 + 7 \times 10^0 + 1 \times 10^{-1} + 4 \times 10^{-2}$

9. (i) 87652.0407 (ii) 5050.505 (iii) 0.000000000025

10. (i) 4.678×10^{11} (ii) 1.972×10^{-6} (iii) 1.642398×10^3
(iv) 1.083×10^{12} cu. km (v) 1.6×10^{-24}

11. (B) $\frac{-2}{5}$ 12. (A) $\frac{-1}{32}$ 13. (D) $-\left(\frac{1}{4}\right)^2 = 16^{-1}$ 14. (C) 6 15. (D) 2.02×10^{-10}

Exercise 1.7

Miscellaneous Practice Problems

1. 4 kg 300 gm 2. $10\frac{2}{5}$ litre 3. both are correct 4. 6 m 5. 552 cm^2

6. 9 cm and 10 cm 7. 15 decimetre 8. 625 9. 8

10. (i) 4.8×10^3 (ii) 1.152×10^5 (iii) 4.2048×10^7 (iv) 4.2048×10^9

Challenging Problems

11. 900 km 13. 320 gm 14. $\frac{9}{20}$ 15. $x = 3$ 16. $\frac{12}{11}$ 17. No, 64

18. 58.85 19. 7.979×10^5 20. $8^{100}, 2^{600}, 3^{500}, 4^{400}, 16^{25}$

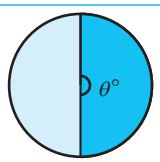
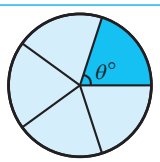
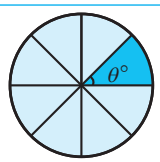
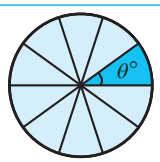
2. MEASUREMENTS

Exercise 2.1

1. (*i*) π (ii) chord (iii) diameter (iv) 12 *cm* (v) circular arc

2. (i) c (ii) d (iii) e (iv) b (v) a

3.

Sectors				
Central angle of each sector (θ)	180°	72°	45°	36°

4.

	Length of arc (approx.)	Area of sector (approx.)	Perimeter of sector (approx.)
(i)	12.56 *cm*	100.48 cm^2	44.56 *cm*
(ii)	13.19 *cm*	41.54 cm^2	25.79 *cm*

5. (i) 240 m^2 (ii) 337.5 cm^2 6. (*i*) $\theta = 120°$ (*ii*) $\theta = 72°$ 7. $30\pi\, m$

8. $980\pi\, cm^2$ 9. 706.5 cm^2 (approximately) 10. 1232 cm^2 (approximately)

Exercise 2.2

1. (i) 38 m, 50.75 m^2 (approximately) (ii) 30 cm,40.25 cm^2 (approximately)

2. (i) 21.5 cm^2 (approximately) (ii) 27.93 cm^2 (approximately)

3. 48 cm^2 4. 41.13 cm^2 (approximately) 5. 5600 cm^2 6. 3500 cm^2 7. 244 m^2

Exercise 2.3

1. (i) length, breadth and height (ii) Vertex (iii) Six (iv) Circle (v) Cube

2. (i) (b) (ii) (a) (iii) (d) (iv) (c)

3. (i) Cube (ii) Cuboid (iii) Triangular Prism (iv) Square Pyramid (v) Cylinder

4. (i) F, T, S (ii) T, S, F (iii) S, F, T

5. (i) Yes (ii) No (iii) Yes (iv) No (v) Yes

Exercise 2.4

Miscellaneous Practice Problems

1. 9.42 *feet* 2. 314 m 3. 128 cm^2

4. (i) Top View Front View Side View (ii) Top View Front View Side View

Challenging Problems

5. double door requires the minimum area 6. 63.48 m^2 (approximately)

7. 1.46 cm^2 (approximately) 8. (i) F = 10 (ii) V = 4 (iii) E= 28

3. ALGEBRA

Exercise 3.1

1.

x	$2x^2$	$-2xy$	x^4y^3	$2xyz$	$-5xz^2$
x^4	$2x^6$	$-2x^5y$	x^8y^3	$2x^5yz$	$-5x^5z^2$
$4xy$	$8x^3y$	$-8x^2y^2$	$4x^5y^4$	$8x^2y^2z$	$-20x^2yz^2$
$-x^2y$	$-2x^4y$	$2x^3y^2$	$-x^6y^3$	$-2x^3y^2z$	$5x^3yz^2$
$2y^2z$	$4x^2y^2z$	$-4xy^3z$	$2x^4y^5z$	$4xy^3z^2$	$-10xy^2z^3$
$-3xyz$	$-6x^3yz$	$6x^2y^2z$	$-3x^5y^4z$	$-6x^2y^2z^2$	$15x^2yz^3$
$-7z$	$-14x^2z$	$14xyz$	$-7x^4y^3z$	$-14xyz^2$	$35xz^3$

2. (*i*) $24m^4n^2$ (*ii*) $-9x^5y^6$ 3. $-24p^6q^6$

4. (*i*) $10xy-15x$ (*ii*) $-10p^3+6p^2-14p$ (*iii*) $3m^4n^4-15m^3n^2+21m^2n^3$

(*iv*) $x^3+y^3-z^3+x^2y+x^2z+xy^2+zy^2+xz^2-yz^2$ 5. (*i*) $4x^2-2x-12$

(*ii*) $2y^4+3y^3-8y^2-12y$ (*iii*) $5m^4n^2-m^2n^2-5m^2n^3+n^3$ (*iv*) $6x^2-36x+30$

6. (*i*) $-2x^2$ *ii*) $-3mp$ *iii*) $2y(5x^2y-x+3y^2)$ 7. (C) iv, v, ii, iii, i 8. $xy+2x+30y+60$

9. (B) $28p^7$ 10. (D) mn^2, -27 11. (C) $6x^2y$ 12. (A) 6 mn 13. (B) $(a+b)$

Exercise 3.2

1. (*i*) $\frac{18m^4(n^8)}{2m^{(3)}n^3}=9mn^5$ (*ii*) $\frac{l^4m^5n^{(7)}}{2lm^{(3)}n^6}=\frac{l^3m^2n}{2}$ (*iii*) $\frac{42a^4b^5(c^2)}{6(a)^4(b)^2}=(7)b^3c^2$

2. (i) True (ii) False 3. (*i*) $9y^2$ (*ii*) xy (*iii*) $-3x^2yz^3$ (*iv*) xy

4. (*i*) $5m$ (*ii*) $3p^3q^2$ 5. (*i*) $16y-4z$ (*ii*) $2mn^2+8m^3n-\frac{1}{2}$ (*iii*) $\frac{5}{6}y-3xy^2+1$
(*iv*) $=9p^2r+18pq-45$

6. (*i*) $9y^2$ (*ii*) $9xy$ (*iii*) $4m^2-3m$ (*iv*) $16n^2-2n+3$ (*v*) x^2+1x-6 (vi) $-(3p^2-4p+7)$

7. (ii) A is true but B is false 8. (i) both A and B are true

Exercise 3.3

1. (*i*) $9m^2+30m+25$ (*ii*) $25p^2-10p+1$ (*iii*) $4n^2+4n-3$ (*iv*) $(2p+5q)(2p-5q)$

2. (*i*) $m^3+9m^2+27m+27$ (*ii*) $8a^3+60a^2+150a+125$ (*iii*) $27p^3+108p^2q+144pq^2+64q^3$
(iv) 14,0608 (v) 1124864

3. (*i*) $125-75x+15x^2-x^3$ (*ii*) $8x^3-48x^2y+96xy^2-64y^3$ (*iii*) $a^3b^3-3a^2b^2c+3abc^3-c^3$
(iv) 110, 592 (*v*) $912673x^3y^3$

4. p^3-5p^2+2p+8 5. x^3+3x^2+3x+1 6. x^3-2x^2-5x+6

7. (C) 2 8. (B) a^2+ab+b^2 9. (A) p^3+q^3 10. (D) 72 11. (D) $3ab(a+b)$

Exercise 3.4

1. (*i*) $6y(3x-2z)$ (*ii*) $3x^2y(3x^3y^2+2xy-6)$ (*iii*) $(b-2c)(x+y)$

(*iv*) $(x+y)(a+b)$ (*v*) $(4x-1)(2x^2-1)$ (*vi*) $(x-2)[3y(x-2)+2]$

(*vii*) $2y(9x-2y-zx)$ (*viii*) $(a-3)(a^2+1)$ (ix) $3y(y+4)(y-4)$ (x) $(b-1)(ab-c^2)$

2. (*i*) $(x+7)^2$ (*ii*) $(y-5)^2$ (*iii*) $(c+2)(c-6)$

(*iv*) $(m+9)(m-8)$ (*v*) $(2x-3)(2x-1)$ 3. (i) $(4x+3)^3$ (ii) $(3p+2q)^3$

4. (i) $(y-6)^3$ (ii) $(2m-5n)^3$ 5. (D) $3x$, $(3x+2y)$ 6. (C) $(2+m)(2-m)$

7. (D) x^2-x-20 8. (B) 3 9. (C) $(1-m)(1+m+m^2)$ 10. (B) $(x+y)$

Exercise 3.5

Miscellaneous Practice Problems

1. $7x^2y^5+4x^4y^3+60x^2y^2$ 2. $12x^3-8x^2+27x-18$ 3. $S.I=\frac{7}{5}a^3b^4$
4. $\frac{1}{2}a+2b+4$ 5. $(y-3)(7y+2)$

Challenging Problems

6. $4x+3$ 7. $x+2$ 8. $(y+7)(y+8)$ 9. $(4p^2+1)(2p+1)(2p-1)$ 10. $3(x-5y)^3$

Exercise 3.6

1. (i) $x=7$ (ii) $y=11$ (iii) $m=7$ (iv) $p=15$ (v) One
2. (i) True (ii) False 3. (c) (iii),(i), (iv), (v), (ii)
4. (i) $x=11$ (ii) $y=\frac{1}{-4}$ (iii) $x=-1$ 5. (i) $x=-4$ (ii) $p=-1$ (iii) $x=-11$
6. (i) $x=-2$ (ii) $m=-4$

Exercise 3.7

1. (i). $x=-\frac{b}{a}$ (ii) Positive (iii) $x=30$ (iv) 40° (v) b=9
2. (i) True (ii) False (iii) False 3. 3, 21 4. 27
5. $l=8$ cm, $b=24$ cm 6. (80,10) notes 7. Murali's is 15 years old, Thenmozhi's age is 20 years old 8. 63 9. $\frac{13}{21}$
10. 37.4km 11. (B) 20 12. (A) 62° 13. (C) 10000 14. (C) 4 15. (D) $(x-1)$

Exercise 3.8

1. (i) Origin (0,0) (ii) negative (iii) y-axis (iv) zero (v) X-Coordinate
2. (i) True (ii) True (iii) False

Exercise 3.9

1. (i) Origin (ii) (4,-4) (iii) x-axis 1*cm*=3 units, y-axis 1*cm*=25 units
2. (i) True (ii) False

Exercise 3.10

Miscellaneous Practice Problems

1. $x = 20$ 2. 60°, 40°, 80° 3. y = 11 units p=133 units 4. 116°,64°

Challenging Problems

6. 7,8,9 7. 54 8. 12 pencils

4. LIFE MATHEMATICS

Exercise 4.1

1. (i) $x = 500$ (ii) $3\frac{1}{3}\%$ (iii) $x = 50$ (iv) 70% (v) 52.52%
2. (i) 50% (ii) 75% (iii) 100% (iv) 96% (v) $66\frac{2}{3}\%$
3. $x = 150$ 4. 30 5. $33\frac{1}{3}\%$ 6. 110 7. $x = 200$ 8. $x = 100$ 9. No change
10. 87% 11. (C) 20% 12. (B) 49% 13. (A) 375 14. (D) 200 15. (D) 36

Exercise 4.2

1. (i) Cost price (ii) ₹7000 (iii) ₹600 (iv) 8% (v) ₹945
2. ₹902 3. ₹670 4. 50% 5. $11\frac{1}{9}\%$ 6. ₹1152
7. (i) x = ₹207 (ii) y = ₹12600 (iii) z = 18% 8. ₹2836
9. Discount of 8% is better 10. ₹5400
11. (C) 25% 12. (B) 550 13. (B) 168 14. (D) ₹250 15. (A) 40%

Exercise 4.3

1. (i) ₹1272 (ii) ₹820 (iii) ₹20,000 (iv) $A = P\left(1+\frac{r}{400}\right)^{4n}$ (v) ₹32
2. (i) True (ii) False (iii) True (iv) False (v) True
3. ₹162 4. ₹936.80 5. ₹1875 6. $1\frac{1}{2}$ years 7. ₹10,875 8. ₹0.50
9. 5% 10. ₹ 16000 11. (C) 6 12. (B) 1 year 13. (B) ₹ 12500
14. (A) ₹2000 15. (D) ₹2500

Exercise 4.4

1. (i) 2 (ii) 5 (iii) 8 (iv) 25 (v) ₹1,20,000 2. 162 men
3. 7000 cement bags 4. $7\frac{1}{2}$ days 5. 4 more lorries 6. 4 hours
7. A- 30 days, B -20 days, C-60 days 8. 180 minutes 9. 16 days 10. 6 days

Exercise 4.5
Miscellaneous Practice Problems

1. 400 2. 300 3. ₹38163 4. 20% 5. $2\frac{2}{9}$% loss 6. 48 men 7. 6 days 8. 8 days

Challenging Problems

9. $\frac{8}{25}$ 10. ₹15000 11. 20 % 12. 30 % 13. 20 more men 14. 3 days 15. ₹ 6000

5. GEOMETRY

Exercise 5.1

1. (i) in proportion (ii) shape (iii) equal (iv) congruent (v) similar

6. Yes, RHS Congruence 8. $HE = 18, TE = 16$ 9. $\angle T = \angle N = 75°, \angle E = \angle B = 35°, \angle A = \angle U = 70°$

11. (D) matching 12. (A) $\angle Q = \angle Y$ 13. (D) 93 m 14. (A) 50° 15. (C) AC = CD

Exercise 5.2

1. (i) Q (ii) $n^2 - m^2$ (iii) a right angled triangle (iv) centroid (v) 2:1
2. (i) True (ii) True (iii) True (iv) True (v) False
3. (i) Yes (ii) No (iii) Yes (iv) Yes (v) Yes
4. (i) $x = 41$ (ii) $y = 16$ (iii) $z = 15$ 5. 5cm 6. 170m
7. 10cm 8. W 9. P 10. 9cm 11. 40°
12. (c) 45° 13. (b) 20cm 14. (c) 420cm^2 15.(d) 20, 48, 52

Exercise 5.3
Miscellaneous Practice Problems

3. 48 ft 4. 25 ft 5. No, the wide of cabinet is lesser than the wide of TV

Challenging Problems

8. 40cm 9. 28 ft 10. (i) 22 (ii) 6 (iii) 16 (iv) 24

Exercise 5.4

1. (i) Orthocentre (ii) Centroid (iii) Incentre (iv) Circumcentre (v) 2:1
2. (i) True (ii) True (iii) false
3. (a) (i) Interior (ii) Exterior (iii) On the hypotenuse
   (b) (i) Interior (ii) Exterior (iii) On the vertices containing 90°
4. (i) BE (ii) AD (iii) CF

5. AB=5cm 6. $\lfloor XYM = \lfloor ZYM = 50°$ 7. 7 cm 8. 10 cm

Exercise 5.5

1. W 2. P 3. 9cm 4. 12 feet 5. 40°
6. (i) 22 (ii) 6 (iii) 16 (iv) 24

6. STATISTICS

Exercise 6.1

1. (i) Secondary (ii) 35 (iii)197 (iv) 8 (v) Circular
2. (i) False (ii) True (iii) True (iv) True
5. (i) 20% (ii) 75 (iii) 1/4 (iv) 400 (v) 275 (vi) 500
8. (i) ₹8000 (ii) ₹40000 (iii) ₹12000

Exercise 6.2

1. (i) Yes (ii) No (iii) No (iv) Yes (v) Yes
2. (i) Proportional (ii) Histogram (iii) Grouped
3. (i) 330 (ii) 150 (iii) No
9. (d) all the three 10. (c) Frequency 11. (a) range 12. (b) grouped
13. (b) discontinuous 14. (b) Exclusive 15. (c) pie chart 16. (a) continuous
17. (a) frequency polygon 18. (d) histogram

7. INFORMATION PROCESSING

Exercise 7.1

1. 5 2. 9 3. 8 4. 6 5. 4995 6. 10000 7. 15 8. 20

9. (i)

10. (i)

more possible ways

(ii)

more possible ways

11. (A) 41 12. (B) 8 13. (C) 12 14. (C) 19

Exercise 7.2

1. (i) 13 (ii) 8 (iii) 5 (iv) 46
2. (i) 14 (ii) 4 (iii) 140 (iv) 6
3. (i) 4 (ii) 5 4. 14 5. 28
6. (C) 89 7) (B) F(8) = F(7)+F(6) 8. (A) 2 9. (C) 6th 10) (D) 987
11. (A) 2×5 12. (B) 2×3 13. (D) 1

Exercise 7.3

1. (i) TAMIL (ii) ENGLISH (iii) MATHEMATICS
(iv) SCIENCE (v) SOCIAL SCIENCE

2 (i) c (ii) d (iii) a (iv) e (v) b

3.

Plain Text	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
Cipher Text	04	05	06	07	08	09	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	00	01	02	03

4. To understand that mathematics can be experienced everywhere in nature and real life.
5. (i) 28 (ii) CHAIR (iii) GIFT VOUCHER
6. (c) [**ɐ 8 ƨ Z H Z ꓘ И T**] 7. (i) (a) C R D T (ii) (d) A D G J
8. (ii) 5 6 3 4 2 1 9. (i) (c) R F U Q N P C (ii) (c) U D G L R

Exercise 7.4

1. (i) 5 Chocolate bars for ₹ 175b (ii) 15 eggs for ₹ 64.50
2. ₹ 634 3. ₹ 39.25 4. ₹ 809 5. (d) all the above
6. (c) something that I need to purchase
7. (b) Compare the choices before buying

Mathematical Terms

Algebraic expression	இயற்கணிதக் கோவை
Altitude	குத்துக்கோடு
Angle bisector	கோண இருசமவெட்டி
Associative property	சேர்ப்புப் பண்பு
Binomial	ஈருறுப்புக் கோவை
Central angle	மையக்கோணம்
Centroid	நடுக்கோட்டு மையம்
CGST	மத்திய சரக்கு மற்றும் சேவை வரி
Circular arc	வட்ட வில்
Circular sector	வட்டக்கோணப் பகுதி
Circular segment	வட்டத்துண்டு
Circumcentre	சுற்றுவட்ட மையம்
Class interval	பிரிவு இடைவெளி
Closure property	அடைவுப் பண்பு
Co-ordinate axes	ஆய அச்சுகள்
Code	குறியீடு
Coincide	ஒருங்கமைவு
Commutative property	பரிமாற்றுப் பண்பு
Compound interest	கூட்டு வட்டி
Compound variation	கலப்பு மாறல்
Congruent	சர்வசமம்
Conversion period	மாற்று காலம்
Corresponding sides	ஒத்தப் பக்கங்கள்
Cost price	அடக்க / வாங்கிய விலை
Cryptology	குறியாக்கவியல்
Cube root	கனமூலம்
Cubic identities	கன முற்றொருமைகள்
Concurrent lines	ஒரு புள்ளி வழிச் செல்லும் கோடுகள்
Data	தரவு
Deposit	முதலீடு
Depreciation value	தேய்மான மதிப்பு
Direct proportion	நேர் விகிதம்
Discount	தள்ளுபடி
Distributive property	பங்கீட்டுப் பண்பு
Edges	விளிம்புகள்
Encryption	குறியாக்கம்
Estimate	உத்தேச மதிப்பு
Extremes	கடை உறுப்புகள்
Frequency distribution	நிகழ்வெண் பரவல்
Frequency polygon	நிகழ்வு பலகோணம்
Front view	முகப்புத்தோற்றம்
Graph sheet	வரைபடத்தாள்
Grouped data	தொகுக்கப்பட்ட தரவு
Histogram	நிகழ்வு செவ்வகம்
Hypotenuse	கர்ணம்
Identity	சமனி
Incentre	உள்வட்ட மையம்
Interest	வட்டி
Inverse	நேர்மாறு
Inverse proportion	எதிர் விகிதம்
Irregular polygon	ஒழுங்கற்ற பலகோணம்

Laws of exponent	அடுக்கு விதிகள்
Linear equation	ஒருபடிச் சமன்பாடு/ நேரியச் சமன்பாடு
Logical	தருக்கரீதியான
Loss	நட்டம்
Lower limit	கீழ் எல்லை
Marked price	குறித்த விலை
Median	நடுக்கோடு
Monomial	ஒருறுப்புக் கோவை
Ordered pair	வரிசை சோடி
Origin	ஆதிப்புள்ளி
Orthocentre	செங்கோட்டு மையம்
Overhead expenses	இதரச் செலவுகள்
Packing	அடைத்தல்
Parallel lines	இணைக்கோடுகள்
Parallelogram	இணைகரம்
Perfect cube numbers	முழு கன எண்கள்
Perfect square	முழு வர்க்கம்
Perpendicular bisector	மையக்குத்துக்கோடு
Pie chart	வட்ட விளக்கப்படம்
Point of concurrency	ஒருங்கமைப் புள்ளி
Polyhedron	பன்முக வடிவம்
Polynomial	பல்லுறுப்புக் கோவை
Primary data	முதல்நிலைத் தரவு
Prime factorisation	பகாக் காரணிப்படுத்துதல்
Principal	அசல்
Profit	இலாபம்
Quadrant	கால்பகுதி
Range	வீச்சு
Rational number	விகிதமுறு எண்
Real number line	மெய்யெண் கோடு
Regular polygon	ஒழுங்குப் பலகோணம்
Scientific notation	அறிவியல் குறியீடு
Secondary data	இரண்டாம் நிலைத் தரவு
Selling price	விற்ற / விற்பனை விலை
SGST	மாநில சரக்கு மற்றும் சேவை வரி
Side view	பக்கவாட்டுத்தோற்றம்
Simple interest	தனி வட்டி
Solid shapes	திண்ம வடிவங்கள்
Square root	வர்க்கமூலம்
Supplementary angles	மிகை நிரப்புக் கோணங்கள்
Tally marks	நேர்கோட்டுக் குறிகள்
Terminating	முடிவுறு
Three dimensional shapes	முப்பரிமாண வடிவங்கள்
Top view	மேற்பக்கத் தோற்றம்
Transposition	இடமாற்று முறை
Trinomial	மூவுறுப்புக் கோவை
Upper limit	மேல் எல்லை
Vertically opposite	குத்தெதிர்
Vertices	உச்சிகள்

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